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QUASIPARTICLES IN PEARSHAPED ROTATING NUCLEI

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Recent calculations of equilibrium deformation combining Strutinsky's shell correction approach with a folded Yukawa [1], Nilsson [2] or a Woods-Saxon single particle potential [3] predict pearshape for the light actinides. Significant octupole deformation is obtained for the restricted region $131 \leq N \leq 138$, $83 \leq Z \leq 90$. Experimental binding energies and spectroscopic information about low spin states [4] seem to support these predictions, although the data is still too scarce to be conclusive. The reaction of quasiparticles to the inertial forces is very sensitive to details of the shape of a rotating nucleus. Analysing experimental rotational bands in terms of the Cranked Shell Model [5] (CSM), ref. [6] found evidence for the existence of strong hexadecapole deformation in the ^{180}Os region and refs. [7] for configuration dependent triaxial quadrupole deformations. In the present publication we show that an octupole component of the deformed potential causes typical pattern of the quasiparticle Routhians e' as functions of the rotational frequency of ω. The CSM relates the $e'(\omega)$ directly to the experimental rotational bands. Therefore, we suggest to measure the rotational bands up to significant spin in order to decide the question whether the light actinides have indeed pearshape as predicted.

We follow ref. [5] and solve the quasiparticle equations given by the quasiparticle Routhian h':

$$h'|\sigma\mu\rangle = e'_{\sigma\mu}|\sigma\mu\rangle ,$$
$$h' = h_{def}(\alpha,\alpha_3,\alpha_4) - \lambda n + \Delta - \omega j_x . \qquad (1)$$

This provides the quasiparticle Routhians $e'_{\sigma\mu}$ as functions of the rotational frequency ω and the quasiparticle aligned angular momenta $i_{\sigma\mu}$ (the negative slope of the trajectories $e'_{\sigma\mu}(\omega)$). Here h_{def} is the deformed Woods-Saxon Hamiltonian as described in ref. [8]. The shape is parametrized by axial symmetric Cassini ovaloids which are modulated by multipoles of different order. The parameters α, α_3, α_4 are approximately equal to ϵ_2, ϵ_3, $-\epsilon_4$ of the more frequently used shape parametrization of ref. [9]. The remainder of eq. (1) are the chemical potential λ, the particle number n, the monopole pair field Δ and the x-component of the angular momentum j_x.

The interpretation of rotational bands in terms of the $e'_{\sigma\mu}(\omega)$ is described in detail in ref. [5]. Here, we only discuss the new aspects of the nonconservation of parity and signature. The Routhian (1) is symmetric with respect to the reflection through the y-z -plane, which may also be expressed by means of the space inversion $\mathcal{P}$ and the rotation $\mathcal{R}_x$ about the x-axis by an angle of π. Bohr and Mottelson /10/ call this operation $\mathcal{S}$.

$$\mathcal{S}h'\mathcal{S}^{-1} = h', \quad \mathcal{S} = \mathcal{P}\mathcal{R}_x. \tag{2}$$

This symmetry implies the quantum number σ defined by

$$\mathcal{S}\,|\,\sigma\mu\rangle = e^{-i\pi\sigma}\,|\,\sigma\mu\rangle, \quad \sigma = \left\{\begin{array}{ccc} \pm\frac{1}{2} & & \text{odd} \\ & \text{for} & \\ 0,1 & & \text{even} \end{array}\right\} \text{particle number}, \tag{3}$$

which we call combined signature * . For systems with D_2-symmetry (no octupole) the combined signature may be expressed by means of the parity π (impled by $\mathcal{P}$) and the signature α /5/ (impled by $\mathcal{R}_x$).

$$\sigma = \alpha + \left\{\begin{array}{ccc} 1 & & -1 \\ & \text{for} \quad \pi = & \\ 0 & & +1 \end{array}\right\}. \tag{4}$$

Let us now consider the relation between the total values of combined signature, spin and parity. Since σ is conserved for the quasiparticles ($\sigma = \pm 1/2$), each intrinsic configuration corresponds to a total σ_t, which is also the σ_t of the total wavefunction and, therefore, measurable. It may be expressed by π, I and the total signature α, which are conserved for the total wavefunction only. The pertinent relations are the connection between I and α /5/

$$\alpha = I + \quad \text{even number} \tag{5}$$

and eq. (4) connecting σ_t and α. The possible σ_t-values correspond to the rotational sequences

$$\begin{array}{lcc} 0^+,\ 1^-,\ 2^+,\ 3^-,\ \ldots & & 0 \\ 0^-,\ 1^+,\ 2^-,\ 3^+,\ \ldots & \text{for} \quad \sigma_t = & 1 \\ 1/2^+,\ 3/2^-,\ 5/2^+,\ 7/2^-,\ \ldots & & 1/2 \\ 1/2^-,\ 3/2^+,\ 5/2^-,\ 7/2^+,\ \ldots & & -1/2. \end{array} \tag{6}$$

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* We suggest this term in analogy to the "combined parity" applied by Landau and Lifshits (v. IVb) in the context of supposed invariance of weak interaction with respect to the combination of space inversion and charge conjugation.

Following ref. /5/ we take the ground state rotational band (g-band) as quasiparticle vacuum. In an octupole deformed nucleus it corresponds to $\sigma = 0$. Then, σ_t of a quasiparticle configuration is equal to the sum of the σ's of the excited quasiparticles. A calculated quasiparticle configuration as function of ω is associated with a rotational band of the appropriate σ_t .

Experimentally, a rotational band of good σ will show up as a regular sequence of enhanced stretched E1-transitions. The enhancement is caused by a displacement of the centre of charge relative to the centre of gravity which is generally expected for octupole nuclei. The resulting rotating dipole generates the E1-radiation. Strutinsky /11/ and Bohr and Mottelson /12/ give the estimate for the dipole moment

$$D = D_o\,\beta_2\,\beta_3\,, \tag{7}$$

where $D_o \approx 10$ e fm for the $N = 134$ region and β_2 and β_3 are the usual /12/ coefficients of the multipole expansion of the nuclear shape. The corresponding B(E1)-values are several orders of magnitude larger than typical low energy B(E1)-single particle values.

The CSM-analysis /5/ is based on the angular momentum $I(\omega)$ and the Routhian $E'(\omega)$ generated from the experimental levels of a band to which a quasiparticle configuration is assigned. The straightforward extension of this concept to pearshape would be to calculated these functions from the adjacent levels of a σ-band (i.e., $\hbar\omega = E(I^\pi) - E((I-1)^{-\pi})$). However, in realistic nuclei there is a tunnelling between the left- and right-hand orientation of the pear, which causes a staggering between the $\pi = +$ and $\pi = -$ members of the band. Since this tunnelling motion is not taken into account in the CSM the parity staggering should be eliminated from the data. Assuming a smooth ω-dependence of the energy shift this is achieved by the following treatment of the data: Each group of states of given π has also a good α. With the help of the CSM-prescriptions of ref. /5/ one determines the functions $I_{x\pi}(\omega)$ and $E'_\pi(\omega)$ for each parity separately. The functions $I_x(\omega)$ and $E'(\omega)$ for the configuration with good σ are calculated by averaging over the two parities

$$\begin{aligned} I_x(\omega) &= \frac{1}{2}\left(I_{x+}(\omega) + I_{x-}(\omega)\right), \\ E'(\omega) &= \frac{1}{2}\left(E'_+(\omega) + E'_-(\omega)\right). \end{aligned} \tag{8}$$

The functions $E'_o(\omega)$ and $I_{xo}(\omega)$ obtained in this way from the levels of the g-band ($I = 0^+, 1^-, 2^+, 3^-, \ldots$) may serve as a reference to construct the relative aligned angular momentum $i(\omega)$ and the Routhian $e'(\omega)$ in complete analogy to ref. /5/

$$i(\omega) = I_x(\omega) - I_{xo}(\omega),$$
$$e'(\omega) = E'(\omega) - E'_o(\omega). \qquad (9)$$

Other types of references are also possible. Once the functions (8) are defined it is quite straightforward to extend the considerations of refs. /13/ to the considered case. The relative quantities (9) may be directly interpreted in terms of configurations of quasiparticles occupying the trajectories $e'_{\sigma\mu}(\omega)$.

Figures 1-3 show a selection of single particle and quasiparticle diagrams $e'_{\sigma\mu}(\omega)$ for the light actinide region. The deformations $\alpha = 0.2$, $\alpha_4 = 0.075$, $\alpha_3 = 0.1$ correspond to the minimum of the ground state deformation energy /2/. We have varied $0.0 \le \alpha_3 \le 0.2$ in order to demonstrate the development of the typical pattern indicating the octupole shape.

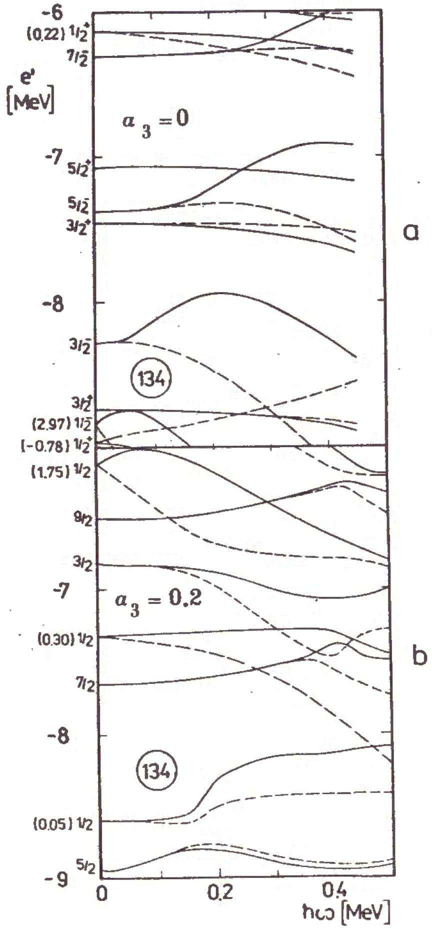


Fig.1. The single neutron Routhian at the fixed octupole deformation $\alpha_3 = 0$ (above) and $\alpha_3 = 0.2$ (below). The remaining deformation parameters are $\alpha = 0.2$ and $\alpha_4 = 0.075$. The parameters of the deformed Woods-Saxon potential are given in ref. /16/. Full drawn and dashed lines correspond to combined signature $\sigma = -1/2$ and $1/2$, respectively. For $\omega = 0$ the angular momentum projection K onto the symmetry axis is indicated. For $\alpha_3 = 0$ the parity is added in the form K^{π}. For the $K = 1/2$ levels one half of the decoupling parameter (slope of the $\sigma = -1/2$-level) is given in parenthesis. The $\pi = -$ levels belong to the $j_{15/2}$ -shell.

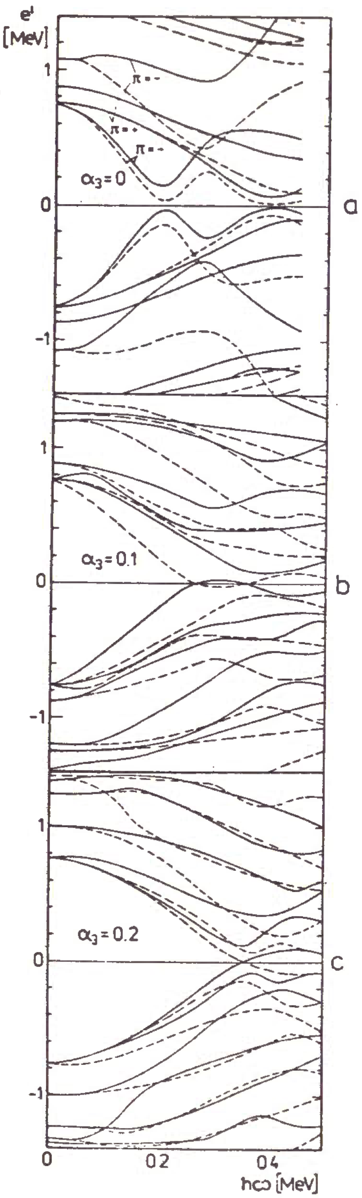
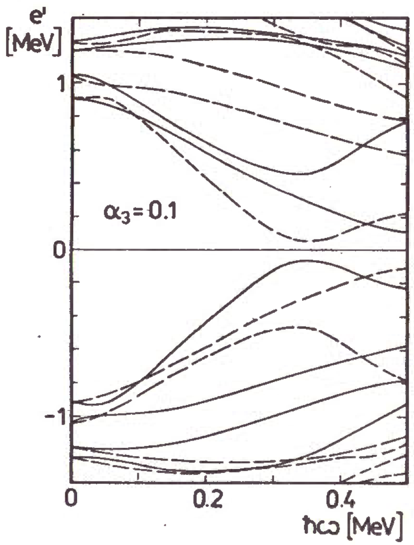


Fig.2. Dependence of the quasineutron Routhians on the octupole deformation α_3. The pairing gap $\Delta = 0.75$ MeV is about 80% of the even-odd-mass-difference /5/. The chemical potential $\lambda = -7.5$ MeV corresponds to $\langle N \rangle = 137.9$, 136.5, 136.4 for figs. 2a, 2b, 2c, respectively, at $\omega = 0$.

Fig.3. Quasiproton Routhians for $\alpha_3 = 0.1$. Line convention and parameters as in fig.1 except $\Delta = 0.9$ MeV and $\lambda = -5.0$ MeV corresponding to $\langle Z \rangle = 86.7$ at $\omega = 0$.



The consequences of pearshape for slow rotation (strong coupling limit) have been discussed by Bohr and Mottelson /12/ and more recently by refs. /4/. Let us express their conclusions in terms of the quasiparticle rajectories. As is seen in the figure for $\omega \to 0$ there are degenerate pairs $e'_{\sigma = \pm 1/2,\, K\mu}(\omega)$, where

K is the angular momentum projection onto the symmetry axis. Since each σ corresponds to a $\Delta I = 1$ - one quasiparticle band with alternating π (c.f. eq. (6)) one may alternatively say that there are two $\Delta I = 1$ - bands having opposite π [12]. The σ-degeneracy of the $e'_{\sigma K\mu}(\omega)$ also implies degenerate pairs of multiquasiparticle configurations each of which corresponding to parity doublet of $\Delta I = 1$-bands. For $K = 1/2$ the two trajectories $e'_{\sigma=\pm 1/2, 1/2, \mu}(\omega)$ have a finite opposite slope at $\omega = 0$. In strong coupling terminology this means that the two degenerate $K = 1/2$-bands have decoupling parameters of the opposite sign but the same absolute value [4,12]. In summary, the strong coupling limit as discussed by Bohr and Mottelson [12] is equivalent with σ-degeneracy of the quasiparticle trajectories. As is demonstrated by the figures this coupling is only realized for sufficiently small frequency. The more general feature of rotating pearshaped nuclei are nondegenerate bands with a fixed combined signature. The frequency at which deviations from the strong coupling arise depends strongly on the state.

The main features of the quasiparticle spectrum at finite ω are a consequence of the parity mixing caused by the octupole component of the potential. For $\alpha_3 = 0$ the conservation of parity results in the well-known intruder-host-structure: Due to the spin orbit splitting intruder states carrying the maximal angular momentum of a shell are embedded into the next lower shell of opposite parity. These host states have moderate or low angular momentum, since the quadrupole deformation causes an appreciable redistribution of angular momentum damping the angular momentum fluctuations in a spherical potential. As a consequence of their different angular momenta intruder and host states react rather differently to the inertial forces at finite ω. This is clearly seen for the single neutron Routhians shown in fig. 1a. The $j_{15/2}$ - intruder trajectories (states with $\pi = -$) rapidly translate into the Rotational Aligned Coupling Scheme characterized by large alignment (slope) and splitting between states of different signature α (cf., e.g., ref. [13]). The host states of positive parity change much less with ω. As is seen in fig. 1b an octupole deformation of $\alpha_3 = 0.2$ melts the host-intruder-pattern. The angular momentum is much more evenly distributed over the states resulting in trajectories $e'(\omega)$ with similar alignment and splitting of the σ-pairs. In the low ω region the angular momentum redistribution shows up as drastic changes of the decoupling parameter [4].

Pairing is introduced in figs. 2 and 3. The α_3-dependence demonstrated by figs. 2a,b,c is also a consequence of the angular momentum redistribution caused by the parity mixing. As is well known, for $\omega = 0$ the pairfield pushes the states away from the Fermi surface ($e' = 0$) such that there is a group of closely spaced levels just above the pairing gap Δ. For $\alpha_3 = 0$ (fig. 2a) this group is quickly resolved with growing ω since the intruder states achieve a much higher alignment (slope) than the hosts. For $\alpha_3 = 0.2$ (fig. 2c) the clustering above the gap persists over a large ω-interval since the similar alignment prevents the spreading. For the intermediate octupole deformation of $\alpha_3 = 0.1$ (fig. 2b) which is suggested by the calculations as the ground state equilibrium value [1-4], the clustering is less prominent but still present. The same is true for the quasiproton spectrum shown in fig. 3.

Another obvious consequence of the angular momentum redistribution is the different pattern of quasicrossings (backbends) between the trajectories $e' > 0$ and $e' < 0$. For $\alpha_3 = 0$ one has the familiar situation [5] of a sharp early crossing for the intruder and later and very smooth crossing for the host. For $\alpha_3 = 0.2$ there is a whole group of smooth crossing at intermediate frequency.

A comparison of figs. 2 shows that $e'(\omega, \alpha_3)$ of the lowest quasiparticle trajectories increases with α_3 for the ones descenting from the intruders ($|\Delta e'(\hbar\omega = 0.25\ \mathrm{MeV})| \sim 0.4$ MeV) and decreases for the ones descending from the hosts ($|\Delta e'(\hbar\omega = 0.25\ \mathrm{MeV})| \sim 0.1$ MeV). Refs. [1-4] calculate ground state deformation energies of the order of 1-2 MeV for $133 \leq N \leq 137$. Hence, the excitation of rotating quasiparticles is not expected to quench the octupole deformation.

In summary, for the $N = 134$ region octupole deformed rotational bands are expected. Doing a CSM analysis the pearshape should be revealed by the following finger-prints:

i) The rotational levels form band with fixed combined signature and are interconnected by enhanced stretched E1-transitions.

ii) There is a cluster of quasiparticle states of both σ above a gap in the spectrum. This structure persists up to about $\hbar\omega = 0.35$ MeV where the gap disappears.

iii) Pronounced backbending is absent at frequencies where it is observed in reflection symmetric nuclei. A delayed smooth upbend is expected instead.

In refs. [14] were the yrast levels of ^{222}Th measured up to $I = 17\hbar$. They form a $\sigma = 0$ sequence connected by enhanced E1-transitions. At low ω there is a pronounced parity staggering that gradually disappears with ω. The experimental function $I(\omega)$ (fig. 12 of ref. [14]) does not show any backbend up to $\hbar\omega = 0.22$ MeV. Our $\alpha_3 = 0$ calculation predicts a pronounced $\nu j_{15/2}$-backbend at $\hbar\omega = 0.20$ MeV (in accordance with the results of ref. [15]). For $\alpha_3 \geq 0.1$ the first irregularity is delayed to $\hbar\omega > 0.30$ MeV. Hence, the ^{222}Th data seems to be consistent with the assumption of a substantial octupole deformation for $I > 10\hbar$. Let us stress the importance of measuring other high spin spectra in the

N = 134 region in order to check in a more conclusive way whether all the above mentioned finger-prints of a stable octupole deformation are observed. In particular, the clustering of bands with both combined signatures and similar alignment predicted for the odd neutron nuclei would be a strong positive evidence.

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Фрауендорф С., Пашкевич В.В. E4-84-180
Квазичастичные возбуждения во вращающихся ядрах грушевидной формы

Собственные значения одночастичного радиуса во вращающемся квадрупольно-октупольно-деформированном аксиально-симметричном потенциале типа Вудса-Саксона обнаруживают типичную зависимость от угловой частоты, что может быть использовано для идентификации формы такого характера. Обсуждаются как следствия для экспериментальных ротационных полос, так и их классификация в соответствии с появляющимся благодаря симметрии квантовым числом, названным комбинированной сигнатурой. В области N = 134 ожидаются октупольно-деформированные ротационные полосы. Анализ, основанный на вращательной оболочечной модели, свидетельствует о том, что грушевидная форма должна проявляться в следующих явлениях: а/ вращательные уровни образуют две полосы с определенной комбинированной сигнатурой, которые связаны между собой усиленными выстроенными E1 -переходами; б/ имеется группа квазичастичных состояний обеих комбинированных сигнатур над щелью в спектре, которая сохраняется вплоть до $\hbar\omega = 0{,}35$ МэВ, после чего щель исчезает; в/ четко выраженный обратный изгиб отсутствует при тех частотах, при которых он наблюдается в зеркально-симметричных ядрах, вместо этого ожидается запаздывающий плавный подъем.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Frauendorf S., Pashkevich V.V. E4-84-180
Quasiparticles in Pearshaped Rotating Nuclei

The quasiparticle Routhians in a rotating octupole-quadrupole deformed axial Woods-Saxon potential show typical pattern that may be used to identify this shape.The implications for experimental rotational bands as well as their classification with respect to the new symmetry quantum number combined signature are discussed. For the N = 134 region octupole deformed rotational bands are expected. Doing a CSM analysis the pearshape should be revealed by the following finger-prints: i) The rotational levels form band with fixed combined signature and are interconnected by enhanced stretched E1-transitions. ii) There is a cluster of quasiparticle states of both combined signatures above a gap in the spectrum. This structure persists up to about $\hbar\omega = 0.35$ MeV where the gap disappears. iii) Pronounced backbending is absent at frequencies where it is observed in reflection symmetric nuclei. A delayed smooth upbend is expected instead.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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