объединенный
институт
ядерных
исследований
дубна

E4-84-340

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πd-SCATTERING LENGTHS IN VIEW OF THE PION AND NUCLEON MASS SPLITTINGS

Submitted to "ЯФ"

1984

INTRODUCTION

Isotopic invariance of strong interactions significantly simplifies both an experimental-data analysis and a theoretical consideration of hadron systems.

However, in experiments one has charged particles, as a rule.

Electromagnetic interactions between the particles break the isotopic invariance. Therefore, in many cases one has to handle with the so-called pure nuclear variables (phase-shifts, amplitudes, cross-sections, etc.) which are isotopically invariant and are related to the corresponding observables by different electromagnetic corrections [1].

Strictly speaking, "switching-off" the Coulomb forces is not sufficient to repair the invariance. It remains broken by the mass differences in charged multiplets.

This effect with the other electromagnetic corrections sometimes is taken into account by means of perturbation theory.

For example, such an account is used in the analysis of pion [2] and kaon [3] scattering from a nucleon.

In many-body problems one usually neglects the mass differences. However, the inclusion of it into consideration may produce qualitatively new results in some cases.

For example, as has been noted by Kolybasov and Kudryavtsev [4], the energy release of virtual πN-charge-exchange processes originated by the mass differences gets a small πd-scattering length imaginary part which is not concerned with the pion absorption.

As another example there are, may be, the investigations of the πd-scattering charge asymmetry in P_{33}-region. Their results strongly depend on a Δ-isobar mass splitting value and on whether account of the nucleon mass difference is taken or not [5-9].

Our paper is devoted to the influence of mass splittings. $\Delta_\pi = m_{\pi^\pm} - m_{\pi^0} \approx 4.6$ MeV and $\Delta_N = m_n - m_p = 1.3$ MeV on low-energy πd-scattering observables, namely, the scattering length and s-phase.

The values of Δ_π and Δ_N are comparable with the deuteron binding energy $|\epsilon_0| \approx 2.2$ MeV. Hence, one cannot a priori say how it is important to take account of them.

The Kolybasov and Kudryavtsev work was a first and unique one till now, which had included Δ_π and Δ_N into a low-energy πd-calculation. They exploited a nonrelativistic diagram technique and zero-range πN-forces. Thereby, their result may only be considered just as a first estimation.

Our calculation is based on a finite-rank approximation of a nuclear-subsystem Hamiltonian [10,11] and uses a more realistic πN-potential.

To check a quality of the approximation, we compare our results, in the case $\Delta_\pi = \Delta_N = 0$, with the corresponding solutions of the Faddeev equations.

In some limiting case we reproduce the result of Kolybasov and Kudryavtsev.

1. ISOTOPICALLY NONINVARIANT HAMILTONIAN

Consider a system of a pion and two nucleons. Let $H = K + W$ be its nonrelativistic Hamiltonian with pure nuclear interactions, where K describes the free motion, and W is a sum of all the two-body potentials.

If we assume the vacuum-energy as an origin of the energy scale, then

$$K = \sum_\theta K^\theta P(\theta), \tag{1}$$

$$K^\theta = \frac{k^2}{2\mu_\theta} + \frac{\kappa^2}{2n_\theta} + m_\theta, \tag{2}$$

where $\theta = (r_\pi, r_1, r_2)$ is a combination of the isospin third components of the particles, k and κ are Jacoby momenta of the relative $\pi(NN)$ and NN motions, μ_θ and n_θ are the corresponding reduced masses, m_θ is a sum of masses of three particles is the isotopic state $|\theta>$, $P(\theta)$ is an operator of projecting onto that state. Let $P(\eta)$ be an isotopic-space projector onto the state $|\eta> = |(t_\pi(t_1 t_2)t_{12})tr>$ which is an eigenstate of the total isospin $\vec{t} = \vec{t}_\pi + (\vec{t}_1 + \vec{t}_2)$, where t_π, t_1, t_2 are isospins of the particles.

Using the property of a projection operator $\sum_\eta P(\eta) = 1$ we write $K = \sum_{\eta'\eta} P(\eta')KP(\eta) = \sum_{\eta'\eta\theta} K^\theta P(\eta')P(\theta)P(\eta)$. Representing the product of the projectors in the form

$$P(\eta')P(\theta)P(\eta) = |\eta'><\eta'|\theta><\theta|\eta><\eta| = |\eta'> M^\theta_{\eta'\eta} <\eta|,$$

$$M^\theta_{\eta'\eta} = \sum_{r'_{12} r_{12}} C^{t'r'}_{t_\pi r_\pi t'_{12} r'_{12}} C^{t'_{12} r'_{12}}_{t_1 r_1 t_2 r_2} C^{t_{12} r_{12}}_{t_1 r_1 t_2 r_2} C^{tr}_{t_\pi r_\pi t_{12} r_{12}} \tag{3}$$

we derive

$$K = \sum_{\eta'\eta\theta} |\eta'> K^\theta M^\theta_{\eta'\eta} <\eta|. \tag{4}$$

From expression (3) of $M^\theta_{\eta'\eta}$ we see that the operator K is nondiagonal over the states $|\eta>$, i.e., it does not conserve the two-body t_{12} and total t isospins.

But as can be seen from the same formula (3), the third component of the total isospin is conserved. This reflects the following fact. Defining the free-motion Hamiltonian by (1) and (2), we demand its diagonality over $|\theta>$, i.e., the particle sort to be conserved. And the latter in particular, means the total charge (or r) conservation.

If we have $m_n = m_p$ and $m_{\pi^\pm} = m_{\pi^0}$, then $K^{\theta'} = K^\theta$. Further, using the equality $\sum_\theta M^\theta_{\eta'\eta} = \delta_{\eta'\eta}$, we obtain the free Hamiltonian (4) to be invariant in isotopic space.

So, the mass-splitting breaks isotopic invariance of the Hamiltonian H. In a sense this breaking may be called as a kinematical one because it is originated by the free Hamiltonian, and the interaction W, being a pure strong operator, is isotopically invariant.

2. CALCULATIONAL METHOD

To calculate the πd-elastic-scattering amplitude, we use approximate three-body equations, based on the finite-rank approximation of a nuclear Hamiltonian [10,11].

Let us extract, off the total Hamiltonian $H = K + W$, the terms H_0 and V which describe the free relative πd-motion and the sum of πN-potentials $V = V_1 + V_2$, respectively.

$$H = H_0 + V + H_A, \tag{5}$$

where the remainder H_A is the Hamiltonian of the NN-subsystem.

We need the following Green-functions:

$$G(z) = (z - H)^{-1}, \quad G_A(z) = (z - H_0 - H_A)^{-1}, \quad G_0(z) = (z - H_0)^{-1}.$$

The elastic scattering amplitude is an asymptotic-state average of the operator $T(z) = V + VG(z)V$ which obeys the equation

$$T = T^\circ + T^\circ G_0 H_A G_A T, \tag{6}$$

where T° is the amplitude of scattering from fixed centers.

$$T^{\circ} = V + VG_0T^{\circ}. \tag{7}$$

If in the spectral expansion of H_A we retain only the ground state $H_A \simeq \epsilon_0|\psi_0\rangle\langle\psi_0|$, then for the elastic amplitude $\langle \vec{k}',\psi_0|T|\vec{k},\psi_0\rangle$ we derive from (6) a one-dimensional (over the variable k) integral-equation [11]

$$\langle T\rangle = \langle T^{\circ}\rangle + \epsilon_0\langle T^{\circ}\rangle G_0(z)\,G_0(z-\epsilon_0)\langle T\rangle . \tag{8}$$

Here brackets $\langle\ \rangle$ denote the average over $|\psi_0\rangle$.

If we have in mind the fixed-scatterer approximation $\langle \vec{k}',\psi_0|T^{\circ}|\vec{k},\psi_0\rangle$ as a basic one, then eq.(8) gives us a possibility of taking approximately into account the nucleon motion inside the nucleus.

Let $\vec{r}$ be a relative NN-coordinate to be considered as a parameter, when one solves eq.(7).

Without performing a partial-wave decomposition of the state $|\vec{r}\rangle$, we, thereby, take into account all partial waves of the relative NN-motion.

Really, as is pointed out in ref. [12], the diagonality of the operator V over $|\vec{r}\rangle$ implies its diaginality with respect to variable r of the state $|rLM\rangle$ obtained by the partial-wave decomposition

$$|\vec{r}\rangle = \sqrt{4\pi}\sum_{LM}|rLM\rangle\, Y^{*}_{LM}(\hat{r}) . \tag{9}$$

However, it is not diagonal with respect to the angular momentum.

Hence, averaging T° over s-wave function ψ_0, we get the projection $L=0$ only for the initial and final states. Meanwhile, in intermediate states we have all values of L.

3. FREE GREEN FUNCTION

Doing the split (5) of the total Hamiltonian H, we include the three-particle mass sum into the term H_0. Then, according to (1) and (2), we obtain $H_0 = \sum_{\theta}|\theta\rangle(\frac{k^2}{2\mu_\theta} + m_\theta)\langle\theta|$.

By using diagonality of H_0 over the states $|\theta\rangle$, it is easy to invert the matrix $(z-H_0)$ and to wtite the Green function $G_0(z) = (z-H_0)^{-1}$ of a free relative πNN -motion in the form

$$G_0(z) = \sum_{\theta}|\theta\rangle G_0^{\theta}(z)\langle\theta|,\quad G_0^{\theta}(z) = (z - m_\theta - \frac{k^2}{2\mu_\theta})^{-1} .$$

After the transition to the isospin basis $|\eta\rangle$ the matrix G_0, as H_0, loses the diagonality

$$G_0(z) = \sum_{\eta'\eta\theta}|\eta'\rangle G_0^{\theta}(z)\, M^{\theta}_{\eta'\eta}\langle\eta| . \tag{10}$$

So, the mass differences in the particle isomultiplets get the noninvariance of G_0, and consequently, of the amplitudes T° and T with respect to rotations in the isospin space.

This leads to a difference among the lengths of scattering of π^{+}-, π°- and π^{-}-mesons from the deuteron. Moreover, there arises an imaginary part of the lengths in the case of the charged pion. The origin of that becomes obvious if one considers the Green function (10) at a physical value of z.

Having in mind that the zero-energy corresponds to the vacuum state, we write the πd-collision total energy as follows $z = E_k - |\epsilon_0| + m_0$, where E_k is the kinetic energy of relative πd-motion, $|\epsilon_0|$ is the deuteron binding energy, $m_0 = m_\pi + m_n + m_p$ is the sum of the masses in an incoming channel.

In a scattering length calculation $E_k = 0$, hence in eq.(7) we have G_0 in the form

$$G_0 = \sum_{\eta'\eta\theta}\frac{|\eta'\rangle M^{\theta}_{\eta'\eta}\langle\eta|}{m_0 - m_\theta - |\epsilon_0| - \frac{k^2}{2\mu_\theta} + i0} .$$

If in the incoming channel we have π^{+}- or π^{-}-meson, then for θ, corresponding to an intermediate charge exchange state $\pi^{\circ}pp$ or $\pi^{\circ}nn$, we obtain $m_0 - m_\theta - |\epsilon_0| > 0$. Therefore, in such a situation, function G_0 has a pole which, in view of the formula $(x+i0)^{-1} = -i\pi\delta(x) + P\cdot 1/x$, gets an imaginary part of the amplitude T°.

4. POTENTIALS

Being a three-body problem, low-energy πd-scattering is well studied on the basis of exact three-particle equations [13]. This gives us an opportunity to use results of exact calculations to check quality of our approximation.

Also, as in paper [14] we assume the πN-interaction being described by a separable s-wave potential acting in the channels S_{11} and S_{31} of the form

$$v_\mu(k',k) = \lambda_\mu/(k'^2+\beta_\mu^2)/(k^2+\beta_\mu^2), \tag{11}$$

where μ denotes the πN-isospin states 1/2 and 3/2.

In ref. [14] on the basis of the Faddeev equations a sensitivity is studied of the πd-scattering length to variations of the depth λ_μ and range β_μ of the πN-potential. And the parameters λ_μ and β_μ suffer the condition to reproduce experimental πN-scattering lengths. In this calculation four different sets of πN-lengths [15-18] are used.

NN-interaction is described in ref.[14] by the s-wave potential which in our case corresponds to averaging of the fixed-scatterer amplitude $<\vec{k}', \vec{r}'|T^\circ|\vec{k}, \vec{r}> = \delta(\vec{r}' - \vec{r}) <\vec{k}'|T^\circ(\vec{r})|\vec{k}>$ over the Hulthen wave function $<\vec{k}', \psi_0|T^\circ|\vec{k}, \psi_0> = \int d\vec{r} |\psi_0(\vec{r})|^2 <\vec{k}'|T^\circ(\vec{r})|\vec{k}>$.

Therefore, comparing the πd-lengths calculated for the equal masses $m_{\pi^\pm} = m_{\pi^0}$ and $m_n = m_p$ with the corresponding result of ref.[14] we can estimate a deviation of our calculation from the exact one.

Further, we recalculate the πN-potential parameters, using in their relation to experimental πN-lengths the experimental masses ($m_{\pi^\pm} \neq m_{\pi^0}$, $m_n \neq m_p$). Such a recalculation takes into account, to an extent, the mass splitting influence on the two-body potential.

5. NUMERICAL RESULTS

When the Faddeev equations are solved with the s-wave potentials, only s-component is retained of the partial wave expansion of the relative NN-motion. Thereby, to compare our results for $\Delta_\pi = \Delta_N = 0$ with the corresponding solutions of exact equations, we carry out an additional calculation, retaining in decomposition (9) only the first term (L = 0).

The method of solving eq.(7) in such an approximation is in detail described in ref. [12].

The πd-scattering length a as a function of the πN-potential range-parameter β with the fixed πN-lengths[15]

$$a_{1/2} = -0.257 \text{ fm}, \quad a_{3/2} = 0.154 \text{ fm} \qquad (12)$$

is compared in Table 1 with the result a^F of paper[14].

Table 1

$\beta_{1/2} = \beta_{3/2}$ (fm^{-1})	a^F (fm)	a (fm)	$\left\lvert\frac{a^F - a}{a^F}\right\rvert \cdot 100\%$
0.1	0.0250	0.0385	54
0.3	0.0430	0.0457	6
0.5	0.0509	0.0517	2
1.0	0.0614	0.0625	2
3.0	0.0736	0.0795	8
5.0	0.0762	0.0829	9
10.0	0.0776	0.0785	2

A rather significant deviation of our value from the exact one at a small β, may be explained by the importance, in such a situation (long range $1/\beta$ of the πN-potential), of the NN-continuum states which are omitted ($H_A = \epsilon_0 |\psi_0><\psi_0|$).

The length a represented in Table 1 is a solution of eq.(8). For all β its difference from the fixed-scatterer approximation (7) is found to be less than 2%.

Comparison of our result for a with a^F shows that for $\Delta_\pi \neq 0$ and $\Delta_N \neq 0$ we may also hope to obtain (with $\beta \sim 1$ fm^{-1}) an accuracy of the calculation not worse than 10%. Equation (7) has, for separable πN-potentials, an analytical solution to be obtained without the use of expansion (9), i.e., allowing for all the values of L.

Further, all results are obtained by solving equations (7), (8) without this expansion.

Taking the experimental masses of particles, we explore the dependence of the πd-lengths 1) on the parameter β with the fixed πN-lengths (12) (Table 2); 2) on the used πN-lengths with $\beta_{1/2} = \beta_{3/2} = 3$ fm^{-1} (Table 3).

Table 2

β (fm^{-1})	$a(\pi d) \cdot 10^5$ (fm)	$a(\pi^- d) \cdot 10^5$ (fm)	$a(\pi^0 d) \cdot 10^5$ (fm)	$a(\pi^+ d) \cdot 10^5$ (fm)
0.1	4022	3847 - i2.08	3850	3849 - i3.09
0.3	4688	4505 - i5.17	4539	4499 - i22.3
0.5	5194	5000 - i5.64	5070	4981 - i29.5
1.0	5965	5749 - i5.81	5889	5708 - i33.5
3.0	6587	6311 - i5.92	6573	6240 - i35.1
5.0	6323	5996 - i6.04	6303	5913 - i35.9
10.0	5174	4709 - i6.36	5072	4614 - i37.9

The second column of Table 2 and the first of Table 3 contain the corresponding results $a(\pi d)$ in the degenerated case $\Delta_\pi = \Delta_N = 0$.

The first line of Table 3 corresponds to the πN-lengths (12), and next three to the sets (-0.257, 0.126) fm[16], (-0.264, 0.148) fm[17] and (-0.240, 0.130) fm[18].

One sees that taking account of the mass differences decreases real parts of the $\pi^\pm d$-lengths by about 0.003 fm and induces imaginary parts of them. The imaginary parts are weakly sensitive to πN-potential-parameter variations.

Table 3

πN	$a(\pi d)\cdot 10^5$ (fm)	$a(\pi^- d)\cdot 10^5$ (fm)	$a(\pi^\circ d)\cdot 10$ (fm)	$a(\pi^+ d)\cdot 10^5$ (fm)
/16/	6587	6312 - i5.92	6573	6240 - i35.1
/17/	2318	2046 - i5.22	2275	1982 - i30.9
/18/	5292	5008 - i5.97	5271	4936 - i35.6
/19/	3899	3649 - i4.85	3864	3591 - i28.8

Table 4

E_k (MeV)	$\delta(\pi d)\cdot 10^3$	$\delta(\pi^- d)\cdot 10^3$	$\delta(\pi^\circ d)\cdot 10^3$	$\delta(\pi^+ d)\cdot 10^3$
0.3	-168	-161	-167	-159
0.6	-237	-227	-236	-224
0.9	-289	-277	-287	-273
1.2	-332	-318	-330	-315
1.5	-369	-353	-366	-350
1.8	-403	-386	-400	-382
2.1	-434	-415	-430	-411

The inequality $|Im\, a(\pi^- d)| < |Im\, a(\pi^+ d)|$ is due to a larger value of the energy release (~5.9 MeV) of the process $\pi^+ n \to pp$ as compared with that (~3.3 MeV) of the charge exchange $\pi^- p \to \pi^\circ n$.

Fixing the πN-lengths used in ref./4/ and simulating a zero-range πN-potential ($\beta \to \infty$), we obtain at $\beta = 25$ fm^{-1} the value of πd-lengths (0.041 - i0.00036) fm, which is close to the result (0.047 - i0.0004) fm of the mentioned paper.

Real parts of our πd-lengths are in a sarisfactory agreement with the measured value of it/19/ $Re\, a(\pi^- d) = 0.073^{+0.031}_{-0.024}$ fm.

The imaginary part originated by the charge exchange processes is very small as compared with the theoretical estimations of the one connected with the pion absorption (-0.007 ± 0.002) fm/18/.

Therefore, exploring the pion absorption processes one can, with a sufficient accuracy, neglect the isomultiplets mass splittings and the isotopic invariance violation.

Table 4 contains the elastic s-wave phase shifts of πd-scattering (in degrees) as functions of the kinetic energy E_k of relative πd-motion. The calculation of them is performed with $\beta_{1/2} = \beta_{3/2} = 3$ fm^{-1} and πN-lengths (12).

The second column of the table displays the phase shifts $\delta(\pi d)$ calculated with $\Delta_\pi = \Delta_N = 0$.

Taking into consideration of the mass differences leads to the splitting of the πd-phase-shifts.

The presence of opened channels $\pi^- d \to \pi^\circ nn$ and $\pi^+ d \to \pi^\circ pp$ decreases the elastic scattering cross-section as compared with the equal-mass case, and consequently, causes the inequality $|\delta(\pi d)| < |\delta(\pi^- d)| < |\delta(\pi^+ d)|$.

At a first sight such a negligible contribution of the mass differences to the low-energy πd-observables seems to be surprising, especially in view of the energy release exceeding the deuteron break-up threshold by factor 1.5 for the negative pion and by more than twice for the positive one.

But scrutinizing the on-shell charge-exchange processes one sees that the Pauli principle and the parity conservation law demand the transition of the particles into P-state of both the relative NN and π(NN) motions.

The overlap of such a πNN-state with an asymptotic one for the elastic s-wave πd-scattering is small. Thereby, virtual charge-exchange processes put a little contribution to the s-wave scattering.

In conclusion we thank Prof. V.B.Belyaev and Prof. A.L.Zubarev for stimulating discussions and interest in our work.

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Received by Publishing Department
on May 18, 1984.

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Пупышев В.В., Ракитянский С.А. E4-84-340
Длины πd -рассеяния с учетом разностей пионных и нуклонных масс

Длины и s-фазы упругого πd -рассеяния вычислены в рамках изотопически неинвариантного подхода, учитывающего расщепление пионного и нуклонного изомультиплетов по массам. Показано, что разность масс частиц приводит к возникновению мнимых частей у длин πd -рассеяния /$\sim 10^{-4}$ фм/, не связанных с поглощением пионов. Исследована чувствительность эффектов разностей масс к вариациям параметров πN -потенциала, которая оказалась слабой.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Pupyshev V.V., Rakityansky S.A. E4-84-340
πd -Scattering Lengths in View of the Pion and Nucleon Mass Splittings

The πd -scattering lengths and s-wave phase-shifts are calculated in the framework of an isotopically noninvariant approach allowing for the pion- and nucleon-isomultiplet mass splittings. It is shown that the mass differences of particles get imaginary parts ($\sim 10^{-4}$ fm) of πd -scattering lengths without inclusion of the pion absorption. The mass splitting influence on the lengths and s-phases is found to be weakly dependent on the πN-potential parameter variations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984