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EXCITATION OF STATES WITH A DIFFERENT λ^{π} IN THE INELASTIC ELECTRON SCATTERING AT BACKWARD ANGLES

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The inelastic electron scattering is nowdays the most fruitful tool for the investigation of excited nuclei states. Rich information on the low lying states (for example, ref. [1]) and giant resonances, electric (for example, refs. [2,3]) and magnetic (for example, refs. [4,5]), is available. Most of these experiments have been performed at low momentum transfer ($q \lesssim 0.5\ fm^{-1}$); these conditions are suitable for the excitation of states with small λ. With increasing q additional states with higher λ begin to excite strongly, and the spectrum of excitation becomes more complicated. So, the question arises what are we going to find at a higher q: a complex mixture of states with different λ^{π} in a wide energy range or there will be some resonance structures in the (e,e′) cross section. The more complicated situation is expected for backward scattering because both electric (due to the transversal form factor) and magnetic states give a comparable contribution to the cross section. On the other hand, large angles are of special interest because of an ever increasing interest in all that is related with spin excitation.

In this paper we consider changes in the (e,e′) backward scattering on ^{90}Zr with increasing q. The structure of excited states is described in the framework of the quasiparticle-phonon nuclear model (QPM) reviewed in detail in ref. [6]. The model Hamiltonian includes the average field for neutrons and protons, the pairing interaction (for ^{90}Zr there is the pairing for protons only) and effective separable forces which generate the phonon excitations with different λ^{π}. The electric excitations (natural parity states) are generated by miltipole forces:

$$V_{\lambda}(\vec{r}_1, \vec{r}_2) = \frac{1}{2}(\kappa_0^{(\lambda)} + \vec{\tau}_1 \vec{\tau}_2 \kappa_1^{(\lambda)}) \frac{\partial U}{\partial r_1} \frac{\partial U}{\partial r_2} \sum_{\mu} Y_{\lambda\mu}(\Omega_1) Y^{*}_{\lambda\mu}(\Omega_2) \qquad (1)$$

and magnetic excitations (unnatural parity states) are generated by spin-multipole forces:

$$V^{\lambda}_{\sigma,\lambda-1}(\vec{r}_1, \vec{r}_2) = \frac{1}{2}(\kappa_0^{(\lambda-1,\lambda)} + \vec{\tau}_1 \vec{\tau}_2 \kappa_1^{(\lambda-1,\lambda)}) \frac{\partial U}{\partial r_1} \frac{\partial U}{\partial r_2} \sum_{\mu} [\vec{\sigma}_1 Y_{\lambda-1}(\Omega_1)]_{\lambda\mu} [\vec{\sigma}_2 Y_{\lambda-1}(\Omega_2)]^{+}_{\lambda\mu}, \qquad (2)$$

where U is the central part of the average potential; κ_0 and κ_1 are the constants of the isoscalar and isovector interactions, respectively. The parameters of the average potential in the Saxon-Woods form and the pairing constants are taken

from ref. [7]. For κ_0 and κ_1 we use the procedure described in ref. [8].

To begin with let us consider how to estimate the contribution of states with different λ^{π} to the (e, e′) cross section without huge numerical calculations. The (e, e′) cross section in the plane wave Born approximation (PWBA) has the form [9]:

$$\frac{d\sigma}{d\Omega} = \frac{2Z^2 e^4}{q_{\mu}^4} \cdot \frac{p_f}{p_i} \{ V_L(\theta) \sum_{\lambda} |F_{\lambda}^{C}(q^2)|^2 + V_T(\theta) \sum_{\lambda} (|F_{\lambda}^{M}(q^2)|^2 + |F_{\lambda}^{E}(q^2)|^2) \}. \qquad (3)$$

The Coulomb $F^C_{\lambda}(q^2)$ and magnetic $F^M_{\lambda}(q^2)$ form factors are the functions of charge $\rho_{\lambda}(r)$ and current $\rho_{\lambda\lambda}(r)$ transition densities:

$$F^C_{\lambda}(q^2) = \frac{\sqrt{4\pi}(2\lambda+1)}{Z} \int \rho_{\lambda}(r) j_{\lambda}(qr) r^2 dr,$$

$$F^M_{\lambda}(q^2) = \frac{\sqrt{4\pi}(2\lambda+1)}{Z} \int \rho_{\lambda\lambda}(r) j_{\lambda}(qr) r^2 dr. \qquad (4)$$

The expressions for the transition densities in the QPM are given in ref. [6]. There are two parameters for the neutron and proton part of the current transition densities, g_s^{eff} and g_{ℓ}^{eff}. In our calculations we use $g_s^{eff} = 0.8\, g_s^{free}$ and $g_{\ell}^{eff} = g_{\ell}^{free}$.

At a small momentum transfer, when $qR \lesssim 1$ (R is the nuclear size), we can write the form factors in terms of the B(Eλ) - or B(Mλ) -values:

$$|F^C_{\lambda}(q^2)|^2 = \frac{4\pi}{Z} \cdot \frac{q^{2\lambda}}{[(2\lambda-1)!!]^2} \cdot \frac{B(E\lambda)}{e^2(2\lambda+1)},$$

$$|F^M_{\lambda}(q^2)|^2 = \frac{4\pi}{Z} \cdot \frac{q^{2\lambda}}{[(2\lambda-1)!!]^2} \cdot \frac{B(M\lambda)}{e^2(2\lambda+1)} \cdot \frac{\lambda+1}{\lambda}. \qquad (5)$$

In this long wave approximation we can also use Siegert′s theorem resulting in:

$$F^E_{\lambda}(q^2) \simeq \frac{k}{q} \sqrt{\frac{\lambda+1}{\lambda}}\, F^C_{\lambda}(q^2), \qquad (6)$$

where k is an energy transferred to the nucleus in units of q. Combining (3), (5), and (6) we get:

$$\left(\frac{d\sigma}{d\Omega}\right)_{E\lambda} = \frac{8\pi e^2}{q_{\mu}^4} \cdot \frac{p_f}{p_i} \frac{q^{2\lambda}}{(2\lambda+1)[(2\lambda-1)!!]^2} \{ V_L(\theta) + \frac{k}{q}\sqrt{\frac{\lambda+1}{\lambda}} V_T(\theta) \} B(E\lambda), \qquad (7)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{M\lambda} = \frac{8\pi e^2}{q_{\mu}^4} \cdot \frac{p_f}{p_i} \frac{q^{2\lambda}(\lambda+1)}{(2\lambda+1)[(2\lambda-1)!!]^2\, \lambda} V_T(\theta)\, B(M\lambda). \qquad (8)$$

Table 1

The ratios of the 1^-, 2^+, 1^+ and 2^- states contributions to the inelastic electron scattering at $\theta = 160°$ cross section. The values $\sigma_{\lambda\pi}(\theta)$ are defined by Eq.(9): $E_{min} = 5$ MeV, $E_{max} = 20$ MeV. The calculations are in the long wave PWBA. To compare with the same values in DWBA are in brackets.

E_0 (MeV)	$\sigma_{1^-}(\theta)/\sigma_{2^+}(\theta)$	$\sigma_{1^+}(\theta)/\sigma_{2^-}(\theta)$	$\sigma_{1^-}(\theta)/\sigma_{1^+}(\theta)$	$\frac{k}{q}\cdot\frac{V_T}{V_L}$
20	9.96	3.19	51.32	14.04
30	3.59(4.04)	1.15(1.35)	30.83(11.30)	7.21
50	1.11(0.70)	0.36(0.39)	17.14(8.61)	3.78

So, to calculate the (e, e′) cross section in the long wave PWBA we need only the B(Eλ)- and B(Mλ) -values.

To compare the contribution of states with different λ^π to the (e, e′) cross section, we introduce the $\sigma_{\lambda^\pi}(\theta)$ cross section as follows:

$$\sigma_{\lambda^\pi}(\theta) = \int_{E_{min}}^{E_{max}} \left(\frac{d^2\sigma}{d\Omega\, dE_x}\right)_{\lambda^\pi} dE_x \qquad (9)$$

and integrate over all one-phonon states with the definite λ^π from the energy range $5 \le E_x \le 20$ MeV (for $d^2\sigma/d\Omega\, dE_x$ we use (7) or (8)). The ratios $\sigma_{\lambda_1^{\pi_1}}(\theta)/\sigma_{\lambda_2^{\pi_2}}(\theta)$ for the lowest λ and electron energy $E_0 = 20$, 30, 50 MeV and $\theta = 160°$ are presented in table 1. Some conclusions can be made even on the basis of these qualitative estimations. First, at a low momentum transfer the most intensively excited states in backward scattering are the 1^--states, but with increasing E_0 their contribution to the cross section reduces. Second, the electroexcitation of the magnetic states is weaker in comparison with the electric states. Third, at small E_0 and backward angles the Eλ -states are excited mainly due to the electric form factor $F_\lambda^E(q^2)$ - this is obvious from the value $(k/q)\cdot(V_T/V_L)$. With increasing E_0 this value becomes smaller but it remains larger than unity. When $\theta \to 180°$ this value tends in PWBA to infinity because in this case $V_L(\theta) \to 0$ and $V_T(\theta) \to 2p_i p_f$.

For comparison we put in brackets the ratios obtained by means of microscopic calculations in the distorted wave Born approximation (DWBA); these calculations will be presented later. The results are in agreement.

Equations (7)-(8) are very simple for calculations, but the range of their validity is rather limited: $qR \lesssim 1$ - at a larger q we cannot use neither the long wave approximation nor Siegert′s theorem. For backward scattering $qR = 1$ at a rather small electron energy. For example, this equivalence is reached for ^{90}Zr(e, e′) with $E_0 = 24$ MeV and $\theta = 160°$. That is why the form factors will be further computed directly through the transition densities of states.

Now, we will describe the inelastic electron scattering in DWBA according to ref. /10/. For the electric states both Coulomb and electric form factors will be taken into account. The results of calculations are presented in figs.1 and 2 as histograms and in table 2.

The cross sections of the inelastic electron scattering with energy from 30 to 140 MeV are shown in fig.1. The scattering angle $\theta = 160°$ has been chosen according to the following reasons. This angle must be close to 180° in order to damp the Coulomb form factor contribution. On the other hand, there are some numerical problems in computing the electric form factor at $\theta \simeq 180°$ in the DWBA. It must be pointed out that most of the experiments on (e, e′) backward scattering have been done for $\theta = 160° \div 165°$.

Each column of the white histogram in fig.1a)-e) contains the sum of excitation probabilities of the states with different λ^π from the interval $\Delta E_x = 1$ MeV:

$$\frac{d^2\sigma}{d\Omega\, dE_x} = \frac{1}{\Delta E_x} \sum \int_{E_x}^{E_x+\Delta E_x} \left(\frac{d^2\sigma}{d\Omega\, dE}\right)_{\lambda^\pi} dE. \qquad (10)$$

All one-phonon states with $\lambda^\pi = 1^\pm \div 6^\pm$- are taken into account here. The contribution of states with $\lambda \ge 7$ to the cross section can be neglected even for $E_0 = 140$ MeV. The black histograms in these figures mean electroexcitation of the magnetic states (M1-M6) only. Figure 1 demonstrates the shape of the (e,e′) cross section as a function of the energy transferred. The contribution of the states with different λ^π to these histograms are in table 2, where the values $\sigma_{\lambda^\pi}(\theta)/\Sigma\sigma_{\lambda^\pi}(\theta)\cdot 100\%$ for all λ^π are given.

At the lowest q the isovector E1-resonance (1^- states from the energy range 15-18 MeV) is the most intensively excited mode, the other bump in fig.1a) is caused by the isoscalar E2-resonance (2^+ states from the energy range 12-14 MeV) excita-

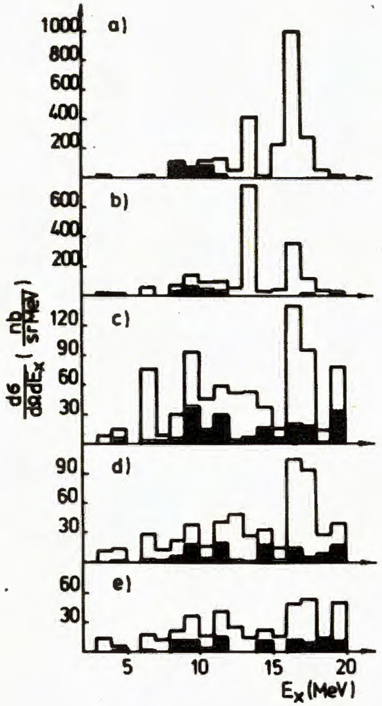

Fig.1. The $^{90}Zr(e,e')$ cross section for E_0 = 30(a), 50(b), 80(c), 110(d), 140(e) MeV and θ = 160°. Black histograms show the contribution of magnetic states.

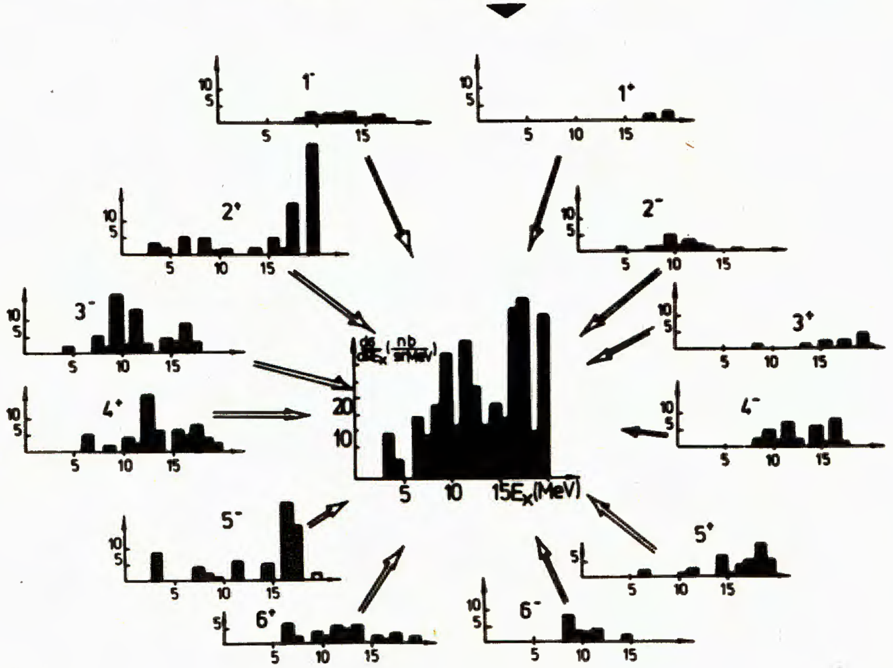

Fig.2. The contribution of the states with different λ^{π} to the $^{90}Zr(e,e')$ cross section for E_0 = 140 MeV and θ = 160°.

Table 2

A relative contribution of the states with different λ^{π} to the $^{90}Zr(e,e')$ cross section: $\sigma_{\lambda\pi}(\theta)/\Sigma\sigma_{\lambda\pi}(\theta)\cdot 100\%$ The values $\sigma_{\lambda\pi}(\theta)$ are defined by Eq. (9): E_{min} = 5 MeV, E_{max} = 20 MeV. The calculstions are in DWBA. The scattering angle θ = 160°. The empty cells mean the contribution less than 1%. The momentum transferred q is calculated for E_x = 10 MeV.

λ^{π}	E_0(MeV)	30	50	80	110	140	
	q(fm^{-1})	0.25	0.45	0.75	1.05	1.35	
1^-		71%	34%	18%	5%	4%	Eλ
2^+		17%	47%	25%	21%	16%	
3^-			5%	24%	20%	14%	
4^+				6%	19%	14%	
5^-				1%	9%	15%	
6^+					2%	9%	
1^+		7%	4%	2%		1%	Mλ
2^-		5%	10%	13%	6%	3%	
3^+				7%	7%	2%	
4^-				2%	6%	8%	
5^+					4%	9%	
6^-						5%	
$\sum^{M\lambda}\sigma_{\lambda\pi}(\theta)$		12%	14%	25%	24%	28%	

tion. With increasing E_0 the excitation probability of 1^- and 1^+ states decreases while the excitation probability of states with $\lambda > 1$ increases. As a result, the major bump in the (e,e′) cross section for E_0 = 50 MeV is due to the E2-resonance, fig.1b).

Let us consider the energy range 8-11 MeV at low q. It is the only range in ^{90}Zr where the predominantly magnetic states (1^+ and 1^-) are excited, an excitation probability of a few 1^-, 2^+ and 3^- states in this range is rather weak. The energy range 8-10 MeV in ^{90}Zr was investigated by A.Richter et al.[11] by using of electrons with energy from 24 to 66 MeV (θ = 165°). The spin and parity of states were defined by the behaviour of form factors. A lot of 2^- and 1^+ states were observed, besides them there were three 1^- and probably two 3^- states. This is the only experiment we can compare with our calculations. The comparison shows a qualitative agreement.

The increasing contribution of states with $\lambda > 2$ for higher q is seen from table 2. For E_0 = 80 MeV the 3^- states give already 24% contribution to the cross section. A bump in the cross section at E_x = 6-7 MeV in fig.1c) is caused by LEOR excitation. The 4^+ states are excited strongly by the 110 MeV electrons and so on. We also see the damping of 1^- states. Table 2 shows that for $q \gtrsim 1$ fm^{-1} it is impossible to reveal the multipolarity which is predominantly excited, because many states with different λ^π give a comparable contribution to the cross section. At higher q the contribution of the main multipolarity decreases: for E_0 = 30 MeV the 1^- states contribution is equal to 71% and for E_0 = 140 MeV 2^+ state contribution is equal only to 16%. As a result, the (e, e′) cross section as a function of the energy transferred E_x becomes more flat with increasing q. For E_0 = 140 MeV there are no such sharp bumps in the cross section as for E_0 = 30, 50, and 80 MeV.

How the cross section is formed at $q \geq 1$ fm^{-1} is seen from fig.2, where the histograms for all λ^π states are presented separately. It must be pointed out that the (e,e′) cross section differs from the B(Eλ)- or B(Mλ) -value distribution, especially for small λ. In the (e,e′) spectrum at $q > 1$ fm^{-1} we find neither giant E1-resonance nor isoscalar quadrupole resonance, nor LEOR, that are clearly seen at lower q. Analysing the behaviour of the form factor squared, we find the reason. The form factor squared is proportional to the B(Eλ) (or B(Mλ)) value up to its first maximum, after the first maximum its behaviour is defined exclusively by the individual properties of the transition density. As a result, some states with small B(Eλ) or B(Mλ) can be excited rather intensively at a definite q and vice versa. These problems concerning the 1^+ and 2^- state excitation have been discussed in detail in ref./12/.

Figure 2 shows that for E_0 = 140 MeV not only a lot of states with different λ^π give a visible contribution to the cross section (this is also seen from the last column of table 2), but even in each energy range with ΔE_x = 1 MeV we find a complex mixture of states with different λ^π. If we take into account the interaction between one-phonon states with more complex ones, it will lead to the fragmentation of the one-phonon strength, and the mixture of states will become stronger. That is why we conclude that there will be no resonance structures with a definite λ^π in the backward electron scattering for $q > 1$ fm^{-1}.

Another question to discuss is a relative contribution of all magnetic states. The previous qualitative estimations show that at low q the magnetic states are excited weaker as com-

pared with electric states. The exact calculation corroborates with this conclusion. The last line of table 2 presents a relative contribution of all magnetic states at a different E_0. With increasing q the magnetic state contribution increases, but it is still less than 30%. Thus, we see that even for angles close to 180° the electric states on the whole are excited by electrons more stronger than the magnetic ones.

Concluding we will repeat in short the main aspects. At low q the (e, e′) backward cross section has few isolated bumps caused by E1, E2, E3, and M2 resonance excitation. With increasing q the cross section as a function of E_0 becomes more flat, much more states with different λ^π give a visible contribution. The contribution from the different λ^π states is hardly mixed and for $q > 1$ fm^{-1} no energy range, in which only the states with a definite λ^π are excited strongly, has been found. As for magnetic states, their contribution on the whole is not large than 30% ever for the backward scattering. The energy range 8-11 MeV is the only in ^{90}Zr where we can expect the predominant excitation of the magnetic states at low q, with increasing q we must separate them from the electric states. Thus, the experimental conditions of refs. /4,5,11/ are the best ones for the investigation of spin modes.

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Пономарев В.Ю. E4-84-72

Возбуждение состояний различной мультипольности в реакции неупругого рассеяния электронов на большие углы

В рамках метода искаженных волн рассчитаны сечения электровозбуждения однофононных состояний ядра ^{90}Zr. Энергия налетающих электронов изменялась в интервале от 30 до 140 МэВ; угол рассеяния $\theta = 160°$. Проанализирован вклад состояний различной мультипольности /$\lambda^{\pi} = 1^{\pm} \div 6^{\pm}$/ в сечение реакций.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Ponomarev V.Yu. E4-84-72

Excitation of States with a Different λ^{π} in the Inelastic Electron Scattering at Backward Angles

The cross section of the one-phonon state electroexcitation in ^{90}Zr is calculated in DWBA. The electron energy varies from 30 to 140 MeV while a scattering angle is fixed: $\theta = 160°$. A contribution of the states with different λ^{π} /$\lambda^{\pi} = 1^{\pm} \div 6^{\pm}$/ to the (e, e′) cross section is analysed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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