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RADIATIVE MUON CAPTURE ON  $^{16}\text{O}$

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As thoroughly emphasized in the literature<sup>/1/</sup> radiative muon capture (RMC) represents among the low energy weak processes the most favourable kinematical conditions to enhance the role of the pseudoscalar form factor  $g_p$  of the weak axial current. As a matter of fact, at the high energy end of the observed photon spectrum, where no energy is given to the neutrino, the pion propagator dominating  $g_p$  has a four-momentum transfer  $q_\mu^2 = q_0^2 - \vec{q}^2 = (\mu + \nu)^2 \approx \mu^2$ , i.e., rather near to the pion mass singularity.

On the experimental side, due to the low counting rate, data for the high-energy ( $E_\gamma \geq 57$  MeV) photon spectrum exist only for nuclei, namely for  $^{40}\text{Ca}$  from Hart et al.<sup>/2/</sup> and from the Lausanne-Munich-Zurich collaboration<sup>/3,4/</sup>, and in a preliminary form for  $^{16}\text{O}$ <sup>/4/</sup>.

It has been repeatedly stressed that either partial transitions in RMC are resolved or the correct knowledge of the nuclear excitation spectrum is crucial<sup>/5,6/</sup> for the extraction from experiment of the  $g_p$  value. As a matter of fact, if the final nuclear state is not detected (inclusive process), which is the case of the above-mentioned experiments, a sum over all final states has to be performed. This entails, because of energy conservation, that the higher the photon momentum the smaller the part of the physically accessible spectrum. It is therefore clear, because of strong dependence of the photon spectrum on the excitation energies, that the closure approximation is at its worst<sup>/5/</sup> in RMC. From the same argument, the contribution connected with the low-lying nuclear states is relatively enhanced with respect to the one due to the high-lying excitation branch.

In the present Letter we address ourselves to the problem of the excitation spectrum and to the prediction of the photon spectrum for RMC on  $^{16}\text{O}$ .

The RMC photon spectrum is proportional to the nuclear excitation function  $r(E, \mathbf{s}) = \sum_b |\langle b | \sum_i r_i^3 \exp(-i\vec{s} \cdot \vec{r}_i) | a \rangle|^2 \delta(E_{ba} - E)$ , where  $\vec{s} = \vec{k} + \vec{\nu}$  is the sum of photon ( $\vec{k}$ ) and neutrino ( $\vec{\nu}$ ) momenta. In refs.<sup>/6,7/</sup> the nuclear matrix element  $\langle b | \sum_i r_i^3 \exp(-i\vec{s} \cdot \vec{r}_i) | a \rangle$  for the RMC on  $^{40}\text{Ca}$  has been obtained in a rather schematic way relying fully on the method of Foldy and Walecka<sup>/8/</sup>. Though keeping a phenomenological approach, the main difference of the present calculation as compared with the previous ones is a more detailed treatment of the nuclear excitation spectrum (this time for the RMC on  $^{16}\text{O}$ )

using the presently available experimental and theoretical information about it.

The spectrum of the final nucleus  $^{16}\text{N}$  can be schematized as being composed of three collective bands: the quartet of bound states ( $0^-$ ,  $1^-$ ,  $2^-$ , and  $3^-$ ), the giant dipole resonance (GDR), and the quadrupole branch. They will be described as follows:

(i) The GDR is treated in the style of ref.<sup>/8/</sup>, i.e., the dipole state excited in RMC is the one observed in photoreactions times the square of the nuclear elastic form factor. The underlying hypothesis of SU(4) symmetry, i.e., the fact that the states excited by the group generators  $r^- \exp(-i\vec{s} \cdot \vec{r})$  and  $\sigma r^- \exp(-i\vec{s} \cdot \vec{r})$  are degenerate, will not be questioned, since for this nucleus it is well enough supported by the detailed microscopical calculations<sup>/9/</sup>.

(ii) The treatment of quadrupole is much more uncertain. In Foldy and Walecka's approach it is essentially out of control, and moreover, it is taken at the harmonic oscillator energy, i.e., at an energy degenerate with the experimental  $E_{\text{GDR}}$ . Therefore, in practice their spectrum would correspond to a closure approximation at  $E = E_{\text{GDR}}$ . The correct consideration of the quadrupole excitation energy in RMC has been shown<sup>/6/</sup> to be relevant, because of the previously given arguments about energy conservation, in decreasing the high-energy contribution, to RMC. This would be of particular importance in a nucleus where quadrupole is sizeable like in  $^{40}\text{Ca}$ . It is, however, not the case in  $^{16}\text{O}$  where its contribution in ordinary muon capture (OMC) is about 10%, see the Table.

Table  
Ordinary and radiative muon capture results on  $^{16}\text{O}$  for the three excitation bands of  $^{16}\text{N}$ . The value R stands for the integrated photon yield for  $k \geq 57$  MeV divided by the calculated ordinary capture rate.  $R_{\text{exp}} = (6.2 \pm 0.8) \cdot 10^{-5}$  (ref.<sup>/4/</sup>, preliminary).

State	$\bar{E}$ (MeV)	$\Gamma$ (MeV)	$\Lambda_{\text{OMC}}^{\text{exp}}$ ( $10^3 \text{ s}^{-1}$ )	$\Lambda_{\text{OMC}}(\Delta=1)$ ( $10^3 \text{ s}^{-1}$ )	$R(\Delta=1)$ ( $10^{-5}$ )	$R(\Delta=2)^b$ ( $10^{-5}$ )
Bound levels ( $0^-$ , $1^-$ , $2^-$ )	12	0.5	9 ÷ 11	14	0.43	0.80
Dipole	24	4.0	✓	70	1.54	2.74
Quadrupole	35	15.0	-83	10	0.11	0.30
Sum			$93 \pm 10^a$	94	2.08	3.84

a) Ref.<sup>/16/</sup>.

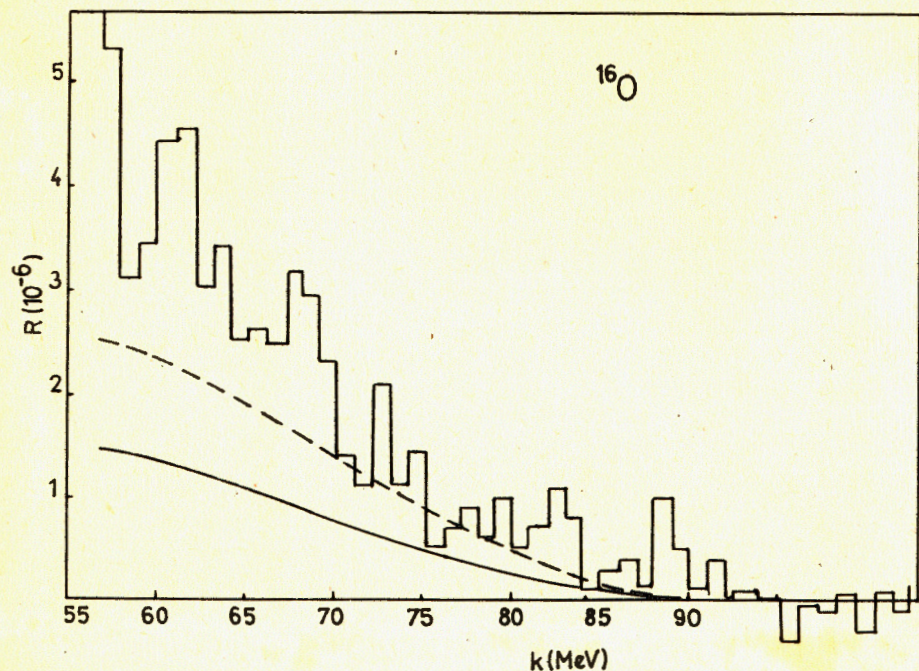
b) Calculated value  $\Lambda_{\text{OMC}}(\Delta=2) = 85 \cdot 470 \text{ s}^{-1}$  has been used here.



(iii) The  $0^-$ ,  $1^-$ ,  $2^-$  and  $3^-$  ( $T=1$ ) bound levels have their isobar analogues in  $^{16}\text{O}$  at about 12 MeV, see the Table. Information about them comes from OMC, radiative pion capture,  $(e, e')$  reactions and microscopic calculations. In OMC in particular their calculated contribution is about 15%<sup>/9/</sup>. Data from the above-mentioned processes have been discussed in Ref.<sup>/10/</sup> and shown to support the overall picture. This gives us confidence in the use of the inelastic form factors which must govern the momentum transfer behaviour of this transition. We will use in the following for "the 12 MeV levels" the form factor suggested by Graves et al.<sup>/11/</sup>.

The overall picture of the excitation bands in OMC on  $^{16}\text{O}$  is summarized in the Table. As for RMC their contribution is relatively hindered for the quadrupole, is obtained as previously mentioned for the dipole, and is relatively enhanced for the low-lying states.

The results of the present calculation for the photon spectrum are plotted in the Figure. The full line is obtained for  $\Delta = 1$  ( $g_p = 2Mg_A \Delta / (q^2 - m_\pi^2)$ ), i.e., at the standard value of  $g_p$  and the dot-dashed line for  $\Delta = 2$ . The results obtained for



Ratio R of the integrated RMC photon spectrum for  $k \geq 57$  MeV to the OMC rate for the cases  $\Delta=1$  (full line) and  $\Delta=2$  (dashed line). Histogram is for the data of ref.<sup>/4/</sup>.

$\Delta = 1$  by assuming that all strength is concentrated in the giant dipole resonance (an absolute theoretical lower limit) are very close to the full line. In the last two columns of the Table the calculated contributions of different terms divided by the OMC rate ( $\Lambda_{\text{OMC}} = 94 \cdot 000 \text{ s}^{-1}$  for  $\Delta=1$ ;  $\Lambda_{\text{OMC}} = 85 \cdot 470 \text{ s}^{-1}$  for  $\Delta=2$ ) are reported under the entries R. Note that  $\Lambda_{\text{OMC}}(\Delta=2)$  does not reproduce data of OMC. In this particular case of  $^{16}\text{O}$  we see that the effects of a realistic nuclear excitation spectrum appear to be of small numerical importance when compared to a naive closure at the GDR energy. Therefore, the partial compensation between enhancement (with respect to OMC) of the low-lying states and depression of quadrupole states shows a stability of our theoretical predictions.

The comparison of the present results with the microscopic calculation of ref.<sup>/9/</sup>, which uses basically identical nuclear structure information, shows an appreciable difference: the RMC rate obtained in the present phenomenological approach is by about 40% less. We would like to mention that preliminary results obtained within the modified impulse-approximation scheme<sup>/12/</sup> in the shell model, have shown a strong reduction of the photon yield bringing the phenomenological and microscopic calculations towards an agreement. This strengthens our confidence in the proper treatment of the nuclear effects in  $^{16}\text{O}$  and therefore, in the present predictions.

With the reservation that data for RMC on  $^{16}\text{O}$  have been reported<sup>/4/</sup> as preliminary ones, we can conclude that their comparison with the present calculation points towards a higher value of  $g_p$  ( $g_p \geq 15$ ) that is in the direction of the results of OMC partial transitions in  $A=12, 16, 28$  nuclei<sup>/13,14,15/</sup>. On the other hand, the analysis<sup>/3/</sup> suggests that  $g_p(\text{RMC on } ^{40}\text{Ca}) < g_p$  (other processes). It is therefore clear that further theoretical (more refined treatment of  $^{40}\text{Ca}$  spectrum beyond SU(4) or simple particle-hole model) and experimental (on  $^{16}\text{O}$ ) work is needed to clarify the contradictory situation of RMC.

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Христиллин П., Гмитро М.  
Радиационный  $\mu$ -захват на  $^{16}\text{O}$

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В рамках феноменологического подхода вычисляется спектр фотонов радиационного захвата мюонов ядром  $^{16}\text{O}$ . Предсказание для спектра получено с использованием реалистического спектра ядерных возбуждений, включающего низколежащие состояния конечного ядра. Предварительные экспериментальные данные не могут быть согласованы с результатами расчета, выполненного для канонического значения индуцированной константы псевдоскалярной связи  $g_p$ .

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Christillin P., Gmitro M.  
Radiative Muon Capture on  $^{16}\text{O}$

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Predictions of the photon spectrum of radiative muon capture on  $^{16}\text{O}$  are given within a phenomenological approach for a realistic nuclear excitation spectrum which includes low-lying excited states. Preliminary experimental results are totally at variance with the predictions based on the canonical value of the induced pseudoscalar coupling constant  $g_p$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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