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LENGTHS OF $\pi^{\pm}\,{}^{3}H$, $\pi^{\pm}\,{}^{3}He$ SCATTERING AND MASS DIFFERENCES OF PIONS, NUCLEONS, AND THE NUCLEI

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1. INTRODUCTION

In a theoretical description of pion-nucleus interactions one usually neglects mass differences of pions, nucleons, and isobar-nuclei because of their smallness with respect to the masses itself or collision energy. But such simple arguments cannot be considered as sufficiently convincing. In a number of problems taking into account of the differences may be essential or even indispensable.

In the first place this is the investigation of strong-interaction charge-symmetry-breaking in pion-nucleus scattering. It is clear that an experimental-data analysis, aimed to obtain information on this subtle effect, demands a careful treatment both of pure electromagnetic corrections and charged-multiplet mass-splittings explicitly breaking the symmetry even of the free Hamiltonian.

Probably, a disregard or insufficiently correct taking into account of the mass differences is just a cause of the contradictions appeared in that investigation area. On the one hand, in the experiments on pion scattering from deuteron [1-5] and 3-nucleon nuclei [6] one sees a significant deviation from the charge-symmetry. On the other hand, for example, in elastic $\pi\,^{40}Ca$ scattering one group finds the symmetry-breaking [7] meanwhile the other does not [8,9]. Moreover, the charge-asymmetry-parameter value changes in a wide region from paper to paper.

The mass differences may also be essential in problems, where the energy-release of the processes $\pi^+ n \to \pi^o p + 5.9$ MeV and $\pi^- p \to \pi^o n + 3.3$ MeV is comparable with collision energy or there are variable strongly depending on energy (for example, in a vicinity of a resonance). As a result of the energy release the inelastic channels with thresholds above the collision energy not more than 5.9 MeV become open. Presence of open channels may essentially change the scattering picture. For example, the scattering length turns out to have an imaginary part not concerned with pion-absorption.

The more channels are open, the greater influence they can get upon the scattering process. Consequently, due to the growth of the nuclear level density with increasing mass number of the nucleus, we may expect that at low collision energies taking into account of the mass differences becomes more important for heavy nuclei.

The mass differences might be a cause of the anomalies of heavy pionic atom spectra discovered for ${}^{181}_{73}\mathrm{Ta}$, ${}^{209}_{83}\mathrm{Bi}$ and some neighbouring nuclei /10,11/. The anomalies consist in much smaller values of the measured shifts and widths of the atomic 3d-states as compared with predictions of the optical-potential model very successfully describing light and medium pionic atoms. In paper/12/ an attempt is made to accomodate the theory to the experiment with the use of the strong energy dependence of the isoscalar πN amplitude near the threshold. The shifts and widths of the 3d-states are very sensitive to the magnitude of that amplitude. Replacing the energy ω by $(\omega - V_c)$ like the minimal coupling in the electrodynamics (here V_c is the Coulomb-potential), the author /12/ has obtained a good agreement for the shifts, but for the widths there has remained a very substantial anomaly by a factor of about two.

The examples, presented here, do not exhaust apparently the list of problems, where taking into account of the mass differences may be important.

Untill now there has been analysed the influence of the mass differences only on the S-wave low-energy πd scattering/13-15/. It has been found that the mass differences decrease the real parts of $\pi^{\pm} d$ lengths by about 0.003 fm and induce imaginary parts of them $\sim 10^{-4}$ fm/15/, that is, the role of the mass differences in πd system turns out to be negligible.

However, being based on that result, one cannot make the conclusion about their small role in scattering of pions from any other nucleus. The point is that the virtual charge-exchange-process contribution to the S-wave πd scattering is suppressed by demanding the transition of the particles into P-state for that processes due to the Pauli principle and the parity conservation law/16/. In other words, this suppression is caused by a very small number of nucleons in the deuteron, i.e., by very few possible spin-isospin states.

For 3-nucleon and more heavier nuclei this suppression is taken off. Hence, one cannot a priori say how it is important to take into consideration the mass differences for pion-nucleus scattering with $A \geq 3$.

In this paper, using the formalism proposed earlier /15/, we calculate the lengths of pion scattering from ${}^3\mathrm{H}$ and ${}^3\mathrm{He}$ in view of the mass splittings of the pion, nucleon, and nucleus isomultiplets and investigate a sensitivity of the results to variations of the πN-potential parameters.

2. FORMALISM

In a theoretical description of nonrelativistic quantum systems a constant term corresponding to the sum of masses of considered particles as usual is not included into the total ener-

gy. In our case it is more convenient to take the vacuum energy as an origin of the energy-scale. Then in the Hamiltonian one has a supplementary addendum, namely, the mass operator.

As eigenstates this operator has states with definite values of the isospin-third-components of particles because a particle of a charged multiplet and its mass are uniquely defined just by its isospin-third-component. And the eigenvalues are equal to the sum of masses of particles in the corresponding isotopic states.

Thereby the mass operator $\hat{m}$ is diagonal in the basis $|\theta\rangle = |t_\pi \tau_\pi, t_1\tau_1, t_2\tau_2, t_3\tau_3\rangle$, where t_π, t_i are pion and i-th nucleon isospins, and τ_π, τ_i are their third components.

If one couples all the isospins to produce the state $|\eta\rangle$ with total isospin $\vec{I} = [(\vec{t}_1 + \vec{t}_2) + \vec{t}_3] + \vec{t}_\pi$ and its third component τ, then in the basis $|\eta\rangle = |(((t_1 t_2)\, t_{12} t_3)\, t t_\pi)\, I\tau\rangle$ operator $\hat{m}$ loses the diagonality $\hat{m} = \sum_{\eta'\eta\theta} |\eta'\rangle\, m_\theta\, M^\theta_{\eta'\eta} \langle\eta|$,

where m_θ is an eigenvalues of $\hat{m}$ in the state $|\theta\rangle$ and $M^\theta_{\eta'\eta} = \langle\eta'|\theta\rangle\langle\theta|\eta\rangle$ are the recoupling coefficients.

The cause of nondiagonality consists in the dependence of m_θ on the quantum numbers of the state $|\theta\rangle$

$$m_\theta = m_0 + \Delta_\pi |\tau_\pi| + \Delta_N \left(\frac{3}{2} - \tau + \tau_\pi\right),$$

where $\Delta_N = m_n - m_p \approx 1.3$ MeV, $\Delta_\pi = m_{\pi^+} - m_{\pi^0} \approx 4.6$ MeV, $m = 3m_p + m_{\pi^0}$. That dependence disappears in the degenerated case $\Delta_N = \Delta_\pi = 0$.

Let $H = H_0 + V + H_A$ be the Hamiltonian of the pion-nucleus system, where H_0 describes a free relative pion-nucleus motion, V is the sum of the πN-potentials, H_A is the nucleus Hamiltonian. The mass operator is included into the free-motion Hamiltonian

$$H_0 = \sum_{\eta'\eta\theta} |\eta'\rangle \left(\frac{k^2}{2\mu_\theta} + m_\theta\right) M^\theta_{\eta'\eta} \langle\eta|,$$

where k is the relative pion-nucleus momentum, μ_θ is the corresponding reduced mass in the state $|\theta\rangle$.

The amplitude of scattering $\pi(3N) \to \pi(3N)$ is an asymptotic-state average of the operator $T(Z) = V + V(Z - H)^{-1} V$, that obeys the equation /17/

$$T = T^\circ + T^\circ G_0 H_A G_A T, \tag{1}$$

where $G_0(Z) = (Z - H_0)^{-1}$, $G_A(Z) = (Z - H_0 - H_A)^{-1}$, and T° is the amplitude of scattering from fixed centers

$$T^\circ = V + V G_0 T^\circ. \tag{2}$$

Inverting the operator $(Z - H_0)$ in the basis $|\theta\rangle$ and then transforming it to the basis $|\eta\rangle$, we obtain the free Green function $G_0(Z)$ in the form

$$G_0(Z) = -\sum_{\eta'\eta\theta} |\eta'\rangle (Z - m_\theta - \frac{k^2}{2\mu_\theta})^{-1} \langle\eta| .$$

The sum over θ formally includes summation over four indices τ_1, τ_2, τ_3, τ_π, but the total-isospin third component $\tau = \tau_1 + \tau_2 + \tau_3 + \tau_\pi$ is conserved. Therefore there are only three independent indices, for example, τ_1, τ_2, τ_π. Values of m_θ and μ_θ depend only on τ and τ_π. Hence $M^\theta_{\eta'\eta}$ may be summed over τ_1 and τ_2. As one can easily see, this implies

$$K^{\tau_\pi}_{\eta'\eta} = \sum_{\tau_1\tau_2} M^\theta_{\eta'\eta} = \delta_{\tau'\tau}\, \delta_{t'_{12}t_{12}}\, \delta_{t't}\, C^{I'\tau}_{t,\tau-\tau_\pi, t_\pi\tau_\pi}\, C^{I\tau}_{t,\tau-\tau_\pi, t_\pi\tau_\pi} .$$

So, the operator G_0 is diagonal in the quantum numbers t_{12}, t, τ of the state $|\eta\rangle$. There remains only the sum over τ_π instead of the θ

$$G_0(Z) = \sum_{\eta'\eta} \sum_{\tau_\pi} |\eta'\rangle (Z - m_{\tau\tau_\pi} - \frac{k^2}{2\mu_{\tau\tau_\pi}})^{-1} K^{\tau_\pi}_{\eta'\eta} \langle\eta| , \qquad (3)$$

i.e., G_0 is a linear combination of the following propagators

$$G_0^{\tau\tau_\pi}(Z) = [Z - m_0 - \Delta_\pi |\tau_\pi| - \Delta_N (\frac{3}{2} - \tau + \tau_\pi) - \frac{k^2}{2\mu_{\tau\tau_\pi}}]^{-1} , \qquad (4)$$

corresponding to charged $(\tau_\pi = \pm 1)$ and neutral $(\tau_\pi = 0)$ pions in an intermediate state.

The nonzero mass differences Δ_π and Δ_N break the isotopic invariance of H. In a sense this breaking may be called a kinematical one because it is originated by the free Hamiltonian, and the interaction V is assumed to be isotopically invariant. Both the free Green function G_0 and the amplitude become dependent on τ and nondiagonal in I. However, the matrix elements of G_0, connecting states of different I, are much smaller than the diagonal ones. This can be seen if one explicitly writes sum (3). The diagonal elements have the form

$$\alpha^2 G_0^{\tau,\pm 1}(Z) + \gamma^2 G_0^{\tau,0}(Z) , \qquad (5)$$

while the nondiagonal ones are proportional to the difference $G_0^{\tau,\pm 1}(Z) - G_0^{\tau,0}(Z)$, and the denominators of $G_0^{\tau,\pm 1}$ and $G_0^{\tau,0}$ are close to each other due to smallness of Δ_π and Δ_N with respect to m_0.

Having in mind this fact, let us assume the total isospin I to be conserved that significantly simplifies the calculation. Then the isotopic-invariance breaking is to be found only as the charge asymmetry, i.e., the dependence of the amplitude on τ.

The total energy Z for scattering of pions from ^3H and ^3He is calculated by using the quantum numbers τ_π and ξ of the incoming channel state as

$$Z = E_{kin} + m_0 + \Delta_\pi |\tau_\pi| + \Delta_N (\frac{3}{2} - \xi) + \mathcal{E}_\xi ,$$

where E_{kin} is the kinetic energy, $\xi = \tau - \tau_\pi$ is the third component of the nucleus isospin, and $-\mathcal{E}_{1/2} = 7.718$ MeV and $-\mathcal{E}_{-1/2} = 8.482$ MeV are the binding energies of ^3He and ^3H.

Setting into (4) the energy Z, corresponding to incoming channels $\pi^+\,{}^3He$, $\pi^+\,{}^3H$, $\pi^-\,{}^3He$, $\pi^-\,{}^3H$ we see that $G_0(Z)$ has no poles in the region $k \in [0, \infty)$, if E_{kin} is less than 1.815 MeV, 2.579 MeV, 4.401 MeV, and 5.165 MeV, respectively.

Therefore in the scattering-length calculation (when $E_{kin} = 0$) eq.(2) has pure real solutions, i.e., in the fixed-scatterer-approximation π(3N) lengths have no imaginary parts caused by the charge-exchange processes, since the energy release of them is less than the (3N)-binding energy.

To solve eq.(1), we approximate the nuclear Hamiltonian by a finite-rank operator /17/

$$H_A \approx H_A^{(2)} = \sum_\xi \mathcal{E}_\xi |\chi_\xi \phi_\xi\rangle\langle\chi_\xi \phi_\xi| , \qquad (6)$$

where $|\chi_\xi\rangle$ are the fully antisymmetrized spin-isospin functions of the nuclei ^3He $(\xi = 1/2)$ and ^3H $(\xi = -1/2)$, and $|\phi_\xi\rangle$ are the corresponding symmetric space S-components.

As is pointed out in ref./18/, a sufficient condition of applicability of such an approximation is the absence of open nucleus-breakup channels at a given collision energy. For the three-nucleon nuclei lowest breakup thresholds are

$$q_+(^3He \to p + d) = |\mathcal{E}_{1/2}| - |\mathcal{E}_d| = 6.257 \text{ MeV},$$

$$q_-(^3H \to n + d) = |\mathcal{E}_{-1/2}| - |\mathcal{E}_d| = 5.493 \text{ MeV}.$$

In the charge-exchange processes $\pi^-\,{}^3He \to \pi^0\,{}^3H$ and $\pi^+\,{}^3H \to \pi^0\,{}^3He$ due to the mass differences of the particles and nuclei, energy $Q_- = 4.081$ MeV and $Q_+ = 5.139$ MeV is released. All the other charge-exchange channels get an absorption of energy. Hence, if the kinetic energy $E_{kin} < (q_+ - Q_+) = 1.118$ MeV, then all breakup channels are closed for all the $\pi(3N) \to \pi(3N)$ processes. Thereby, for our calculation of the scattering lengths approximation (6) is quite applicable.

To simplify the calculations, we assume the space components of the nuclear functions to be equal to each other $|\phi_{1/2}\rangle = |\phi_{-1/2}\rangle$. As ϕ we choose the function of ^3H or ^3He in accordance to what target pion is scattered from.

Physical amplitudes of the processes $\pi(3N) \to \pi(3N)$ are the T-operator matrix-elements of the form

$$\langle \vec{k}', \phi_{\xi'} \chi_{\xi' r'_\pi} | T(Z) | \vec{k}, \phi_\xi \chi_{\xi r_\pi} \rangle ,$$

where $|\chi_{\xi r_\pi}\rangle$ is a direct product of a nucleus spin-isospin function with three-particle isospin $t = 1/2$ and a pion isospin-function with the third component r_π, $\vec{k}$ is the relative pion-nucleus momentum.

From (1) and (6) we obtain for these matrix elements the following set of integral equations

$$\langle \phi_{\xi'} \chi_{\xi' r'_\pi} | T(Z) | \phi_\xi \chi_{\xi r_\pi} \rangle = \langle \phi_{\xi'} \chi_{\xi' r'_\pi} | T^o(Z) | \phi_\xi \chi_{\xi r_\pi} \rangle +$$

$$+ \sum_{\xi'' r''_\pi} \mathcal{E}_{\xi''} \langle \phi_{\xi'} \chi_{\xi' r'_\pi} | T^o(Z) | \phi_{\xi''} \chi_{\xi'' r''_\pi} \rangle G_0^{r, r-\xi''}(Z)\, G_0^{r, r-\xi''}(Z - \mathcal{E}_{\xi''}) *$$

$$\langle \phi_{\xi''} \chi_{\xi'' r''_\pi} | T(Z) | \phi_\xi \chi_{\xi r_\pi} \rangle .$$

Performing a transformation to the isotopic basis $|\eta\rangle$ and again retaining only the matrix elements diagonal in the total isospin, we obtain for the S-wave amplitude the one-dimensional integral equation

$$\langle k' | \bar{T}^{Ir}(Z) | k \rangle = \langle k' | \bar{T}^{oIr}(Z) | k \rangle +$$

$$+ \frac{1}{2\pi^2} \int_0^\infty dk'' k''^2 \langle k' | \bar{T}^{oIr}(Z) | k'' \rangle R^{Ir}(Z) \langle k'' | \bar{T}^{Ir}(Z) | k \rangle , \qquad (7)$$

where the bar denotes the average over the nucleon coordinates, and

$$R^{Ir}(Z) = \sum_\xi \mathcal{E}_\xi G_0^{r, r-\xi}(Z)\, G_0^{r, r-\xi}(Z - \mathcal{E}_\xi) [C^{Ir}_{\frac{1}{2}\xi, 1 r-\xi}]^2 .$$

In contrast with eq.(2), the kernel of eq.(7) for $\pi^+ {}^3H$- and $\pi^- {}^3He$ scattering has a pole at $k^2/(2\mu_{\frac{1}{2}0}) = Q_+ + E_{kin}$ and $k^2/(2\mu_{-\frac{1}{2}0}) = Q_- + E_{kin}$, because it contains the free propagator taken at energy $(Z - \mathcal{E}_\xi)$.

Presence of the pole even for $E_{kin} = 0$ gives rise to imaginary parts of $\pi^+ {}^3H$ and $\pi^- {}^3He$ scattering lengths. The physical cause of them consists in the mass differences which generate the energy release of the charge-exchange processes.

We assume πN interaction to be isotopically invariant and to be described by a separable S-wave potential acting in the channels $S_{11}(n=1)$ and $S_{31}(n=3)$ of the form

$$V_n(k', k) = \lambda_n / (k'^2 + \beta_n^2)/(k^2 + \beta_n^2). \qquad (8)$$

The use of a separable πN potential enables us to solve eq.(2) analytically, i.e., to obtain a complete sum of series of the pion multiple scattering from fixed nucleons.

The method of solving eq.(2) is in detail described in paper /19/.

Equation (7), containing, as an inhomogeneous term, a solution of eq.(2) averaged over a nuclear wave-function, allows us to some extent to take into account the Fermi-motion of the nucleons.

As wave-functions ϕ_ξ we use the functions of Irving /20/ reproducing the experimental values of mean square radii of ^3H and ^3He, namely, 1.70 fm and 1.88 fm.

3. RESULTS

The results of our calculation are represented in tables 1,2 and in figures 1-6.

Table 1 contains the complex lengths of $\pi^\pm {}^3He$, $\pi^\pm {}^3H$-scattering calculated for different values of the range-parameter $\beta = \beta_1 = \beta_3$ of the πN potential (8), with the depth-parameters λ_1 and λ_3 being taken as

$$\lambda_n = \frac{2\pi}{\mu_{\pi N}} a_n \beta_n^4 (1 - a_n \beta_n / 2)^{-1} , \qquad (9)$$

which enables us to reproduce the experimental πN lengths /21/

$$a_1 = -0.257 \text{ fm}, \quad a_3 = 0.154 \text{ fm}. \qquad (10)$$

Here the reduced pion-nucleon mass $\mu_{\pi N}$ is calculated for the doublet and quartet states as a weighted average

$$\mu_{\pi N} = \frac{1}{2t_{\pi N} + 1} \sum_{r_\pi r_N r_{\pi N}} [C^{t_{\pi N} r_{\pi N}}_{1 r_\pi \frac{1}{2} r_N}]^2 \mu^{\pi N}_{r_\pi r_N} .$$

For comparison, in the fouth column of Table 1 there are placed the corresponding values $\tilde{A}$ of $\pi^- {}^3He$ lengths calculated in the equal-masses case. As a particle mass in this case we take their values averaged over the multiplets.

The results of Table 1 are graphically shown in figs.1 and 2. In fig.1 there are shown the lengths for the doublet and quar-

Table 1

β fm^{-1}	$A(\pi^{+}\,{}^{3}He)\cdot 10^{4}$ fm	$A(\pi^{-}\,{}^{3}He)\cdot 10^{4}$ fm	$\tilde{A}(\pi^{-}\,{}^{3}He)\cdot 10^{4}$ fm	$A(\pi^{-}\,{}^{3}H)\cdot 10^{4}$ fm	$A(\pi^{+}\,{}^{3}H)\cdot 10^{4}$ fm
0.1	2108	-896 - i 2	-884	2109	-897 - i 3
0.3	2165	-888 - i15	-715	2166	-892 - i31
0.5	2219	-727 - i20	-599	2219	-731 - i43
1.0	2365	-496 - i21	-347	2357	-507 - i47
1.5	2524	-238 - i19	- 83	2508	-256 - i43
2.0	2695	31 - i18	191	2671	87 - i39
2.5	2877	304 - i17	474	2844	277 - i38
3.0	3068	577 - i17	763	3026	548 - i38
3.5	3269	852 - i19	1059	3218	819 - i40
4.0	3482	1129 - i21	1364	3421	1093 - i44
4.5	3708	1411 - i24	1682	3636	1370 - i51
5.0	3948	1698 - i28	2014	3854	1654 - i59
5.5	4207	1999 - i33	2387	4110	1953 - i72
6.0	4486	2314 - i39	2758	4400	2252 - i83

Table 2

$a^{\pi N}_{1/2}$ fm	$a^{\pi N}_{3/2}$ fm	$A(\pi^{+}\,{}^{3}He)\cdot 10^{4}$ fm	$A(\pi^{-}\,{}^{3}He)\cdot 10^{4}$ fm	$\tilde{A}(\pi^{-}\,{}^{3}He)\cdot 10^{4}$ fm	$A(\pi^{-}\,{}^{3}H)\cdot 10^{4}$ fm	$A(\pi^{+}\,{}^{3}H)\cdot 10^{4}$ fm
-0.257	0.126	2239	-179 - i15	-56	2215	-206 - i34
-0.240	0.130	2410	82 - i14	209	2384	58 - i31
-0.264	0.148	2869	353 - i17	522	2833	324 - i37
-0.257	0.154	3068	577 - i18	763	3026	548 - i38

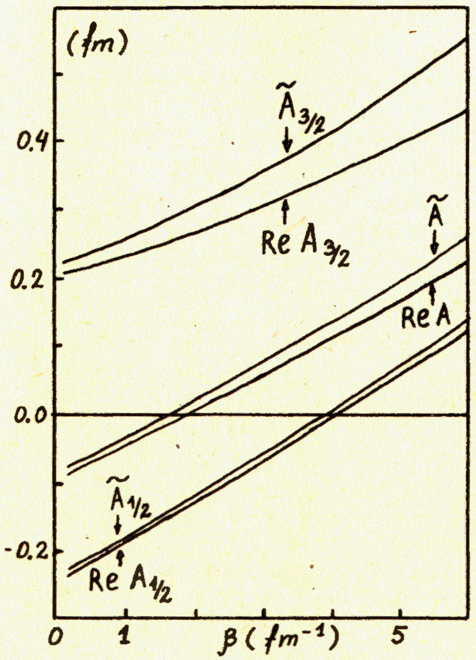

Fig.1

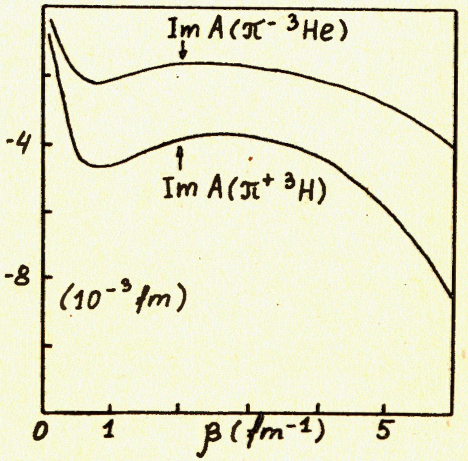

Fig.2

tet states with the total-isospin third component $\tau = -1/2$ and the corresponding physical $\pi^- \, ^3He$ length. A more significant role of the mass differences in the state with $I = 3/2$ is explained by the fact that for the quartet case in formula (5) $\alpha^2 = 1/3$, $\gamma^2 = 2/3$, while for $I = 1/2$ we have $\alpha^2 = 2/3$, $\gamma^2 = 1/3$, i.e., in the quartet state the weight of intermediate charge-exchange states is greater than that for the doublet one.

As one sees in fig.2, $|\mathrm{Im}\, A(\pi^- \, ^3He)| < |\mathrm{Im}\, A(\pi^+ \, ^3H)|$ for all β that correctly reflects the energy-release inequality $Q_- < Q_+$.

In Table 2 there are displayed the lengths calculated for $\beta_1 = \beta_3 = 3$ fm^{-1} and four different experimental sets of πN lengths [21-24]. The fifth column of this table contains the lengths $\tilde{A}$ calculated without inclusion of the mass differences.

Varying πN lengths in the intervals of their experimental uncertainties, we explore the sensitivity of the $\pi^- \, ^3He$ length to changes of πN-potential depth. In figs.3 and 4 there are shown (at $\beta_1 = \beta_3 = 3$ fm^{-1}) the dependence of $\mathrm{Re}\, A(\pi^- \, ^3He)$ on the doublet and quartet πN lengths. The parallel lines correspond to different values of fixed a_3 and a_1.

Because of the use, by many authors of the zero-range πN potential in pion-nucleus scattering-length calculations we investigate the sensitivity of the $\pi^- \, ^3He$ lengths to variations of the relation between the nucleus-size and πN potential range.

In figs.5 and 6 there are plotted the real and imaginary parts of the $\pi^- \, ^3He$ length as functions of the mean square radius of ^3He. The calculations were performed for three values of $\beta = 1, 3, 5$ fm^{-1} and one set of πN lengths (10).

If the range of forces acting between a projectile and a target-particle is much longer than the target-size, then it is intuitively clear that the total amplitude is an additive sum of the elementary ones, i.e., the impulse approximation is reliable in that case. And, as one can see in fig.5, in our case in the limit $\langle r^2 \rangle \to 0$ the $\pi^- \, ^3He$ lengths indeed is just equal to the impulse approximation value $(4a_1 + 5a_3)/3 = -0.086$ fm. For small β (when the range of πN forces is long) the difference from the impulse approximation is not very large even if we increase $\langle r^2 \rangle$ almost up to its physical value. If we decrease πN-potential range $(1/\beta)$, then rescattering effects become essential even for small $\langle r^2 \rangle$. In the limit $\langle r^2 \rangle \to \infty$ all the curves in figs.5,6, as is to be expected, go together because the potentials do not overlap with each other and their ranges are not substantial.

The large difference from the impulse approximation at the physical value $\langle r^2 \rangle^{1/2} = 1.88$ fm and $1/\beta \le 1/3$ fm means that in low-energy $\pi^- \, ^3He$-scattering problems the multiple-scattering series might poorly converge both for realistic ($\beta \sim 3$ fm^{-1}) and especially for zero-range πN potentials.

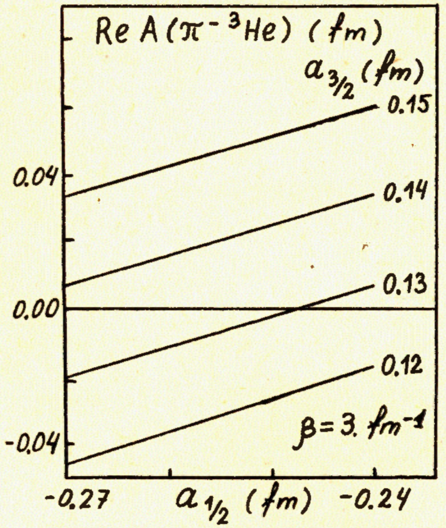

Fig.3

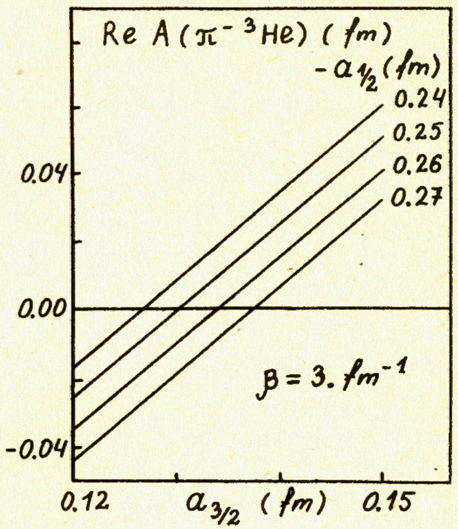

Fig.4

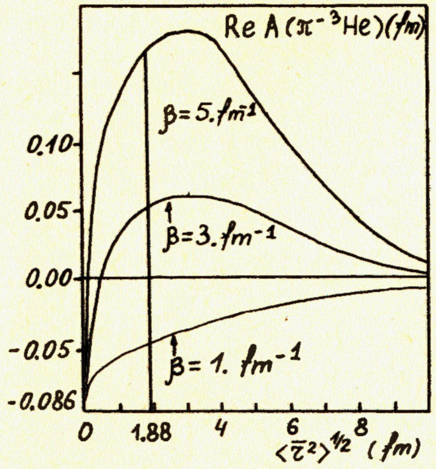

Fig.5

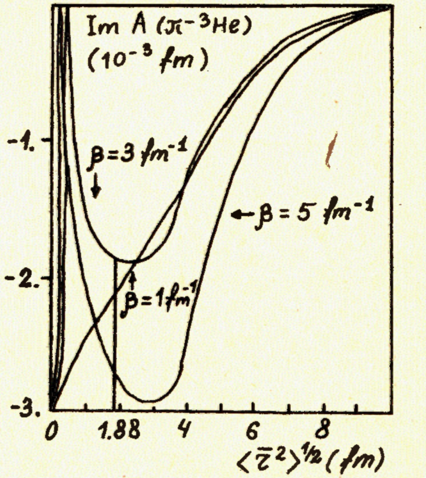

Fig.6

Now it is impossible directly to measure the pion-nucleus lengths. Therefore as their experimental values (A_{exp}) one uses the corresponding lengths calculated with measured shifts ΔE and widths Γ of the S-levels of pionic atoms. As a rule, the calculation is based on different modifications [25-27] of the Deser formula [28].

To compare our $\pi^- \, ^3He$ lengths with the experimental data we apply a Deser-formula in the form [27]

$$A_{exp} = (\Delta E - i\Gamma/2)/[2(\alpha z)^3(1-0.02A)\,m_\pi^2] = \kappa(\Delta E - \frac{i\Gamma}{2}), \qquad (11)$$

where $z = 2$, $A = 3$, $\alpha = 1/137$, $\kappa = 0.001909838$ fm/eV.

Table 3

ref. exp.	$-\Delta E_{1s}$ (eV)	Γ_{1s} (eV)	$-A_{exp}(\pi^- {}^3He)\cdot 10^3$ (fm)	$-A_{exp}(\pi^- {}^3He)\cdot 10^3$ (fm)	$-A_{pot}\cdot 10^3$ (fm)
/29/	50^{+16}_{-20}	89 ± 67	$(71\pm7)+i(48\pm17)$ /25/	$(95^{+31}_{-38})+i(85\pm64)$	280^{+171}_{-177}
/30/	44 ± 5	42 ± 14	$(79\pm9)+i(38\pm12)$ /26/	$(84\pm9)+i(40\pm13)$	171 ± 37
/31/	34 ± 4	36 ± 7		$(65\pm7)+i(34\pm7)$	139 ± 22
/32/	27 ± 5	65 ± 12	$(48\pm9)+i(58\pm11)$ /26/	$(52\pm9)+i(62\pm11)$	187 ± 33
/27/	35.46	36.68		67+i35	143

Table 3 contains experimental data for 1S-level of $\pi^- {}^3He$ atom obtained in the works /29-32/ and also the values A_{exp} calculated with these data by different variants of the Deser-formula /25-27/ (the fourth column) and those we have calculated with formula (11) (the fifth column). The last column of Table 3 contains the lengths A_{pot} obtained by subtraction of the true pion-absorption contribution off A_{exp} with the Brueckner formula

$$\Delta E = \Delta E_{pot} + 1.09\Gamma, \qquad (12)$$

where ΔE_{pot} is the 1S-level shift caused by a pure potential interaction.

Results of our and other potential calculations should be compared just with A_{pot}. However, formula (12) has been obtained by Brueckner for $\pi^- d$ atom, and its application to any more heavier nucleus is an extrapolation based only on qualitative arguments.

At realistic values of the range-parameter ($\beta \sim 3$ fm^{-1}) our $\pi^- {}^3He$ length differs from A_{pot}. This fact may be considered as an indication for inapplicability of formula (12) to $\pi^- {}^3He$ atom.

As one sees from Table 3, different variants of the Deser-formula give essentially different A_{exp} with the same ΔE and Γ. This nonuniqueness of the experimental data trearment turns out to be more explicit, if one compares A_{exp} of the first line of table 3 with

$$A_{exp}(\pi^- {}^3He) = -(0.0522 \pm 0.0037) + i(0.0345 \pm {}^{0.0041}_{0.0040})\ \text{fm},$$

computed in ref. /34/ with the same ΔE and Γ independently by the direct calculation of the scattering lengths in the framework of the Klein-Gordon equation.

Thus, as it follows from Table 3, the experimental value of the $\pi^- {}^3He$ length is known with a great uncertainty, which covers the interval

$$\mathrm{Re}\,A_{exp}(\pi^- {}^3He) \in [-0.126, -0.039]\ \text{fm},$$

$$\mathrm{Im}\,A_{exp}(\pi^- {}^3He) \in [-0.149, -0.021]\ \text{fm}.$$

The theoretical value of this length obtained in multiple scattering theory /35, 26/, by the method of strongly coupled chanels /36, 37/, by the coupling constant evolution method /38/, and by the fixed scatterer approximation /39/, is also contained in the wide interval $A_{th}(\pi^- {}^3He) \in [-0.097, 0.003]$ fm. Such significant variations may be explained by the use of different πN potential parameters which cannot be uniquely deduced from the available πN data, and by many approximations employed in the different papers.

Thus, in ref. /35/ there are taken into account only single and double scattering terms of the multiple series. The author of paper /26/ in the framework of the fixed scatterer approximation includes also the triple term, but he uses zero-range πN forces. In works /36-39/ there are unsatisfactory neglected all intermediate $\pi 3N$ -states with the total isospin of (3N)-subsystem being inequal to 1/2. Moreover in ref. /39/ the midpoint theorem is used to compute the integral in averaging the fixed-center-scattering amplitude over the nuclear wave function; and their results strongly depend on the choice of the midpoint.

As has been already noted in this work we have a complete sum of all the terms of pion multiple scattering from the fixed nucleons and take into account all possible $\pi 3N$ isotopic states and to some extent the Fermi motion of nucleons. Therefore our calculation seems to be at present the most correct theoretical estimation of the $\pi(3N)$ lengths.

Taking into consideration the mass differences of the particles and nuclei enables us to calculate in a consistent way the imaginary parts of $\pi^{-3}He$ and $\pi^{+3}H$ scattering lengths caused by the charge-exchange processes and to obtain the corresponding partial width of 1S-state of $\pi^{-3}He$ atom

$$\Gamma_{ch,ex} = -\frac{2}{\kappa} \operatorname{Im} A(\pi^{-\,3}He).$$

To summarize our results, we say the following. The $\pi(3N)$ scattering lengths strongly depend on πN potential parameters. The multiple scattering series for low energy $\pi(3N)$ scattering seems to poorly cobverge. The Brueckner formula is presumably inapplicable to $\pi^{-\,3}He$ atom. The mass differences of the particles and nuclei give rise to imaginary parts of the $\pi^{-3}He$ and $\pi^{+3}H$ lengths, namely $\sim 10^{-3}$ fm, which is by one order greater than the corresponding effect in πd scattering. Taking into consideration the mass differences substantially changes real parts of $\pi^{-\,3}He$ and $\pi^{+\,3}H$ scattering lengths. For some values of the πN potential parameters these changes are greater than 100%.

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Беляев В.Б., Пупышев В.В., Ракитянский С.А. E4-84-675

Длины $\pi^{\pm}\,{}^3H$-, $\pi^{\pm}\,{}^3He$-рассеяния
и разности пионных, нуклонных и ядерных масс

На основе приближенных четырехчастичных уравнений вычислены длины рассеяния пионов трехнуклонными ядрами с учетом расщепления по массам изомультиплетов пионов, нуклонов и ядер. Показано, что учет разностей масс приводит к появлению у длин $\pi^{-}\,{}^3He$ и $\pi^{+}\,{}^3H$-рассеяния мнимых частей $\sim 10^{-3}$ Фм и существенно меняет их действительные части. Обнаружена сильная зависимость результатов от выбора параметров πN-потенциала. Получено указание на то, что ряд многократного $\pi(3N)$-рассеяния при низких энергиях, по-видимому, сходится плохо.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Belyaev V.B., Pupyshev V.V., Rakityansky S.A. E4-84-675

Lengths of $\pi^{\pm}\,{}^3H$, $\pi^{\pm}\,{}^3He$ Scattering
and Mass Differences of Pions, Nucleons and the Nuclei

In the framework of approximate 4-body equations the lengths of pion scattering from 3-nucleon nuclei are calculated with taking into account the mass-splittings of the pion, nucleon, and nucleus isomultiplets. It is shown that the mass differences generate imaginary parts $\sim 10^{-3}$ fm of the $\pi^{-}\,{}^3He$ and $\pi^{+}\,{}^3H$ lengths and essentially change their real parts. A strong dependence of the results on the choice of πN-potential parameters is found. An indication is obtained for a poor convergence of the multiple $\pi(3N)$-scattering series at low energy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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