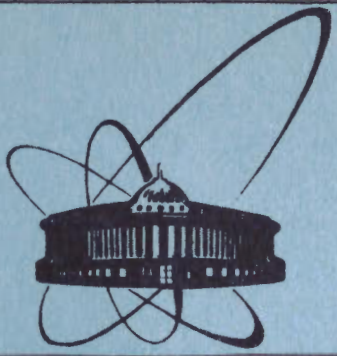

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E4-84-264

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THE SECOND ORDER OPTICAL POTENTIAL FOR πd SYSTEM AT THE THRESHOLD

Submitted to
"Czechoslovak Journal of Physics"

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1984

1. The low energy pion nucleus interaction and the properties of pion-mesic atoms are usually described in the framework of the optical potential model [1,2]. However, the application of the first order Kisslinger type optical potential to the light nuclei in low energy range leads to considerable difficulties. Indeed, the nonlocal term arising from the p-wave πN interaction becomes dominating [3], while the calculations of the pion-light nuclei scattering lengths (d, ^{3}He, ^{4}He) with the help of the various iteration schemes (the Watson multiple scattering theory [4], the non-relativistic Feynman diagram technique [5] and the unitary iteration scheme [6]) show the dominant role of the s-wave πN-interaction in the low energy region. Further, in the Faddeev type calculations of the pion deuteron scattering length $a_{\pi d}$ [7] the p-wave contribution does not exceed 10 percent. Therefore, it is important to study the role of the second order optical potential in the dynamics of the pion-light nuclei systems, which is the aim of the present paper. As an important application of the second order potential we shall consider the properties of π^-d atom. The pion wave function of this system, distorted by the strong interaction, is the input for the calculations of the following important reactions [3,8-10]:

$$\pi^- + d \to 2n + \gamma, \quad \pi^- + d \to 2n, \quad \pi^- + d \to 2n + \pi^0 .$$

The paper is organized as follows. In section 2, we present the general expression for the first and second order optical potentials. In section 3, we discuss the main approximations we use in the practical calculations; here also the local approximation to πd optical potential is given. In section 4, some aspects of numerical technique are briefly described and results for $a_{\pi d}$ and bound 1s-state wave function are presented.

2. In this section we give for completeness a brief derivation of the general formulae for the first and second order pion nuclear optical potential using the Watson version [11] of the optical model, referring for details to [1,12-15]. Unlike in the case of the multiple-scattering formalism of Kerman, McManus and Thaler [12], the pion-nucleus scattering matrix in the Watson theory obeys the standard Lippman-Schwinger equation. The optical potential, therefore, could be simply added to the Coulomb potential in the Schrödinger equation for the pion nucleus system.

The exact equation for the optical potential U(E) reads:

$$U(E) = A\tau(E) + (A-1)\tau(E)\,G(E)\,\hat{Q}\,U(E), \tag{1}$$

where $\hat{Q} = \sum_{n=0} |n\rangle\langle n|$ is projection operator on the excited nuclear states, $G(E) = (E - K_\pi - H_A + i\epsilon)^{-1}$ is the Green function, K_π is the pion kinetic energy operator and H_A is the nuclear Hamiltonian.

The many-body operator $\tau(E)$ for pion scattering on a bound nucleon is defined by:

$$\tau(E) = v + vG(E)\,\hat{Q}\tau(E), \tag{2}$$

where v stands for pion-nucleon potential and E is the total pion-nucleus collision energy. The purpose is to expand potential U(E) in terms of free pion-nucleon scattering matrix $t(\omega)$ satisfying:

$$t(\omega) = v + vg(\omega)\,t(\omega), \quad g(\omega) = (\omega - K_\pi - K_N + i\epsilon)^{-1}, \tag{3}$$

where K_N is the nucleon kinetic energy operator and ω is the collision energy of the πN system. Using eqs. (3), eq. (2) is cast in the standard way into:

$$\tau(E) = t(\omega) + t(\omega)[\,G(E)\,\hat{Q} - g(\omega)]\tau(E). \tag{4}$$

The derived expansion of the optical potential stems from iteration of eqs. (1) and (4). Thus, the first and second order optical potentials are:

$$U^{I} = \sum_{i=1}^{A} t_i, \tag{5}$$

$$U^{II} = \sum_{i \neq j} t_i G(E)\, t_j - \sum_{i,j} t_i \hat{P} G(E)\, \hat{P} t_j + \sum_i t_i [\,G(E) - g(\omega)]\, t_i, \tag{6}$$

where $\hat{P} = 1 - \hat{Q}$ is projection operator on the nuclear ground state. The second order potential U^{II} contains two types of corrections: the first one, arising from iteration of eq. (4), is the correction to the impulse approximation and the second one, which originates from iteration of eq. (1), is the correction to the so-called coherent approximation. The latter correction is of the order of $\sim 1/A$ and should play an essential role in the description of the pion interaction with light nuclei.

3. The explicit expression for the first and second order optical potential follows from the averaging of the operators

(5,6) over the asymptotic wave functions of the pion nucleus system $|\vec{\pi}, n>$, where $\vec{\pi}$ is the pion momentum in the pion-nucleus c.m. system and n labels the nuclear states ($n = 0$ for ground state). Being interested mainly in the properties of πd atom we calculate the optical potential at pion threshold, i.e., in the limit of pion kinetic energy going to zero. According to the results of the Faddeev type calculations [7], we take into account only the s-wave πN interaction:

$$f_{\pi N} = -(2\pi)^2 \mu t_{\pi N} = b_0 + b_1 (\vec{t}\cdot\vec{\tau}), \tag{7}$$

where the parameters b_0, b_1 are expressed in terms of the πN-scattering lengths a_1 and a_3 in the channels with isospin 1/2 and 3/2, respectively: $b_0 = (a_1 + 2a_3)/3$, $b_1 = (a_3 - a_1)/3$, $\mu = (m\cdot M)/(m+M)$ is the reduced mass of the πN system (m and M are pion and nucleon masses), $\frac{1}{2}\vec{\tau}$ and $\vec{t}$ are nucleon and pion isospin operators, respectively. Further in the calculation of the second order optical potential (6) we use the so-called static limit $m/M \to 0$, which is justified for example by the results of ref. [6]. In this limit, the third term of eq.(6) is strictly equal to zero owing to the cancellation of $G(E)$ and $g(\omega)$, because the energy denominators in the Green functions do not depend on nuclear variables for $m/M \to 0$. In the static limit, the closure approximation (the use of the completeness relation for the nuclear states) is automatically satisfied.

Using these approximations we get for the first and second order optical potentials in the momentum space:

$$U^{I}(\vec{p}, \vec{p}') = A<\vec{p}, 0|t_1|\vec{p}', 0>, \tag{8}$$

$$U^{II}(\vec{p}, \vec{p}') = -A(A-1)\int d\vec{q}\{<\vec{p}, 0|t_1|\vec{q}><\vec{q}|t_2|\vec{p}', 0> -$$

$$-\frac{A}{A-1}<\vec{p}, 0|t_1|\vec{q}, 0><\vec{q}, 0|t_1|\vec{p}', 0>\}\frac{2\mathfrak{M}}{q^2 - i\epsilon}, \tag{9}$$

where $\mathfrak{M}$ is the reduced mass of the pion nucleus system and πN t-matrix is defined in eq.(7). Putting eq.(7) into eq.(8) gives the following expression for the first-order potential for the π-d-system at threshold:

$$U^{I}(\vec{p}, \vec{p}') = -\frac{1}{2M}\frac{1}{\pi^2} b_0 \rho_0(\vec{Q}), \tag{10}$$

where $\vec{Q} = \frac{1}{2}(\vec{p}' - \vec{p})$, $\rho_0(\vec{Q})$ is the nuclear form factor:

$$\rho_0(\vec{Q}) = \frac{1}{4\pi}\int d\vec{r}\exp(i\vec{Q}\vec{r})\rho(\vec{r}), \quad \rho(\vec{r}) \equiv 4\pi|\psi_d(\vec{r})|^2. \tag{11}$$

Transformation into the coordinate space leads to the local potential $V^{I}(\vec{r}, \vec{r}') = V(\vec{r})\delta(\vec{r} - \vec{r}')$, with $V(\vec{r})$ given by:

$$V(\vec{r}) = -\frac{1}{2\mathfrak{M}} 2b_0 q(\vec{r}), \quad q(\vec{r}) \equiv 8\rho(2r). \tag{12}$$

The parameter b_0 is negative for the majority of the sets of πN scattering lengths data [16]. Therefore, the potential (12) is a repulsive one. The strength of this potential is strongly reduced due to the cancellation of the quantities a_1 and $2a_3$ (see the table). This increases the importance of the second order potential given by:

$$U^{II}(\vec{p}, \vec{p}') = U_1^{II}(\vec{p}, \vec{p}') + U_2^{II}(\vec{p}, \vec{p}'), \tag{13}$$

$$U_1^{II}(\vec{p}, \vec{p}') = -\frac{1}{2\mathfrak{M}}(b_0^2 - 2b_1^2)\frac{1}{\pi^2} C\left(\frac{\vec{p} + \vec{p}'}{2}\right), \tag{14}$$

$$U_2^{II}(\vec{p}, \vec{p}') = \frac{1}{2\mathfrak{M}} b_0^2 \frac{1}{\pi^2}\int d\vec{q}\rho_0\left(\frac{\vec{p}' - \vec{q}}{2}\right)\rho_0\left(\frac{\vec{q} - \vec{p}}{2}\right)/(q^2 - i\epsilon), \tag{15}$$

where $C(\vec{Q}) = \frac{1}{4\pi}\int d\vec{r}\rho(r)\exp(i\vec{Q}\vec{r})/r$ and ρ_0 is given in eq.(11). In coordinate space $U_{1,2}^{II}(\vec{p}, \vec{p}')$ transforms into $V_{1,2}^{II}(\vec{r}, \vec{r}')$:

$$V_1^{II}(\vec{r}, \vec{r}') = -\frac{1}{2\mathfrak{M}}(b_0^2 - 2b_1^2) q(r)\frac{1}{r}\delta(\vec{r} + \vec{r}'), \tag{16}$$

$$V_2^{II}(\vec{r}, \vec{r}') = \frac{1}{2\mathfrak{M}} b_0^2 q(r) q(r')\frac{1}{\pi|\vec{r} - \vec{r}'|}. \tag{17}$$

The piece V_1^{II} is the local odd potential, which is, similarly to first order potential (12), repulsive (since $(b_0^2 - 2b_1^2) < 0$) in the s-wave of pion nucleus interaction. $V_2^{II}(\vec{r}, \vec{r}')$ is a non-local term, suppressed by small coefficient b_0^2. For practical calculation, it is convenient to make a local approximation to $V_2^{II}(\vec{r}, \vec{r}')$. Acting on the pion wave function the integral $\int^2 V_2^{II}(\vec{r}, \vec{r}')\psi(\vec{r}')d\vec{r}'$ is proportional to $\int d\vec{r}'\rho(2\vec{r}')\psi(\vec{r}')/|\vec{r} - \vec{r}'|$. Since the wave function $\psi(\vec{r})$ does not vary strongly in the nucleus region one can remove it from the integral. Then integration over the angular variables provides the following local approximation to $V_2^{II}(\vec{r}, \vec{r}')$:

$$V_2^{II}(\vec{r}, \vec{r}') = \frac{1}{2M} 4b_0^2 q(r)\frac{1}{r} f(r)\delta(\vec{r} - \vec{r}'), \tag{18}$$

where

$$f(r) = 1 + \int_r^\infty dr'\rho(r')r'(r - r'), \tag{19}$$

Table

Energy shift ΔE and scattering length $a_{\pi d}^{I+II}$ calculated using potential (20) and two sets of parameters a_1, a_s are given. A comparison is made with the first-order optical potential (12) contribution $a_{\pi d}^{I}$ and Faddeev type[7] result $a_{\pi d}^{Fad}$. Experimental value of $a_{\pi d}$ is [21] $a_{\pi d}^{exp} = -0.073^{+0.024}_{-0.031}$ fm. Frequently used ratio $[R_\lambda(0)/R_1(0)]^2$ is here also displayed

πN-scattering lengths, fm			Energy shift, eV	πd-scattering lengths, fm			Ratio of wave functions at the origin
a_1	a_s	Ref.	b_0	ΔE	$a_{\pi d}^{I+II}$	$a_{\pi d}^{I}$	$a_{\pi d}^{Fad}$	$[R_\lambda(0)/R_1(0)]^2$
0.2418	-0.1244	22	-0.0023	2.256	-0.0339	-0.0049	-0.0365	0.92
0.2573	-0.1541	23	-0.0167	4.677	-0.0703	-0.0348	-0.0746	0.85

with obvious limiting values $f(0) = 0$ and $f(\infty) = 1$. Adding (12), (16) and (18) one obtains a simple form of strong πd interaction in the L partial wave:

$$V_L(r) = -\frac{q(r)}{2\mathfrak{M}}\{2b_0^2 + \frac{1}{r}\cdot[(-1)^L(b_0^2 - 2b_1^2) - 4b_0^2 f(r)]\}. \qquad (20)$$

It is easy to show that the first two terms in the Born expansion of the scattering amplitude with the potential (20) reproduce the result of the multiple scattering theory for πd -scattering length [16]:

$$a_{\pi d} = \frac{2\mathfrak{M}}{\mu}[b_0 + (b_0^2 - 2b_1^2)\langle\frac{1}{r}\rangle_d]. \qquad (21)$$

Neglecting the term proportional to b_0, which is rather small, in eq.(20) one gets a simple interpretation of the influence of the strong interaction on the characteristics of the πd atom. Indeed, it follows from eq.(20) for $b_0 \ll b_1$ that:

$$V_L(r) \simeq \frac{(-1)^L}{\mathfrak{M}}\cdot\frac{q(r)\,b_1^2}{r}. \qquad (22)$$

In the Schrödinger equation, potential (22) renormalizes the strength of the Coulomb potential:

$$e^2 \to e_{eff}^2 = e^2 - (-1)^L b_1^2 q(r)/\mathfrak{M}, \qquad (23)$$

which gives a clear physical picture of the πd interaction in terms of the effective r-dependent charge e_{eff}.

4. In this section we describe a solution of the s-wave Schrödinger equation:

$$\{-\frac{\hbar^2}{2\mathfrak{M}}\frac{d^2}{dr^2} - \frac{e^2}{r} + V_0(r) - E\}u(r) = 0, \quad u(r) = r\cdot R(r), \qquad (24)$$

for πd atom. The potential V_0 is given in eq.(20), where we take $\rho(r)$ corresponding to the Reid soft core [17] deuteron wave function.

To obtain the πd scattering length $a_{\pi d}$ with an accuracy of ~1 percent, the energy E is to be computed with relative precision of least $\sim 10^{-8}$. For this reason, it is advantageous to use the following two-step procedure:

a) For the calculation of energy E an advanced numerical method with well defined precision is used based on the transformation of the Schrödinger equation to the first order nonlinear differential equation with the help of the Prüfer trans-

formation[18]. The energy shift $\Delta E = E - E_0$ (where E_0 is pure Coulomb energy) is connected with $a_{\pi d}$ by relation [19]:

$$a_{\pi d} = - \frac{\mathfrak{M} \Delta E}{2\pi |R_1(0)|^2} . \qquad (25)$$

b) With computed energy E, eq.(24) is cast into the form:

$$\left\{ \frac{d^2}{dz^2} + \frac{\lambda}{z} - \frac{1}{4} + \frac{V_0(z/\beta_\lambda)}{4E} \right\} u_\lambda(z) = 0 , \qquad (26)$$

where $\lambda = \frac{e^2}{\hbar c} \left(- \frac{\mathfrak{M} c^2}{2E} \right)^{1/2}$ and $\beta_\lambda = \beta_1 / \lambda = 2\mathfrak{M} e^2 / (\hbar^2 \lambda)$ are standard Coulomb parameters. Further, $\lambda = 1$ for the case of the pure Coulomb interaction and $\lambda = 1 + \epsilon$, $0 < \epsilon \ll 1$ with potential (20) included and $z = \beta_\lambda r$.

For $V_0(r)$ given in eq.(20), solution of eq.(26) in the region $z > 30$ fm is given by the regular Whittaker's function $W_{\lambda, 1/2}(z)$ [20]. To find $u_\lambda(z)$, eq.(26) is integrated numerically outwards (inwards) in the interval (0,20 fm) ((20 fm, 30 fm)) with the fixed step of 0.05 fm (0.1 fm). At the point 20 fm, left and right logarithmic derivatives were calculated and found to be equal within a relative precision of $\sim 10^{-6}$, which checks independently the computed energy value E. Normalized wave function is chosen in the form:

$$R_\lambda(r) = \frac{1}{N_\lambda a^{1/2}} \frac{u_\lambda(\beta_\lambda r)}{r} , \quad \int R_\lambda^2(r) r^2 dr = 1 , \qquad (27)$$

where a is the Bohr radius for πd system.

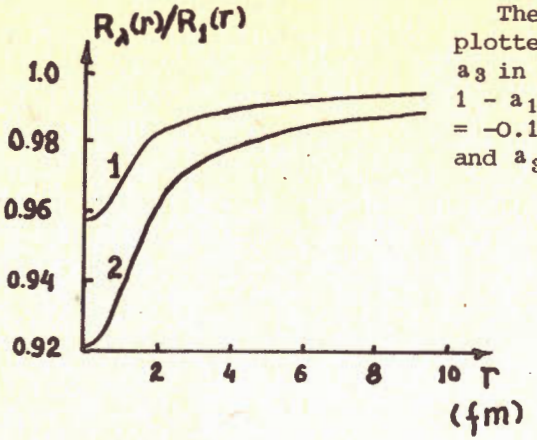
The normalization constant N_λ can be found by numerical integration:

$$N_\lambda^2 = 1 + \int_0^{30} \{ u_\lambda^2(\beta_\lambda r) - u_1^2(\beta_1 r) \} dr + \frac{1}{a} \int_{30}^{3000} \{ W_{\lambda, 1/2}^2(\beta_\lambda r) - W_{1, 1/2}^2(\beta_1 r) \} dr . \qquad (28)$$

Resulting energy shifts ΔE, $a_{\pi d}$ and ratio $(R_\lambda(0)/R_1(0))^2$ are given in the table for two extreme sets of parameters a_1, a_3. Ratios $R_\lambda(r)/R_1(r)$ are plotted in the figure. While near the origin $R_\lambda(r)$ is suppressed by strong interaction, for $r \to \infty$ $R_\lambda(r)/R_1(r) \to \infty$ due to the asymptotic behaviour:

$$R_\lambda(r) \longrightarrow \frac{1}{N_\lambda a^{1/2}} \beta_\lambda^\lambda r^{\lambda - 1} \exp(-\beta_\lambda r/2) . \qquad (29)$$

Taking into account the numerical result: $N_\lambda = 1 + \delta$, $\delta \cong \epsilon$, it is easy to show that the intersection point r_0 of $R_\lambda(r)$



The ratio $R_\lambda(r)/R_1(r)$ is plotted for two sets of a_1 and a_3 in the range of $0 \leq r \leq 10$ fm: 1 - a_1 = 0.2418 fm and a_3 = = -0.1244 fm, 2 - a_1 = 0.2573 fm and a_3 = -0.1541 fm.

and $R_1(r)$ satisfies the equation:

$$\ln \beta_1 r_0 \cong 2 - \beta_1 r_0 / 2 , \qquad (30)$$

from which follows that $r_0 \cong 240$ fm.

Our results for $a_{\pi d}$ displayed in the table are to be compared with the results of the Faddeev type calculations $a_{\pi d}^{Fad}$ [7]. As in ref. [7], we have found that $a_{\pi d}$ are quite sensitive to the changes in πN scattering parameters. The agreement between our $a_{\pi d}$ and $a_{\pi d}^{Fad}$ can be achieved only when taking into account the second order optical potential. Discrepancy of about 10 percent arises mainly from the on-shell approximation for the πN-scattering amplitude adopted in the calculation of the second order optical potential. In more refined calculation the πN p-wave interaction and effects of pion absorption should also be taken into account, the latter effect leading to the broadening of the $\pi^- d$ atomic energy levels. However, it was shown in refs. [7,24] that both those effects give only a small contribution to the $Re a_{\pi d}$ (or to the 1s level shift). Hence the simple optical potential (20) derived in the present paper, being quite convenient from the practical point of view, should be treated as an effective tool for an estimation of the influence of the strong interaction on the properties of the πd system.

The authors wish to thank V.B.Belyaev, R.Mach, M.G.Sapozhnikov and R.A.Eramzhyan for stimulating discussions and helpful advice.

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on April 19, 1984.

Адам И. /мл./, Ханхасаев М.Х., Труглик Э. E4-84-264

Оптический потенциал второго порядка для пион-дейтронного атома

В рамках теории многократного рассеяния Ватсона построен потенциал второго порядка для пион-дейтронного атома. Получено простое локальное приближение для этого потенциала, вычислены сдвиг 1s -уровня и соответствующая ему волновая функция. Рассчитанная длина πd -рассеяния хорошо согласуется с точным значением, вычисленным с помощью уравнений Фаддеева.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

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The Second Order Optical Potential for πd System at the Threshold

The second order optical potential for the πd system is derived in the framework of the Watson formalism and applied to the calculation of 1s -state energy shift of the atom and πd scattering length $a_{\pi d}$. The results are in good agreement with those of the Faddeev type calculations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984