

# Oбъ МНСТИTYT лдвриых <br> исследований <br> дубна 



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THE LOW-LYING DIPOLE STATES
IN SPHERICAL. NUCLEI

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## INTRODUCTION

The elastic scattering of photons by $\mathrm{Sm}, \mathrm{Ba}, \mathrm{Ce}$, and Nd isotopes has been investigated in the experiments reported in refs. $/ 1-3 /$. The angular distribution and linear polarization measurements enable one to identify some low-1ying $1^{-1}$ levels in these nuclei. Partial widths for the E1 transitions from these states to the ground state have been measured. The main feature of the observed $1^{-}$states is that their energies are slightly below the sum of the excitation energies of the lowest $2 \dagger$ and $3_{1}^{+}$states, and naturally one would try to give a theoretical interpretation of these $1^{-}$states as two-phonon excitations $\left[2^{+} * 3_{1}^{-} 1^{-}\right.$. As one can see from the experimental data (3), the reduced dipole transition probabilities $\mathrm{B}\left(\mathbb{E} 1 ; 1^{-} \rightarrow 0_{\mathrm{g} . \mathrm{s} .}^{+}\right)$have a specific dependence on the neutron number N of the stable even Sm isotopes. As $N$ approaches the magic number $N=82$, the $B(E 1)$ values increase and reach a maximum, for $N>82$ they decrease and reach a minimum at $N=86$. If one proceeds further to the deformed region ( $N>86$ ), the $B(E 1)$-values will increase again. At $N=80$ an experimental $B(E 1)$ value is available for ${ }^{136} \mathrm{Ba}$. This value is close to the experimental $\mathrm{B}(\mathrm{E} 1)$ value for ${ }^{144} \mathrm{Nd}(\mathrm{N}=84)$, so one can see the trend for the dependence of the $B(E 1)$ values on $N$ near the magic number $N=82$. In this work an attempt is made to give an explanation for such a behaviour of the reduced E1 transition probabilities.

## 1. CALCULATIONAL details

The dipole excitations in spherical nuclei have been studied within the quasiparticle-phonon nuclear model (QPM) $/ 4 /$ in ref $/ 5 /$. The QPM Hamiltonian includes the Saxon-Woods potential as the average field, pairing interaction, multipole and spin-multipole forces. A detailed description of the Hamiltonian and its parameters is given in ref./4/. In ref/5/ the calculation has been performed with the wave function

where $|0\rangle$ is the phonon vacuum and $Q_{\lambda \mu i}^{+}$is the phonon creation operator which can be written in terms of the quasiparticle creation $a_{j m}^{+}$and annihilation $a_{j m}$ operators:

$$
\begin{equation*}
Q_{\lambda \mu \mathrm{i}}^{+}=\frac{1}{2} \sum_{\mathrm{j}_{1} \mathrm{j}_{2}}\left\{\psi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}\left[a_{\mathrm{j}_{1} \mathrm{~m}_{1}}^{+} a_{\mathrm{j}_{2} \mathrm{~m}_{2}}^{+}\right]_{\lambda_{\mu}}+(-)^{\lambda-\mu+1} \phi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}\left[a_{\mathrm{j}_{2} \mathrm{~m}_{2}} a_{\mathrm{j}_{1} \mathrm{~m} 1}\right]_{\lambda-\mu}\right\} . \tag{2}
\end{equation*}
$$

The phonon amplitudes $\psi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}$ and $\phi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}$ are found from the secular equation in the random-phase approximation (RPA). The RPA equations for the QPM Hamiltonian are given, for example, in ref. ${ }^{\text {/4/. Since the RPA phonons are not ideal bosons, in the }}$ two-phonon components of the wave function (1) there are terms violating the Pauli principle. To take into account the Pauli principle in the calculation of matrix elements, one needs to calculate correctly the commutation relations between phonon operators. The QPM equations with the inclusion of the Pauli principle are given in ref. 6/ . In this case the wave function (1) is normalized as follows


This normalization condition corresponds to the diagonal approximation for $k J$ function, a more general case is considered in ref. $/ 6 /$. For the states prohibited by the Pauli principle $\mathrm{KJ}=$ $=-2$ and they are eliminated automatically from the normalization relation and equations for energy and $R_{i}$ and $P_{i}$ coefficients. The influence of the Pauli principle on the two-phonon components of the low-lying states in spherical nuclei has been investigated in ref. $/ 7 \%$. It has been shown that in most cases considered the corrections are small, especially for octupole states. For two-phonon states with negative parity the corrections due to the Pauli principle are expected to be not important, because these states consist of different quasiparticles and the violation of the Pauli principle is less probable. The RPA calculations ${ }^{5}$ ? show that the lowest $1^{--}$states in heavy spherical nuclei are at the energies ranging from 7 to 8 MeV . The inclusion of anharmonic effects, i.e., the coupling between the one-phonon and two-phonon states, leads to some shift in excitation energies for $1^{-}$states with El-transitions via onephonon components of the wave function (1). Nevertheless, the energies of these states are still higher by $1.0-1.5 \mathrm{MeV}$ than the energies of $1^{-}$states observed in refs. $1-3 /$. This is a further support for the interpretation of the low-lying $1^{-}$states
as two-phonon excitations. To investigate the structure of the lowest $1^{-}$states, we have made the calculations with the wave function (1), including the Pauli principle corrections, for Sm isotopes. The results show that the lowest dipole states practically are the two-phonon $\left[2+\otimes 3_{1}\right]$ states. In ${ }^{142} \mathrm{Sm}$, for example, the contribution of $[2+3-1]$ components to normalization of the wave function (1) is more than $99 \%$ for the lowest dipole state. The energy of this state is lower than the sum of the energies of $2_{1}^{+}$and $3-1$ levels only by 75 keV . Thus, one may conclude that the one-phonon states and two-phonon components $\left[2_{i}^{+} \otimes 3_{j}^{-}\right]$with $i, j \neq 1$ have nearly no influence on the structure of the lowest dipole states. The corrections caused by the Pauli principle are negligible. Therefore, in our calculations of the E1-transition probabilities we only consider the transitions via two-phonon components of the wave function (1). In this case one gets the probability of Ed transition from a state with moment $\lambda$ and parity $\pi$ to the ground state in the form:


where the sum over, is the sum over single-particle indices of the neutron ( $n$ ) and proton ( $p$ ) states, $\hat{j}_{j_{1}}^{\lambda_{2}}$, are the reduced matrix elements of the multipole operators, ${v_{i_{1}} j_{2}}^{(-)}=\mathbf{u}_{j_{1}} \quad u_{j_{2}}-v_{j_{1}} \quad v_{j_{2}}$ is the combination of Bogolubov's-transformation coefficients. For the dipole transitions considered here we take $d_{n}=-Z /(A) e$, $e_{p}^{(I)}=N /(A) e$, where $e$ is the elementary charge.

In the calculation we consider the lowest $1^{-1}$ states only as the two-phonon ones of $\left[2_{1}^{+} * 3_{1}\right]_{1^{--}}$structure. It should be noted that among the components of the wave function (1) there are the so-called generalized neutron particle-hole states introduced in ref. $/ 8 /$ to describe the El-decay of the isobar-analog resonances excited via inelastic proton scattering $/ 9 /$. However, such states, as shown above, do not influence the structure of the lowest $1^{-}$levels.

The calculations have been performed with the modified RPAS code reported in ref. $/ 10 /$, where a detailed description of the procedure of solving the RPA equations is given. We have used the same parameters for the Saxon-Woods potential and pairing
constants as in ref. $11 \%$. To calculate the phonon amplitudes in the. QPM, one has to choose the isoscalar and isovector constants for the multipole-multipole interaction/4/. The set of constants has been chosen so as to fit the calculated reduced probabilities $B(E 2)$ and $B(E 3)$ with the experimental data for $2_{1}^{+}$and $3_{1}^{-1}$ levels, respectively. Under such a choice of constants the RPA- energies for $2^{+}$and $3^{-}$levels are higher than their experimental values, but the calculations with the wave function (1) enable one to describe simultaneously either the energies or the transition probabilities. We have used the experimental data compiled in refs. 12,13 / for the energies and transition probabilities.

## 2. RESULTS OF CALCULATIONS

Let us discuss the results of our calculations. The calculated reduced probabilities $B(E 1)$ and experimental data for Sm isotopes are shown in fig.1. The $B(E 1)$ values are expressed in Weisskopf's single-particle units (s.p.u.)/14/. As one can see from fig.l, there is a typical dependence of $B(E 1)$ values on the neutron number $N$. The $B(E 1)$ value reaches a maximum at $\mathrm{N}=82$, decreases as one moves towards $\mathrm{N}=84$, and increases again at $N=86$. To give an explanation of this behaviour of $B$ (E1) values, we have made some analysis with the expression (4) in the case of Sm isotopes. The relative contributions to (4) from the sums over proton ( $\mathrm{S}_{\mathrm{Z}}$ ) and neutron ( $\mathrm{S}_{\mathrm{N}}$ ) indices are shown in fig.2. As one can see, the sum $\mathrm{S}_{\mathrm{Z}}+\mathrm{S}_{\mathrm{N}}$ has the maximum at the magic neutron number $\mathrm{N}=82$. A qualitative explanation is as follows: at $N=82$ the neutron pairing is absent and $v_{j_{1}}^{(-)} j_{2}=0$ for $j_{1}$ and $j_{2}$ levels located in the opposite sides from the Fermi surface, i.e., for particle-hole couples. And the number of terms with nonzero contributions to $S_{N}$ is much decreased, so $S_{N}$ is substantially less than $S_{Z}$. At $N=80,84$ there are pairing forces between neutrons, and both the neutron and proton states contribute to (4) but with opposite signs. Therefore, even though the $\mathrm{S}_{\mathrm{Z}}$ value increases with increasing collectivity of $2_{1}^{+}$and $3_{1}^{-}$states as one moves from the magic nucleus ${ }^{144} \mathrm{Sm}$ to nonmagic ${ }^{142,146} \mathrm{Sm}$, after the subtraction of $S_{N}$ the $B(E I)$ value decreases. The $2_{1}^{+}$and $3_{1}^{-1}$ levels in ${ }^{148} \mathrm{Sm}$ are strongly collective, so the number of components in the sum (4)

* is greater, and this leads to the increase in the absolute value of $S_{N}$ and $S_{Z}$. Since the absolute value of the effective charge for proton is greater than for neutron, the $\mathrm{S}_{\mathrm{Z}}$ value is dominating over $S_{N}$ and the $B(E 1)$ value increases. It should be noted that the RPA is not much reliable for ${ }^{148} \mathrm{Sm}{ }^{/ 15}$, and only a qualitative trend for the $B(E 1)$ value can be given.


Fig. 1. Dependence of $B(E 1)$ values on the neutron number $N$ for Sm isotopes.


Fig. 2. Dependence of $S_{N}$ and $S_{Z}$ on the neutron number $N$ for Sm isotopes.

We have also made the calculations for the chains of $\mathrm{Nd}, \mathrm{Ba}$, and Ce isotopes. The results of our calculations and the experimental data/1-3/ are shown in fig.3. One can see that the $B$ (E1) values for these isotopes have the same dependence on the neutron number as for the Sm isotopes. Our calculations have shown that this dependence of the $B(E 1)$ value on $N$ is caused by the same effects as in the case of Sm isotopes. Thus, in the region $N=82$ for various chains of isotepes there is a universal mechanism leading to a specific behaviour of the $B(E 1)$ value depending on $N$. It is natural to suppose a similar situation for nuclei with neutron numbers close to another magic number $\mathrm{N}=50$. We have calculated the $\mathrm{B}(\mathrm{E} 1)$ values for the chain of Zr isotopes with $48 \leq \mathrm{N} \leq 58$ and the results are shown in fig.4. As one can see from this figure the dependence of $B$ (E1) values on $N$ in the region $N=50$ is the same as in the region $N=82$. The calculation of $S_{Z}$ and $S_{N}$ has shown that the same explanation can be given for the behaviour of $B$ (E1) values for Zr isotopes as in the case for Sm isotopes. Note, that this dependence of $B(E 1)$ values on $N$ is correlated with the behaviour of the energies for $2+$ levels of all nuclei studied in this work. Thus, the probabilities of E1 transitions to two-phonon states have a specific dependence on the neutron numbers near the magic ones. Here one would ask the question: Will the dependence of $B(E 1)$ values for nuclei with proton numbers close to a magic number be the same? To study this effect, we have calculated the probabilities of E1 transitions to two-phonon states $\left[2_{1}^{+} \otimes 3_{1}^{-}\right] 1_{1}^{-}$for isotones with $N=32$, namely, for nuclei ${ }^{58} \mathrm{Fe},{ }^{60} \mathrm{Ni}$, and ${ }^{62} \mathrm{Zn}$. The results of calcula-

tion are shown in fig.5. As one can see, the $B(E 1)$ values for isotones with proton numbers close to the magic number $Z=28$ have qualitatively the same behaviour as for isotopes with $Z=40,56-82$. Such a behaviour is caused by pairing effects leading to the correlation between the proton and neutron contributions to (4) and the increase of collectivity of the lowest $2{ }_{1}^{+}$and 3 - levels as the nucleon number changes from a magic number to a nonmagic one. Thus, the probabilities of E1 transitions to two-phonon states have a specific dependence on the nucleon number in a near-magic region. Note, that this conclusion may be not so valid for nuclei whose structures cannot be described within the RPA.

Table
Experimental and theoretical $\mathrm{B}\left(\mathrm{E} 1 ; \mathrm{I}^{-} \rightarrow \mathrm{O}_{\mathrm{g} . \mathrm{s.}_{-}^{+}}^{+}\right)$ values (in single-particle units.) for $1^{-}$levelis

| Nucleus | Experimental Data |  | Calculation |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{E}_{1^{-}}{ }^{-}, \mathrm{MeV}$ | $\begin{gathered} 10^{3} \times \mathrm{B}(\mathrm{E} 1) \\ \text { s.p.u. } \end{gathered}$ | $\mathrm{E}_{1^{-}}, \mathrm{MeV}$ | $\begin{gathered} 10^{3} \times \mathrm{B}(\mathrm{E} 1 \\ \text { s.p.u. } \end{gathered}$ |
| 58 Fe |  |  | 5.8 | 0.1 |
| ${ }^{60} \mathrm{Ni}$ |  |  | 6.1 | 28.9 |
| ${ }^{62} \mathrm{zn}$ |  |  | 6.0 | 16.8 |
| 88 zr |  |  | 3.7 | 0.002 |
| 90 zr |  |  | 5.2 | 3.5 |
| ${ }^{92} \mathrm{Zr}$ |  |  | 3.5 | 0.03 |
| $9^{4} \mathrm{Zr}$ |  |  | 3.5 | 0.02 |
| 96 Zr |  |  | 3.9 | 0.7 |
| $98_{\text {Zr }}$ |  |  | 3.9 | 2.3 |
| 136 Ba | 3.044 | 0.4 | 4.0 | 1.4 |
| ${ }^{138} \mathrm{Ba}$ | 4.027 | $3.0 \pm 0.3$ | 4.4 | 2.9 |
| ${ }^{140} \mathrm{Ba}$ |  |  | 4.0 | 2.0 |
| ${ }^{138} \mathrm{Ce}$ |  |  | 3.9 | 1.3 |
| ${ }^{140} \mathrm{Ce}$ | 3.644 | $3.5 \pm 0.5$ | 4.1 | 4.8 |
| 142 Ce |  |  | 3.7 | 2.2 |
| 140 Nd |  |  | 3.4 | 2.4 |
| 142 Nd | 3.426 | 5.0 00.5 | 3.9 | 5.9 |
| 144 Nd | 2.186 | $1.2 \pm 0.2$ | 3.5 | 1.8 |
| 146 Nd |  |  | 3.0 | 3.7 |
| 142 Sm |  |  | 3.5 | 1.1 |
| ${ }^{144} \mathrm{Sm}$ | 3.225 | $3.5 \pm 0.5$ | 4.3 | 3.1 |
| ${ }^{146} \mathrm{Sm}$ |  |  | 3.4 | 1.3 |
| $148{ }^{\text {Sm }}$ | 1.465 | $0.5 \pm 0.1$ | 3.0 | 2.5 |

The results of our calculations and available experimental data are given in the table. As is seen from the table, the cal-
culations fit rather well the experimental data and give a correct description of the behaviour of $B(E 1)$ values for nuclei in the region $N=82$. In our simple approximation the calculated energies for dipole states are higher than the experimental values. Besides, for a ${ }^{136} \mathrm{Ba}$ isotope there are three near-lying $1^{-}$levels observed in experiment ${ }^{/ 3 /}$ with the total probability $\Sigma B(E 1)=2.1 \times 10^{-3} \mathrm{~s} . \mathrm{p} . \mathrm{u}$. Our calculations for three lowest $1^{-}$ states $\left[2_{1}^{+} 3_{1}^{-}\right]_{1-}^{-}$give $\Sigma B(E 1)=2.4 \times 10^{-3} \mathrm{~s} . \mathrm{p} . \mathrm{u}$. and there are also $\left[2_{2}^{+} \otimes 8_{1,2}^{-}\right] 1^{-}$states with very small transition probabilities. This splitting of $1^{-\quad}$ levels and their energies can be described by including into the wave function (1) threephonon components, as has been done/16/ in the calculation of energies for the two-phonon quintet $1^{--5}$ levels in some of nuclei considered here. The excitation of $1^{-}, 4^{-}, 5^{-}$states by inelastic proton scattering on ${ }^{144} \mathrm{Sm} / 17$ / also confirms that the discussed $1^{-}$levels for nuclei in the region $N=82$ belong to the two-phonon quintet. The experimental fact/2/ that the lowest $1^{-1}$ level in ${ }^{138} \mathrm{Ba}$ is excited very weakly in the ( $\mathrm{d}, \mathrm{p}$ ) reaction indicates that one-phonon components in the wave function of this state are very small.

It should be emphasized that the studied El transitions show clearly the difference between the Tamm-Dancoff approximation (TDA) and the RPA. In the TDA $\phi$-amplitudes are equal to zero and the transitions via two-phonon components are prohibited. Therefore, the experimental observation of the discussed above dependence of the $B(E 1)$ value on the neutron number in the magic region shows clearly an important role of the RPA correlations.

## CONCLUSION

In this work we have shown that for El transitions to the low-lying dipole two-phonon states of spherical nuclei with nucleon numbers in the magic region there is a specific dependence of the $B(E I)$ value. From the experimental data available by now and the results of our calculations one can draw the conclusion that the wave functions for the observed $1^{-}$levels of nuclei in the region $N=82$ are mainly consisting of the twophonon $\left[2_{1}^{+} \otimes 3-1 l^{-}\right.$components. The two-phonon nature of these levels would be strongly confirmed by the measurement of $B(E 1)$ values for all chains of isotopes with $N=80,82,84$ and the discovery of the characteristic dependence discussed in this work. The investigations of the low-lying dipole states in other spherical nuclei may also be of interest from our point of view.

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## REFERENCES

1. Metzger F.R. Phys.Rev., 1976, C14, p. 543.
2. Swann C.P. Phys.Rev., 1977, C15, p. 1967.
3. Metzger F.R. Phys.Rev., 1978, C18, p. 2138
4. Soloviev V.G. Particles and Nuclei, 1978, v.9, p. 580; Nucleonika, 1978, v. 23, p. 1149. Malov L.A., Soloviev V.G. Particles and Nuclei, 1980, v.11, p. 301.

Vdovin A.I., Soloviev V.G. Particles and Nuclei, 1983, v. 14 p. 237.

Voronov V.V., Soloviev V.G. Particles and Nuclei, 1983, v.14, p. 1381.
5. Soloviev V.G., Stoyanov Ch., Voronov V.V. Nuc1.Phys., 1978 A304, p. 503; 1983, A399, p. 141.
6. Voronov V.V., Soloviev V.G. TMF, 1983, v. 57, p. 75.
7. Soloviev V.G., Stoyanov Ch., Nikolaeva R. Izv.Akad. Nauk SSSR (ser.fiz.), 1983, v. 47, p. 2082.
8. Wurm J.P. et al. Nucl.Phys., 1969, A128, p. 433.
9. Von Brentano P., Phys.Lett., 1968, v. 26 B, p. 448 Williams N. et al. Phys.Rev., 1970, C2, p. 1539. Mudersbach K. et al. Nuc1. Phys., 1970, A146, p. 477.
10. Stoyanov Ch., Yudin I.P. JINR, P4-11076, Dubna. 1977.
11. Ponomarev V.Yu. et a1. Nuc1. Phys., 1979, A323, p. 446.
12. Christy A., Häusser 0. Nuc1.Data Tab., 1972, v. 11, p. 281
13. Vdovin A.I., Stoyanov Ch. Izv.Akad. Nauk SSSR (ser.fiz.), 1974, v. 38, p. 2598.
14. Blatt J.M., Weisskopf V.F. Theoretical Nuclear Physics, 1952, Wiley, New York, chap. 12.
15. Soloviev V.G., Stoyanova O., Stoyanov Ch. Izv. Akad.Nauk SSSR (ser.fiz.), 1980, v. 44, p. 1938.
16. Vogel P., Kocbach L. Nuc1. Phys., 1971, A176, p. 33.
17. Martin R. et al. Nuc1. Phys., 1973, A210, p. 221.

Дао Тиен Кхоа, Лономарев В.Ю., Воронов В.В.
E4-84-27 Низколежащие дипольные состояния сферических ядер

В рамках квазичастично-фононной модели ядра рассчитаны вероятности Е1 - переходов на нияколежание дипольные состояния ряда сферических ядер, наблодавимеся при упругом рассеянии у квантов. Объяснена специфическая зависимость $\mathrm{B}(\mathrm{E} 1)$ величин от числа нейтронов в этих ядрах. Показано, что эти низколежащие дипольные состояния могут быть проинтерпретированы как двухфононные возбуждения, построеные ия первого квадрупольного и октупольного фононов. Е1 -переходы на этн состояния указы вают на принциииальнее раяличие мехду прибликением ТаммаДанкова и приближением стучайных фаз.

Работа виполнена в Лаборатории теоретическол физики ОИЯИ.

## Dao Tien Khoa, Ponomarev V. Yu., Vorontw V.V.

 The Low-Lying Dipole States in Spherical NucleiThe low-lying dipole states observed in elastic photon scattering from some spherical nuclei have been investigated within the quasiparticle-phonon nuclear model. The explanation is given for a specific dependence of $B(E 1)$ values on the neutron number for these nuclei. It has been shown that the low-lying $1^{-}$-states can be interpreted as two-phonon $2 \dagger \otimes 3 \dagger$ excitations. The E1 transitions from these states to the ground states clearly indicate a principal difference between the RPA and TDA.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

