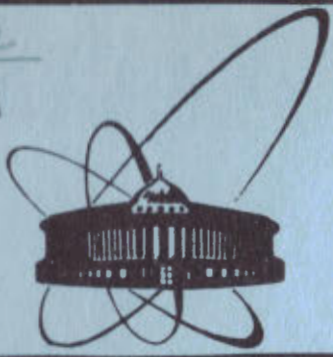

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RECURRENT EQUATIONS OF THE FIXED-SCATTERERS APPROXIMATION

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INTRODUCTION

Theoretical description of the particle scattering from complex systems requires the development of both exact and approximate calculation methods.

The first are based on the Faddeev equations and their different generalizations to systems of more than three particles. However, their practical application is restricted by computational difficulties. Therefore one can't go today beyond the 4-body problem in this direction. Furthermore the 4-body calculations are done for several tasks with simplest interactions only.

For example, so physically interesting processes as pion and antinucleon interactions with 3-nucleon systems are left aside.

The approximate methods cover a wide range of different approaches.

Having no in mind to do a review of them we will note only the methods based on the fixed-scatterers approximation (FSA) which is called sometimes the adiabatic one.

Starting from the Brueckner works [1,2] who has obtained an analytical expression for the amplitude of scattering of a light particle from two heavy ones, and the Foldy and Walecka work [3] who have shown the FSA and the target-states-closure equivalence, this approximation has been investigated itself [4-7] and has been intensively used to study different approximations of the multiple-scattering theory [8-13] and to construct optical potentials of particle-nucleus interactions [14]. One of the important FSA applications was the Sitenco-Glouber model [15-17]. Besides the adiabaticity it used the eikonal two-particle amplitudes.

Equations obtained by the finite-rank approximation of the target Hamiltonian [18-22] became another FSA-applicability region. These integral equations use the FSA amplitude as a kernel and an inhomogeneous term.

A more complete FSA paper list can be found in the reviews [23,24].

Such a wide use of FSA is explained by a physical simplicity of the resulting task. Indeed, the scattering from fixed centres is a 2-body problem of motion in the joint field of scatterers. Therefore, in many cases the multiple-scattering series may be completely summed since the sum in FSA obeys the 2-particle Lippmann-Schwinger equation.

$$\begin{cases} T(Z) = \sum_{i=1}^{A} T_i(Z) \\ T_i(Z) = t_i(Z) + t_i(Z) G_0(Z) \sum_{j \neq i} T_j(Z) \\ t_i(Z) = V_i + V_i G_0(Z) t_i(Z) . \end{cases} \quad (3)$$

Note that the kinetic-energy operator of $G_0(Z)$ contains the reduced-mass of x and the whole target. Therefore $t_i(Z)$ is not the free two-particle amplitude. Increase of the mass takes into account the influence on the i-th particle of the other $(A-1)$ -particles. However, as it has been shown by the $\pi\,^3$He scattering near the P_{33} resonance /21/, this account overestimates the one-nucleon binding with the remaining nucleons of the nucleus. In FSA this binding is hard since the nucleus recoils as a whole.

This problem also was investigated by the model calculations /4,11/. Application of FSA in its traditional form is restricted by computational difficulties.

In the case of separable potentials, as for the two-particle problem, the FSA-amplitude calculation produces a set of linear algebraic equations. To solve the set, one needs to invert the matrix depending on target coordinates. The final amplitude may be obtained by sandwiching the solution between the target states. This procedure demands the integration over $3(A-1)$ variables.

Taking into account of all spin and isospin states produces matrices of a huge dimension.

It is clear that the inversion of a big matrix depending on coordinates with a subsequent integration over these coordinates is a complicated problem of the numerical analysis.

For example, let us consider the elastic πd-scattering. Taking into account of the spin-isospin variables transforms (2) into a set of 8 equations connecting 64 amplitudes $\langle \vec{k}' t'_{12} s'_{12} \sigma' | T(\vec{r}) | \vec{k} t_{12} s_{12} \sigma \rangle$, where $\vec{k}$ is the momentum of the πd relative motion, $\vec{r}$ is the distance between nucleons, t_{12}, s_{12}, σ are their isospin, spin, and spin projection, respectively.

If one takes into consideration the πN channels S_{11}, S_{31}, P_{33}, P_{31} with a separable interaction, then each of the 64 amplitudes is to be constructed of the 16 form factors.

$$\langle \vec{k}', \alpha' | T(\vec{r}) | \vec{k}, \alpha \rangle = \sum_{n'=1}^{16} \sum_{n=1}^{16} g_{n'}(\vec{k}', \vec{r}) \Lambda_{n'n}^{\alpha'\alpha}(\vec{r}) g_n^*(\vec{k}, \vec{r}) ,$$

where $n \equiv (i, \ell, m, \mu)$ is the multi-index; i, the nucleon number; ℓ, m, the angular momentum and its projection in πN-channel with number μ.

$$g_n(\vec{k}, \vec{r}) = \exp(i\vec{k}\vec{z}_1) Y_{\ell m}(\vec{k}) h_\mu(k) ,$$

where $h_\mu(k)$ is the form factor of the πN-potential. Therefore, for $\Lambda_{n'n}^{\alpha'\alpha}(\vec{r})$ one has a set of $\alpha \cdot n = 128$ linear equations. Consequently, one needs to invert 128-dimensioned matrix for each point of the net of numerical integration over $\vec{r}$. This is a rather difficult computational problem.

In view of the same difficulty the authors of /27,28/ have to omit some of the intermediate spin-isospin states from the $\pi\,^4$He-scattering calculations. For integration they had to construct a special procedure on the basis of the Monte-Carlo method.

Referring to these difficulties authors of the reviews /23,24/ deduce that the realization of FSA for nuclei heavier than the deuteron is a very complicated and tedious task, despite its attractiveness and physical simplicity.

Further, in this paper we will propose a new formulation of FSA on the basis of the recurrent equations, which simplifies the problem considerably by using a convenient basis for each of them and makes FSA to be a reliable method for finding the amplitudes of hadron scattering on nuclei.

II. RECURRENT EQUATIONS

Consider the two-particle scattering with a potential v composed of two terms $v = v_1 + v_2$. Introducing amplitudes t_i by $t_i = v_i + v_i g_0 t_i$ one can easily show /29/ that the total amplitude t may be expressed as follows

$$t = t_1 + (1 + t_1 g_0) r (1 + g_0 t_1) , \quad (4)$$

where the auxiliary operator r obeys the equation

$$r = t_2 + t_2 g_0 t_1 g_0 r . \quad (5)$$

This may be proved in the following way. Let us rewrite the Lippmann-Schwinger equation for t in the form $t = (1 - v_1 g_0)^{-1}(v_1 + v_2) + (1 - v_1 g_0)^{-1} v_2 g_0 t$. From the equations $t_1 = v_1 + v_1 g_0 t_1$ and $t_1 = v_1 + t_1 g_0 v_1$ we obtain $(1 - v_1 g_0)^{-1} = (1 + t_1 g_0)$. Then $t = t_1 + (1 + t_1 g_0) v_2 (1 + g_0 t)$. Substituting t of (4), which defines r, into the last equation we obtain (5).

This so-called two-potential formula is often used to extract a Coulomb amplitude in the charged-hadrons-interaction problems /30,31/.

Now look at the FSA equation (2)

$$T_A = \sum_{i=1}^{A} V_i + \sum_{i=1}^{A} V_i G_0 T_A .$$

It is a two-particle equation with a complicated potential. With the notation $v_1 = \sum_{i=1}^{A-1} V_i$ and $v_2 = V_A$ and using (4) and (5), we derive

$$T_A = T_{A-1} + (1 + T_{A-1} G_0) r_A (1 + G_0 T_{A-1}),$$

$$r_A = t_A + t_A G_0 T_{A-1} G_0 r_A ,$$

where T_{A-1} is the FSA-amplitude of scattering from $(A-1)$-target particles.

$$T_{A-1} = \sum_{i=1}^{A-1} V_i + \sum_{i=1}^{A-1} V_i G_0 T_{A-1} .$$

Applying this procedure step by step for the amplitudes T_A, T_{A-1}, T_{A-2}, ... , etc., we arrive at the set of recurrent equations

$$\begin{cases} T_1 = t_1 \\ T_2 = T_1 + (1 + T_1 G_0) r_2 (1 + G_0 T_1) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ T_A = T_{A-1} + (1 + T_{A-1} G_0) r_A (1 + G_0 T_{A-1}) \\ r_2 = t_2 + t_2 G_0 T_1 G_0 r_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ r_A = t_A + t_A G_0 T_{A-1} G_0 r_A \\ t_i = V_i + V_i G_0 t_i . \end{cases} \qquad (6)$$

From (6) we see that the amplitude T_A may be expressed by the finite sum

$$T_A = T_{A-1} + r_A + T_{A-1} G_0 r_A + r_A G_0 T_{A-1} + T_{A-1} G_0 r_A G_0 T_{A-1} .$$

The multiple-scattering series is to be obtained by the substitution of

$$r_A = t_A + t_A G_0 T_{A-1} G_0 t_A + t_A G_0 T_{A-1} G_0 t_A G_0 T_{A-1} G_0 t_A + \ldots , \text{ etc.},$$

into this sum. As a result, we arrive at a partly summed series for T_A, because T_{A-1} takes into account all orders of re-

scattering from (A-1) particles. Let us introduce the notation $\theta_A = (1 + T_{A-1} G_0) r_A (1 + G_0 T_{A-1})$. Extracting the components θ_A, θ_{A-1}, θ_{A-2}, ..., etc., off the amplitudes T_A, T_{A-1}, T_{A-2}, ..., etc., we express T_A as a sum of A terms

$$T_A = t_1 + \sum_{i=2}^{A} \theta_i .$$

First N terms of the sum include all orders of multiple scattering from these N target particles.

This result looks like the so-called spectator expansion [32-34] obtained by rearranging and partly summing of the Watson series. To derive (6) we split V by the step by step extraction of one V_i. More generally we can split it in arbitrary groups.

$$V = \sum_{i=1}^{B} V_i + \sum_{i=B+1}^{A} V_i , \quad T_A = T_B + (1 + T_B G_0) r_{AB} (1 + G_0 T_B) ,$$

$$r_{AB} = T_{A-B} + T_{A-B} G_0 T_B G_0 r_{AB} .$$

Such a splitting may be more suitable if the target particles are identical.

For example, if the target is a nucleus, the number of equations may be reduced considerably by splitting the target into identical groups, since one has to calculate the FSA amplitude for each group only once. Note that such a reduction does not require the clustering, because T_B is not the amplitude of scattering from the cluster of B-nucleons rather it is the scattering amplitude for by nucleons with spacing $|\vec{z}_1, \vec{z}_2, \ldots, \vec{z}_B\rangle$. The advantages of equations (6) as compared with (2) and (3) are the diagonality of $r_2, r_3, \ldots, r_{A-1}$ with respect to quantum numbers of the spectators and the diagonality of an inhomogeneous term of each of the equations for r with respect to variables of all other (A-1) particles. As will be shown later, a special choice of the basis for each of the equations simplifies the consideration to a great extent.

III. CHOICE OF THE BASIS

Ordinarily one solves the FSA equation using the basis

$$|\vec{k}, \vec{r}, \alpha\rangle , \qquad (7)$$

where $\vec{k}$ is the relative-motion momentum of the projectile x and the target - c.m., α stands for all discrete quantum numbers.

In nuclear physics $|\alpha\rangle$ usually denotes spin-isospin-subspace states.

The operators V_i and t_i of (2), (3), (6) are many-body ones. Using the spatial-translation operator one easily can relate them to the corresponding two-body operators [28].

Let $\hat{\vec{P}}$ is the operator corresponding to momentum $\vec{k}$ of (7). Then the operator $Q = \exp(-i\hat{\vec{Z}}\hat{\vec{P}})$ transforms the state (7) to produce the state of the same value and direction of the momentum but related to the point shifted from the target - s.m. by vector $\vec{z}$ [35].

Therefore, $|\vec{k}, \vec{r}, a\rangle = Q_i^+ |\vec{k}_i, \vec{r}, a\rangle$, where $Q_i = \exp(-i\hat{\vec{Z}}_i \hat{\vec{P}})$ and $\vec{k}_i$ is the relative momentum of the projectile and i-th particle. Consequently, one has

$$\langle \vec{k}', \vec{r}', a' | t_i | \vec{k}, \vec{r}, a\rangle = \prod_j \delta(\vec{r}_j' - \vec{r}_j) \exp[i(\vec{k} - \vec{k}')\vec{z}_i] \sum_{\beta'\beta} \langle a'|\beta'\rangle t_i^{\beta'\beta}(\vec{k}', \vec{k}) \langle \beta | a\rangle. \tag{8}$$

Here $|\beta\rangle$ is the spin-isospin subspace state representing the projectile spin and isospin direct coupling to the corresponding i-th particle variables, and $\langle a|\beta\rangle$ is the recoupling coefficient. $t_i^{\beta'\beta}(\vec{k}', \vec{k})$ is a two-body operator diagonal with respect to other (A-1) particles variables of $|\beta\rangle$.

Note that the Green function of (2), (3), (6) has the same form both in the shifted and nonshifted basis, since Q and H_0 commute with each other.

So, in the basis (7) the solution of the FSA equations is an amplitude depending on the (A-1) vectors $\vec{r}_i$. Thereby one has to integrate it over 3(A-1) variables [28].

To reduce the multi-dimensional-integration difficulty, let us take the expansion

$$|\vec{r}_i\rangle = \sqrt{4\pi} \sum_{L_i M_i} |r_i L_i M_i\rangle Y^*_{L_i M_i}(\hat{\vec{r}}_i). \tag{9}$$

The FSA-Hamiltonian is diagonal in $|\vec{r}_i\rangle$. This fact yields its diagonality with respect to r_i of $|r_i L_i M_i\rangle$. But the Hamiltonian is not diagonal in L_i and M_i. To prove the statement, consider the potential V_i of the Hamiltonian.

$$V_i = Q_i v_i Q_i^+, \tag{10}$$

where v_i is a two-particle potential.

$$\langle \vec{k}', r_i' L_i' M_i' | V_i | \vec{k}, r_i L_i M_i\rangle =$$

$$= \frac{1}{4\pi} \int d\vec{r}_i'' \, d\vec{r}_i''' \frac{\delta(r_i'' - r_i')}{r_i''^2} \cdot \frac{\delta(r_i''' - r_i)}{r_i^2} Y^*_{L_i' M_i'}(\hat{\vec{r}}_i'') Y_{L_i M_i}(\hat{\vec{r}}_i''') \times$$

$$\times \langle \vec{k}', \vec{r}_i'' | Q_i v_i Q_i^+ | \vec{k}, \vec{r}_i'''\rangle.$$

Using the diagonality in $|\vec{r}_i\rangle$, we obtain

$$\langle \vec{k}', \vec{r}_i'' | Q_i v_i Q_i^+ | \vec{k}, \vec{r}_i'''\rangle = v_i(\vec{k}', \vec{k}) \exp[i(\vec{k} - \vec{k}')\vec{z}_i''] \delta(\vec{r}_i'' - \vec{r}_i''').$$

Then

$$\langle \vec{k}', r_i' L_i' M_i' | V_i | \vec{k}, r_i L_i M_i\rangle =$$

$$= \frac{1}{4\pi} \cdot \frac{\delta(r_i' - r_i)}{r_i^2} v_i(\vec{k}', \vec{k}) \int d\hat{\vec{r}}_i Y^*_{L_i' M_i'}(\hat{\vec{r}}_i) Y_{L_i M_i}(\hat{\vec{r}}_i) \exp[i(\vec{k} - \vec{k}')\vec{z}_i],$$

that completes the proof.

Performing the Multipole-expansion (9) of the state (7) both in the $|\vec{r}\rangle$ and $|\vec{k}\rangle$ -subspaces, and coupling all spin and angular momenta we get the basis of states with a conserving total angular momentum

$$|k, r, \eta\rangle \equiv |k, r, (\ell s L) JJ_z, (t_z T) tt_z\rangle, \tag{11}$$

where η stands for all discrete quantum numbers.

In the basis (11) the FSA-amplitude is a function of (A-1) scalar variables r_i, thereby this basis leads to a 3-time reduction in integration over target variables.

The diagonality in the total angular momentum $|JJ_z\rangle$ also simplifies the calculations with (11) instead of (7).

Moreover, the use of (7) takes into account all terms of the multipole expansion of both intermediate and initial and final states. The projection onto L_i contributing to the target state is made at the last step performing the integration over $\vec{r}$. As a result, the basis (7) gives no simplification even for a target with the S-state only, while the basis (11) in this case permits one to take into account appropriate values of L_i.

In view of the recurrent structure of the set (6) we can do a further simplification of the problem by the use of a convenient basis for each of the equations of (6) since we have to solve them successively but not all at once.

Consider the equation for r_i

$$r_i = t_i + t_i G_0 T_{i-1} G_0 r_i. \tag{12}$$

Here we note once more that r_i is diagonal with respect to variables of (A-i) spectators. The amplitude r_A connecting all particles has a maximal dimension.

Iterating eq. (12) we see that r_i is the amplitude of the process which begins and finishes at an i-th particle. Thus, to solve (12), let us take the basis shifted by Q_i to the i-th particle.

After the partial-wave decomposition of the shifted state $|\vec{k}_i, \vec{r}, a>$ we couple the angular momenta and isospin to obtain a state with the projectile- and i-th-particle direct coupling

$$|k_i, r, \chi_i>. \qquad (13)$$

In the basis (13) the inhomogeneous term of (12) is diagonal with respect to quantum numbers of all the particles except the i-th one.

As a rule, the 2-body interaction v_i is taken in a finite number of channel states, which are characterized by an orbital ℓ_i and a total j_i angular momenta and by a total isospin. This permits us to take into account just the states of (13) corresponding to the channels just we have to include.

Solving (12) in the basis (13) one can easily transform the solution to the natural unshifted basis (11) by means of the transition matrix $<k', r', \eta' | k_i, r, \chi_i>$. This transformation is a simple sum over the discrete variables.

So, each of the equations of (6) has its own convenient basis which simplifies the solution considerably.

The use of the unique basis (11) for all the equations gives some complications. For example, one has to include the intermediate states of all orbital angular momenta ℓ of the relative motion xA (number of ℓ may be more than of ℓ_i) and to sum implicitly over the two-particle spin-isospin states corresponding to neglected channels. As a result, the dimension of the matrix to be inverted (for a separable-potential case) increases significantly. Moreover, in the basis (11) the inhomogeneous term (8) loses its simple diagonal structure because of the recoupling coefficients $<a|\beta>$ and the operator Q_i.

Note that the possibility of the choice of own basis for each of the equations of (6) is ensured by the recurrent structure of the set (6).

For comparison consider the set (3). Iterating the eq. for T_i we see that this amplitude also (as r_i of (12)) describes the process beginning and finishing at an i-th particle. However, the set (3) couples all A-equations which we have to solve all at once and therefore in a common basis.

IV. NUMERICAL EXAMPLE

As an example of the application of the recurrent equations (6), consider the pion-deuteron scattering. In that case the set (6) becomes

$$\begin{cases} r_2 = t_2 + t_2 G_0 t_1 G_0 r_2, \\ T_2 = t_1 + r_2 + t_1 G_0 r_2 + r_2 G_0 t_1 + t_1 G_0 r_2 G_0 t_1. \end{cases} \qquad (14)$$

To solve (14), we introduce the three bases

$$|k_1 r \nu_1> \equiv |k_1 r((\ell_1 s_1) j_1 (s_2 L) J_2) JJ_z;\ ((t_\pi t_1) t_{\pi 1} t_2) tt_z>,$$

$$|k_2 r \nu_2> \equiv |k_2 r((\ell_2 s_2) j_2 (s_1 L) J_1) JJ_z;\ ((t_\pi t_2) t_{\pi 2} t_1) tt_z>,$$

$$|k r \nu> \equiv |k r(\ell((s_1 s_2) s_{12} L) j_{12}) JJ_z;\ (t_\pi (t_1 t_2) t_{12}) tt_z>.$$

The state $|k_2 r \nu_2>$ is a specified form of (13) appropriate for our problem. The angular-momenta coupling-scheme is as follows:

$$(\vec{\ell}_2 + \vec{s}_2) + (\vec{s}_1 + \vec{L}) = \vec{J}, \quad (\vec{t}_\pi + \vec{t}_2) + \vec{t}_1 = \vec{t},$$

where s_i, t_i, t_π are the i-th nucleon spin and isospin and the pion isospin, and ℓ_2, L are the orbital angular momenta of relative motion of πN_2 and $N_1 N_1$.

Let the πN interaction be described by the separable potential

$$< k_i' \mu_i' | v_i | k_i \mu_i> = \delta_{\mu_i' \mu_i} \lambda_{\mu_i} h_{\mu_i}(k_i') h_{\mu_i}(k_i), \qquad (15)$$

where $\mu_i \equiv (\ell_i j_i t_{\pi i})$ is the set of quantum numbers defining a πN channel state $|\mu_i>$.

From (15) it follows that the matrix elements of t_i and r_2 may be written as

$$<k_i' \mu_i' | t_i | k_i \mu_i> = \delta_{\mu_i' \mu_i} \Gamma_{\mu_i} h_{\mu_i}(k_i') h_{\mu_i}(k_i),$$

$$<k_2' \nu_2' | r_2 | k_2 \nu_2> = h_{\mu_2'}(k_2') Z_{\nu_2' \nu_2}(r) h_{\mu_2}(k_2).$$

Then from (14) we obtain

$$Z_{\nu_2' \nu_2}(r) = \delta_{\nu_2' \nu_2} \Gamma_{\mu_2} + \sum_{\nu_2''} \Gamma_{\mu_2'} P_{\nu_2' \nu_2''}(r) Z_{\nu_2'' \nu_2}(r), \qquad (16)$$

$$P_{\nu_2' \nu_2}(r) = \left(\frac{1}{2\pi^2}\right)^2 \int_0^\infty dk_2' dk_2 (k_2' k_2)^2 h_{\mu_2'}(k_2') <k_2' \nu_2' | G_0 t_1 G_0 | k_2 \nu_2> h_{\mu_2}(k_2).$$

Equation (16) can be easily solved

$$Z_{\nu_2' \nu_2}(r) = [\delta_{\nu_2' \nu_2} / \Gamma_{\mu_2} - P_{\nu_2' \nu_2}(r)]^{-1}. \qquad (17)$$

To calculate $P_{\nu_2' \nu_2}(r)$, we need to make the transition from $|k_2 r \nu_2>$ to $|k_1 r \nu_1>$. The corresponding transition matrix may be easily obtained by the use of the complete set of unshifted states $|\vec{k}, \vec{r}, a>$.

$$\langle k_2' r' \nu_2' | k_1 r \nu_1 \rangle = \sum_a \int \frac{d\vec{k}''}{(2\pi)^3} d\vec{r}'' \langle k_2' r' \nu_2' | \vec{k}'', \vec{r}'', a \rangle \langle \vec{k}'', \vec{r}'', a | k_1 r \nu_1 \rangle =$$

$$= \sum_a \int \frac{d\vec{k}''}{(2\pi)^3} d\vec{r}'' \exp(-i\vec{k}''\vec{r}'') \langle k_2' r' \nu_2' | \vec{k}_2'', \vec{r}'', a \rangle \langle \vec{k}_1'', \vec{r}'', a | k_1 r \nu_1 \rangle .$$

Similarly, the transition is to be done from $|k_2 r \nu_2\rangle$ to $|k r \nu\rangle$. In view of the separability of t_1 and τ_2 we can easily construct the total amplitude T_2 according to (14), since the integrals over intermediate momenta are factorized.

The final expression of T_2 can be found in the Appendix.

Let us estimate the dimension of the matrix (17) which we need to invert. The total angular momentum J and isospin t are integrals of motion. Thus, the dimension of the matrix z is defined by the number of possible combinations of the quantum numbers $(\ell_2\, j_2\, t\, \pi_2 L J_1)$ of the state ν_2. The first three numbers of the set define the πN-channel state $|\mu\rangle$, thereby we have as many combinations of $(\ell_2\, j_2 t \pi_2)$ as the πN-channels to include.

The nucleon-nucleon relative motion angular-momentum L and its sum J_1 with the nucleon spin have some possible combinations depending on the total J and πN-channel j_2 angular momenta.

For simplicity we assume that the nucleons may be in the relative S-state only. Hence the set (L, J_1) has only one combination (0, 1/2), and the dimension of the matrix Z is equal to the number of πN-channels.

We have used the FSA amplitude T_2 to calculate the πd scattering length in the framework of the nuclear-Hamiltonian finite-rank-approximation method [18-22]

The essence of the method is as follows. The transition operator T, corresponding to the Hamiltonian (1) obeys the equation

$$T = T_{FSA} + T_{FSA} G_0 H_A G_A T , \qquad (18)$$

where $T_{FSA} = V + VG_0 T_{FSA}$.

If in the spectral expansion of H_A we retain the ground state only, $H_A \approx E_0 |\psi_0\rangle\langle\psi_0|$, then for the elastic scattering amplitude $\langle \vec{k}', \psi_0 | T | \vec{k}, \psi_0 \rangle$ we obtain from eq. (18) a one-dimensional integral equation with the inhomogeneous term $\langle \vec{k}', \psi_0 | T_{FSA} | \vec{k}, \psi_0 \rangle$.

Therefore, eq. (18) permits one to calculate corrections to FSA that take an approximate account of the nucleon motion.

The elastic πd-scattering being a three-body problem is well-studied on the basis of exact three-particle equations [23]. Our calculation has been performed to demonstrate the applicability of the equations (6).

To get the numerical estimation of the quality of our approximations, we have used the same πN and NN potentials as in [36], where the πd-scattering length had been calculated on the basis of the Faddeev equations. To describe NN-interaction, the authors of [36] have used a separable S-wave potential which in our case corresponded to the averaging of $T_{FSA}(r)$ over the Hulten wave function.

As in refs. [36], we have taken πN-potential in the channels S_{11} and S_{31} in the form $v(k', k) = \lambda / (k'^2 + \beta^2)/(k^2 + \beta^2)$ with $\beta = 3$ fm^{-1}. The depth λ was defined by πN scattering lengths of the corresponding isotopic state.

Table 1 contains the results of calculations which have been performed with four different sets of πN-length of [37-40].

Table

	/37/	/38/	/39/	/40/
$a_{\pi N}^{1/2}$ (fm)	-0.257	-0.257	-0.264	-0.240
$a_{\pi N}^{3/2}$ (fm)	0.154	0.126	0.148	0.130
$a_{\pi d}$ [36] (fm)	0.0736	0.0300	0.0606	0.0453
$a_{\pi d}$ (FSA)	0.0802	0.0353	0.0672	0.0504
$a_{\pi d}$ (eq. 18)	0.0795	0.0351	0.0667	0.0501
$a_{\pi d}$ (iteration)	0.0795	0.0351	0.0667	0.0501

In the third line the exact result is shown for the πd scattering length $a_{\pi d}$ [36]. The fourth and fifth lines contain our corresponding results for FSA and eq. (18). The last line is the solution of eq. (18) obtained by its first iteration. The experimental value of $a_{\pi d}$ [41] is as follows

$$a_{\pi d}(\text{exp}) = 0.073 \, {}^{+\,0.031}_{-\,0.024} \text{ fm.}$$

As one sees from the table, the result of FSA is sufficiently close to the exact value. The smallness of the correction due to the nucleon motion may be explained by the elimination of the nuclear continuum that may produce a main contribution to the correction. The equality of the exact and iterative solutions of eq. (18) is explained by the small value of $a_{\pi d}$ itself.

APPENDIX

If we assume that in the course of scattering the nucleons may form just the S-state only, then the FSA πd-elastic scattering amplitude can be calculated by the following formulae

$$<k'\ell'\psi|T_2^J|k\ell\psi> = \delta_{\ell'\ell}\sum_{\mu'\mu} h_{\mu'}(k')h_{\mu}(k)\int_0^\infty dr\, u^2(r)\, j_0(\frac{k'r}{2})\, j_0(\frac{kr}{2}) M_{\mu'\mu}^{11}(r),$$

$$\int_0^\infty dr\, u^2(r) = 1,$$

$$M_{\mu'\mu}^{a'a}(r) = \delta_{\mu'\mu} X_{\mu}^{a'a} + Y_{\mu'\mu}^{a'a}(r) - W_{\mu'\mu}^{a'a}(r) - W_{\mu\mu'}^{aa'}(r) + Q_{\mu'\mu}^{a'a}(r),$$

$$X_{\mu}^{a'a} = C_{\mu a'}\Gamma_{\mu} C_{\mu a},$$

$$Y_{\mu'\mu}^{a'a} = (-1)^{s_{a'}+s_a+t_{a'}+t_a} C_{\mu'a'} Z_{\mu'\mu} C_{\mu a},$$

$$W_{\mu'\mu}^{a'a} = \sum_{a''\mu''} X_{\mu'}^{a'a''}\tilde{g}_{\mu'\mu''} Y_{\mu''\mu}^{a''a},$$

$$Q_{\mu'\mu}^{a'a} = \sum_{a''\mu''} W_{\mu'\mu''}^{a'a''}\tilde{g}_{\mu''\mu} X_{\mu}^{a''a},$$

$$Z_{\mu'\mu} = (\delta_{\mu'\mu}/\Gamma_{\mu} - P_{\mu'\mu})^{-1},$$

$$P_{\mu'\mu} = \sum_{\mu''} B_{\mu'\mu'} g_{\mu'\mu''}\Gamma_{\mu''} g_{\mu''\mu} B_{\mu''\mu},$$

$$g_{\mu'\mu} = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 h_{\mu'}(k) h_{\mu}(k) j_0(kr)/(E_k - Z),$$

$$\tilde{g}_{\mu'\mu} = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 h_{\mu'}(k) h_{\mu}(k)[j_0(\frac{kr}{2})]^2/(E_k - Z),$$

$$\Gamma_{\mu} = (1/\lambda_{\mu} + \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 h_{\mu}^2(k)/(E_k - Z))^{-1},$$

$$E_k = \frac{k^2}{2\mu_{\pi d}},$$

$$B_{\mu'\mu} = \delta_{\ell_{\mu'}\ell_{\mu}} (-1)^{j_{\mu'}+j_{\mu}+t_{\mu'}+t_{\mu}} \sqrt{(2j_{\mu'}+1)(2j_{\mu}+1)(2t_{\mu'}+1)(2t_{\mu}+1)} \times$$

$$\times \begin{Bmatrix} \frac{1}{2} & \ell_{\mu} & j_{\mu} \\ \frac{1}{2} & J & j_{\mu'} \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & 1 & t_{\mu} \\ \frac{1}{2} & t & t_{\mu'} \end{Bmatrix},$$

$$C_{\mu a} = \delta_{\ell_{\mu}\ell} (-1)^{1+\ell+J+t} \sqrt{(2j_{\mu}+1)(2s_a+1)(2t_{\mu}+1)(2t_a+1)} \times$$

$$\times \begin{Bmatrix} \ell & \frac{1}{2} & j_{\mu} \\ \frac{1}{2} & J & s_a \end{Bmatrix} \begin{Bmatrix} 1 & \frac{1}{2} & t_{\mu} \\ \frac{1}{2} & t & t_a \end{Bmatrix}.$$

Here $u(r)$ is the deuteron wave function, μ labels πN-channel states with quantum numbers (ℓ_μ, j_μ, t_μ), a labels NN-channel states with quantum numbers (s_a, t_a). The deuteron state corresponds to $a = 1$ with $s_a = 1$ and $t_a = 0$.

REFERENCES

1. Brueckner K.A. Phys.Rev., 1953, 89 (4), p. 834.
2. Brueckner K.A. Phys.Rev., 1953, 90 (4), p. 715.
3. Foldy L.L., Walecka J.D. Ann.Phys. (N.Y.), 1969, 54(3), p. 447.
4. Avishai Y., Rinat A.S. Phys.Lett., 1975, B55, p. 153.
5. Benayoun J.J., Gignoux C., Gillespie J. Nucl.Phys., 1976, A274, p. 525.
6. Sparrow D.A. Ann.Phys., (N.Y.), 1975, 91(1), p. 157.
7. Пересыпкин В.В. В сб.: Тезисы докладов Всесоюзной конференции по теории систем нескольких частиц с сильным взаимодействием. Изд-во ЛГУ, Л., 1983, с. 26.
8. Harrington D.R. Phys.Rev., 1964, 135B, p. 358.
9. Kowalski K.L., Pieper S.C. Phys.Rev., 1971, C4(1), p. 74.
10. Gibbs W.R. Phys.Rev., 1971, C3(3), p. 1127.
11. Kujawski E., Lambert E. Ann. Phys. (N.Y.), 1973, 81(2), p. 591.
12. Johnson M.B., Bethe H.A. Nucl.Phys., 1978, A305(2), p. 418.
13. Johnson M.B., Keister B.D. Nucl.Phys., 1978, A305(2),p.461.
14. Piepho M.G., Walker G.E. Phys.Rev., 1974, C9(4), p. 1352.
15. Ситенко А.Г. Укр. Физ.Ж, 1959, 4, с. 152.
16. Glouber R.J. Phys.Rev., 1955, 100, p. 242.
17. Ситенко А.Г. ЭЧАЯ, 1973, 4/2/, с. 546.
18. Rosenberg L. Phys.Rev., 1976, C13(4), p. 1406.

19. Беляев В.Б., Вжеционко Е. ЯФ, 1978, 28, с. 392.
20. Беляев В.Б., Пупышев В.В. ЯФ, 1980, 31, с. 1324.
21. Belyaev V.B., Rakityansky S.A., Wrzecionko J. Nucl.Phys., 1981, A368, p. 394.
22. Belyaev V.B., Pupyshev V.V. JINR, P4-83-173, Dubna, 1983.
23. Thomas A.W., Landau R.H. Phys.Rep., 1980, 58, p. 121.
24. Копалейшвили Т.И. ЭЧАЯ, 1979, 10/2/. с. 429.
25. Agassi D., Gal A., Mandelzweig V. Ann.Phys.(N.Y.), 1975, 91(1), p. 194.
26. Hüfner J. Nucl.Phys., 1973, B58(1), p. 55.
27. Gibbs W.R. et al. Phys.Rev., 1976, C13(6), p. 2433.
28. Gibbs W.R., Hess A.T., Kaufmann W.B. Phys.Rev., 1976, C13(5), p. 1982.
29. Goldberger M.L., Watson K.M. Collision Theory. "John Wiley & Sons", N.Y., 1964.
30. Harrington D.R. Phys.Rev., 1965, 139B, p. 691.
31. Haeringen H., Wageningen R. J.Math.Phys., 1975, 16, p.1441.
32. Ernst D.J. et al. Phys.Rev., 1977, C16(2), p. 537.
33. Siciliano E.R., Thaler R.M. Phys.Rev., 1977, C16(4), p.1322.
34. Siciliano E.R., Thaler R.M. Ann.Phys., 1978, 115, p.191.
35. Messiah A.M.L. Quantum Mechanics. "Nauka", M., 1978.
36. Peresypkin V.V., Petrov N.M. Nucl.Phys., 1974, A220(2), p. 277.
37. Ericson T.E.O. CERN, TH-1093, Geneva, 1969.
38. Samaranayke V.K., Woolcock W.S. Nucl.Phys., 1972, B48, p.205.
39. Sznajder N. Nucl.Phys., 1972, B48, p. 549.
40. Bugg D.V., Carter A.A., Carter J.R. Phys.Lett., 1973, B44, p. 278.
41. Bailey J. et al. Phys.Lett., 1974, B50, p. 403.

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Беляев В.Б., Ракитянский С.А. E4-84-118
Рекуррентные уравнения приближения фиксированных центров

В рамках ПФП получены уравнения, связывающие амплитуду рассеяния на системе из A частиц с соответствующей амплитудой для (A-1) частиц. Показано, что рекуррентная структура этих уравнений позволяет оптимально выбирать базис для каждого из них, что значительно упрощает задачу. В качестве численного примера рассмотрено упругое рассеяния пиона на дейтроне.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Recurrent Equations of the Fixed-Scatterers Approximation

Equations connecting the amplitude of scattering on an A-particle system with the one for an (A-1)-system are obtained in the FSA framework. It is shown that this recurrence permits one for each of the equations to get an optimal choice of the basis, which simplifies the task considerably. As a numerical example the elastic pion-deuteron scattering is considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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