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I.N.Mikhailov, R.Reif

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I.N.Mikhailov, R.Reif *

DENSITY-DEPENDENT EFFECTIVE NUCLEON-NUCLEON INTERACTIONS AND DIRECT NUCLEAR REACTIONS

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* Technical University Dresden, Section of Physics, Dresden (DDR)

1. Introduction

In the recent years many attempts have been made to calculate the properties of finite nuclei using the nucleon-nucleon potentials, derived from the scattering properties of elementary particles [1]. A number of such "semirealistic" potentials depends on the local value of the nucleon density at the point where two particles collide.

Density-dependent forces are particularly used in Hartree-Fock calculations to predict bulk properties of nuclei such as total binding energies, density distributions and mean square radii, see, e.g., [2]. Using the Migdal force [3], Band et al. [4] studies the influence of the density dependence on particle-hole excitations. With the same force Theis and Werner [5] calculated the form factors for inelastic electron scattering from low-lying states of ^{208}Pb. Recently, Sharp and Zamick [6] demonstrated the possibility of using density-dependent interactions in nuclear structure studies based on the shell model to investigate single particle energies, effective nucleon charges for electromagnetic transition probabilities and the excitation of simple configurations. Particular attention has been given to monopole excitations (breathing modes), which are mostly effected by density dependence [7].

There is much evidence from the microscopic description of direct inelastic nucleon scattering that together with the separation method of Scott and Moszkowski [8] realistic interactions such as the Hamada-Johnston potential can be used to reproduce measured angular distribu-

tions as well as the absolute transition strength [9]. So it would be worthwhile to have information as to what extent the density-dependent forces fitted to ground state saturation properties of nuclei can be used not only for investigating the bulk properties and the structure of low-lying excited states but also for nuclear reaction calculations.

It is likely that the effects of density dependence should manifest themselves most clearly in direct reactions, which proceed preferably in the nuclear surface region where the nucleon density changes rapidly. We would like to mention that also from the exciton model of precompound processes there is evidence on the density dependence of intranuclear transition rates [10].

The paper presents some DWBA calculations in a microscopic approach to direct inelastic proton scattering using different density-dependent effective interactions. The results have been given already partly in ref. [11]. The details of the formalism and expressions for the form factors used are described elsewhere [12]. For simplification, exchange terms and spin-flip processes were neglected. The optical parameters and the parameters for calculating bound single-particle wave functions may be found in refs. [9,12,13]. In all cases a uniform density distribution of the Woods-Saxon shape $\rho(0)=0.17$ nucleons/fm^3 and geometrical parameters $r=1.2$fm, $a=0.7$fm, have been employed. It should be emphasized that in all calculations there is no consistency between the nuclear wave functions used and the density dependence of the force.

In sect. 2 a brief discussion of density dependence of nucleon-nucleon forces is given. Sect. 3 contains investigations of collective octupole excitations in the reaction ^{116}Sn(p,p′) and monopole transitions in ^{28}Si(p,p′) using the Migdal force. The results with the finite range interaction of Sprung and Benerjee [14] are summarized in sect. 4. Because the main aim was to test different forces, in most cases oversimplified nuclear wave functions have been used. So we have made only a rough comparison of calculated absolute transition strength with experimental data.

2. The Density Dependence of the Nucleon-Nucleon Force

Here we discuss briefly the origin of the density dependence in the internucleon interaction and also the main features of such a dependence. This allows one to foresee some of the effects of the density dependence in the force on the inelastic scattering process.

2.1. Fundamental properties of the interaction - the field theory effects

The exchange of two π mesons between three nucleons described by the diagram (a) of Fig. 1 where both mesons

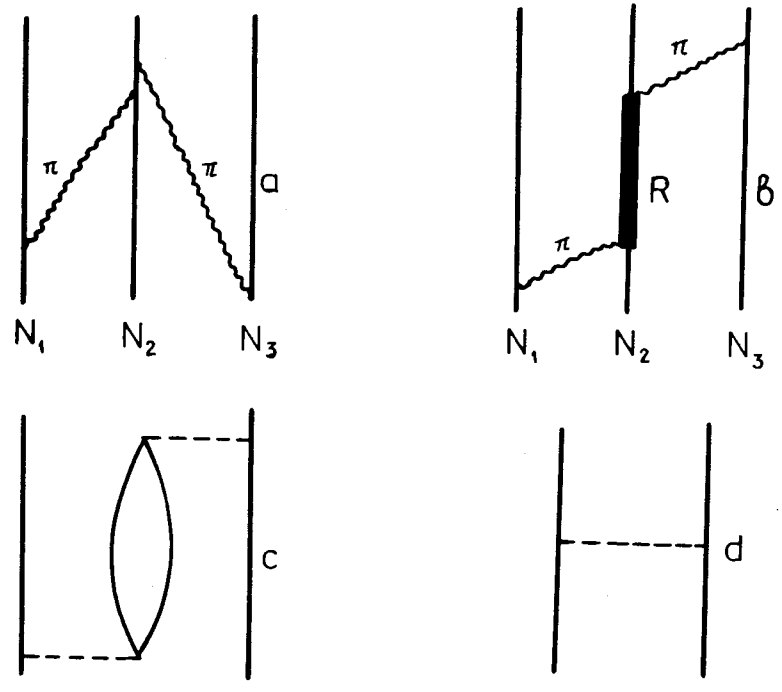

Fig. 1. Diagrams corresponding to some of the processes whose description brings the density dependence into the effective force. Explanation in the text.

are in the air at the same time can be regarded as a density-dependent renormalization of the interaction between each of the three pairs of nucleons [15]. The Compton range for the two π mesons is half of that for one meson and consequently the range of the interaction associated with this process should be very small.

The resonances in the scattering of mesons on nucleons are associated with the contribution to the three-body forces by the graphs of type (b) in Fig. 1 [15]. The range of such a force must be of the order (t_{Res} : half-life)

$$R_{Res} = V_{Res} \cdot t_{Res} = \frac{V_{Res}}{c} \frac{5 \cdot 10^2}{\Gamma_{Res}(MeV)} fm .$$

Especially important is the Δ_{33} resonance in the $\pi - p$ scattering at 200 MeV with a width $\Gamma_{Res} \sim 10^2$ MeV. In nuclei the kinetic energy of colliding nucleons is smaller than the resonance energy and thus the effective value of V_{Res} is smaller than $V_{nucleon} \sim (\frac{20}{10^3})^{1/2} C$.

Thus

$$R_{Res} < \approx 0.7 fm$$

The small range of the three-body force was used by Skyrme [16] to approximate it by a local force proportional to δ -functions of two internucleon distances in the system of three colliding nucleons. This force is in some sense equivalent to the local binary force, acting between each of the three pairs, with the strength proportional to the density at the point where the nucleons collide.

2.2. The semi-fundamental properties - the nature of a smooth effective interaction

In addition to the effects mentioned above there exist some others which also result in a density dependence of the force between two nucleons in nuclear matter. An additional density dependence appears in the nuclear theory as the price for reducing the basis used to construct the many-body wave functions for nuclei. The interaction in

the reduced basis simulating the effects of the realistic interaction working in a large basis is called an effective interaction and differs from the realistic interaction. It is not always easy to establish relations between the bases used in different nuclear models and this consideration brings to life many investigations (including the present one) where the effective interaction taken from one model is used to calculate the properties described by another model. Let us mention some procedures allowing to reduce the basis in the many-body problems.

a) Exclusion of the short-range repulsion between the nucleons is the prime goal of the nuclear matter theory. Different techniques for doing this are known and the resultant effective force is like the force in the Scott-Moszkowski separation method [8], which is the long-range part of the realistic force with a separation distance d increasing with the density of nuclear matter.

b) The tensor force in the closure approximation is taken as the central part of an operator

$$V_T(closure) \approx V_T \frac{P}{e} V_T$$

with the operator P/e, excluding the occupied states and involving the energy denominator, dependent on the density. This dependence lessens the attraction due to the tensor force at high densities.

c) The simplest example of correlation effects is described by the diagram (c) of Fig. 1 containing two external lines. This diagram introduces the state (and consequently density) renormalization of the first order matrix element of the interaction (part (d) of Fig. 1). Such renormalization turned out to be important for the spectrum of low-lying states and might also be important for the scattering process.

d) In the local density approximation the effective interaction between a pair of nucleons in a certain region of a finite nucleus is taken as the G-matrix for infinite nuclear matter of corresponding density, expressed as a local, momentum dependent two-body operator in coordinate space. Averaging this G-matrix over the Fermi sphere as well as over angular momentum states J ,

L of the interacting pair one obtains a local central force for each spin (S)- isospin (T) subspace, which becomes density dependent because of the upper limit of integration over momentum. In this way various density-dependent forces are derived using different nucleon-nucleon potentials or different procedures to connect the local two-body force with the elements of the G-matrix, e.g., Negele [17], Sprung and Benerjee [14] or Nemeth and Ripka [18]. A simple zero-range density-dependent force has been given by Migdal [3].

2.3. The manifestation of the density dependence of the force

The first motivation to use the density-dependent force gave the "1"-dependence of the isomer shifts and quadrupole moments. This dependence indicated that the internucleon attraction is the strongest just outside the nuclear surface [19].

The Galilean invariance conditions relating the properties of the average (self-consistent) field and the interaction are strongly violated when the simple (δ-type) effective interaction was used without the density dependence. These conditions are important when 1^--states are described. They can be accounted for if one assumes that the nucleons interact only outside the nuclear matter [20].

The discrepancy between the calculation of the saturation properties of nuclear matter and the empirical data is also calling for some density dependence in the force [16]. We discuss the saturation properties in some details to estimate the density dependence of the effective δ-force. Most of effects of the density dependence are connected with the small distances and one may expect the effective δ-force to be a reasonable approximation to the realistic case. Following Bochnacki, Holban and Mikhailov [21] we shall use the relation between the energy, potential and scattering amplitude from the Fermi-liquid theory by

Landau. Let us assume that the potential energy of a nucleus is given by the formula

$$\epsilon_{pot} = \int d\vec{r}\, e_{pot}(\rho(\vec{r}))$$

where $\rho(\vec{r})$ is the local value of the particle density at a point $\vec{r}$. We write also

$$e_{pot}(\rho(\vec{r})) = -\frac{a}{2}\rho^2 + \frac{b}{\gamma+2}\rho^{\gamma+2}.$$

The three constants a, b, γ are positive if this formula describes saturation at the density ρ_0 with $e_{pot}(\rho_0) < 0$. The self-consistent potential and the force (scattering amplitude) are given by the variational derivatives of ϵ_{pot} in the density matrix. With ϵ_{pot} given above the self-consistent potential is local

$$U(\vec{r}) = -a\rho(\vec{r}) + b\rho^{\gamma+1}(\vec{r})$$

while the force operator is local and of the contact type:

$$\tilde{f} = \delta(\vec{r}_1 - \vec{r}_2)\{-a + (\gamma+1)b\rho^{\gamma}\}.$$

The two constants a and b can be determined from the saturation conditions in nuclear matter. In ref. [21] the parameter γ was taken arbitrarily to be equal to 1. One can use the additional relation to fix γ so that the self-consistent potential would have the appropriate depth at the center of the nucleus. The conditions then are

$$\mathrm{Min}\ \frac{e_{pot} + e_{kin}}{\rho} = -e_0$$

$$\rho_{min} = \rho_0$$

$$U(\vec{r}_0) = U_0,$$

where $e_0, T_F^{(0)}, U_0$ and ρ_0 denote the empirical values of the binding energy per nucleon, the kinetic energy at the Fermi surface, the depth of the potential in the interior of a nucleus and the saturation density, respectively. The third relation is reasonably well reproduced with $\gamma = 1$ and we take this value in the following. The coeffi-

cients a and $b\rho_0$ are then given by the formula ($\alpha = e_0 / T_F^{(0)}$)

$$a = 4(\alpha + \frac{2}{5}) T_F^{(0)} / \rho_0$$

$$b\rho_0 = 3(\alpha + \frac{1}{5}) T_F^{(0)} / \rho_0 \quad .$$

The most essential property of the density dependent force is its weakness in the interior regions of the nucleus. For example, the strength of the δ-force inside the nucleus is proportional to

$$f_{int} = -a + 2b\rho_0 = 2(\alpha - \frac{1}{5}) T_F^{(0)} / \rho_0 .$$

Taking $\alpha = 1/5$ (which is close to the empirical data without corrections for the surface and Coulomb energy) we obtain $f_{int} = 0$, i.e., there is no interaction between nucleons inside the nuclear matter. In this case the strength of the force in the exterior regions of the nucleus is equal to

$$f_{out} = -a = -\frac{12}{5} T_F^{(0)} / \rho_0 .$$

The practical implication of this is the small amount of correlations between nucleons inside the nucleus which appear in the nuclear models using the density dependent force. One may understand this as a result of the implicit description of various correlations in the approximation, where the effective density dependent force is taken to the lowest order and without exchange terms. The argument for this gives the formula for the energy e_{pot}. The first term in this formula is expected for an arbitrary binary potential and for the uniform distribution of nuclear matter in the Hartree approximation. Then the second term may be interpreted as the correction to the approximation where no processes discussed at the beginning of this section are taken into account. We notice here that the quadratic dependence on the density in e_{pot} is removed by the Pauli principle in the Hartree-Fock theory. Thus we may think that the effects of the exclusion principle are also simulated by the density dependence of the δ-force.

3. Zero-Range Forces of the Migdal Type

3.1. Collective excitations

As an example, the excitation of the 3^-, 2.27 MeV level of ^{116}Sn in the inelastic scattering of 16 MeV protons has been treated with the wave functions of Clement and Baranger [22] using a zero-range force with a term proportional to the density

$$v_{i0} = c(a + b\rho(r_i)/\rho_0)\delta(\vec{r}_i - \vec{r}_0)$$

$$\rho(r) = \rho_0 [1 + e^{(r-R)/a}]^{-1} \quad .$$

Theoretical results are compared with experimental data of Makofske et al. [23].

The angular distribution of inelastically scattered particles is drastically changed by varying the force parameter b (Fig. 2). Weakening the attraction in the nuclear interior, the agreement between measured and calculated angular distributions is improved, in particular in the region of the minimum at about $\theta_{c.m.} \approx 60°$ and after the maximum at about $\theta_{c.m.} \approx 90°$. A reasonable fit up to $\theta_{c.m.} \approx 120°$ is obtained for $b \approx -a$, i.e., for vanishing interaction in the centre of the nucleus. An appreciable repulsion in the inner region gives no acceptable results. This is in a good agreement with the arguments in Sect. 2 and also in [20] obtained from the analysis of isomer shifts in Nd, Sm and Gd isotopes and from electromagnetic moments and transition probabilities in the lead region. A derivation of the force parameters from the condition of translational invariance gives similar results [20].

The cross section is reduced by decreasing the contribution from the inner part of the nucleus. To reproduce the observed transition strength at $b \approx -a$ a depth parameter $c \approx 700$ MeV is necessary which is higher than the estimates $c \approx 386$ MeV and $c \approx 320$ MeV given in ref. [20] and ref. [21]. Such deviations are not unexpec-

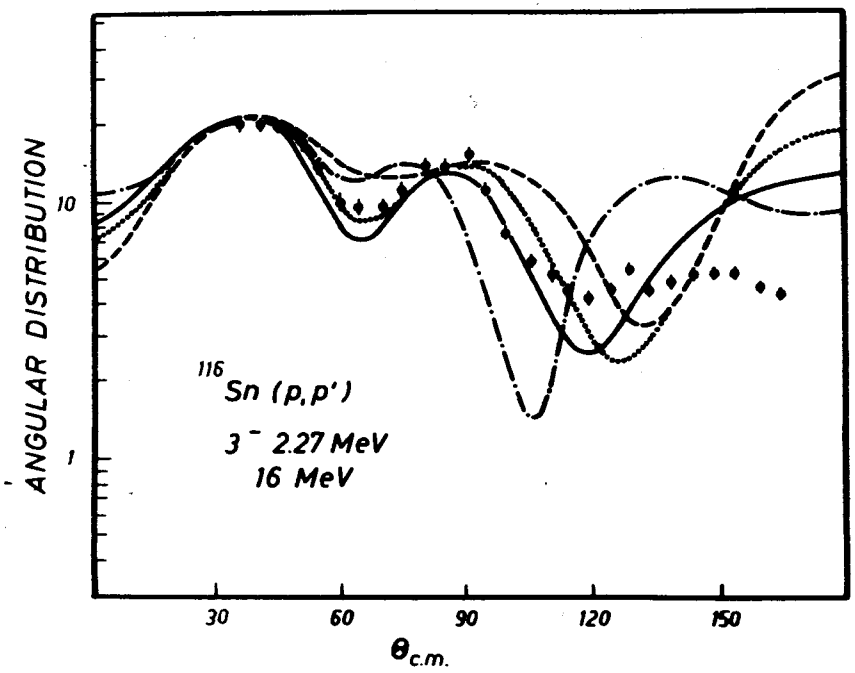

Fig. 2. Angular distribution for the reaction $^{116}Sn(p,p')$. Theoretical curves are calculated for the Migdal force with different sets of parameters (a, b). Experimental data from ref.[23]. - - - (-2.0) · · · · (-2; 1.7) —— (-2.2) -·—·— (-2, 2.3).

ted because the results are very sensitive to the wave functions and to the magnitude of parameters r and a in $\rho(\vec{r})$. The change in these parameters to r = = 1.25 fm, a = 0.65 fm reduces the cross section at b=−a by a factor of two without altering the angular distribution. We mention here the result of [21] according to which the precise values of r and a in the expression of the force are somewhat different than those reproducing the charge and mass distribution.

3.2. Monopole excitations

All attempts to calculate the excitation of 0^+ states in a shell model description with phenomenological Gaussian or Yukawa potentials failed in two respects: (i) Fitting the strength of the effective interaction to $L \neq 0$ excitations the cross section for $L=0$ transitions is overestimated by about one order of magnitude. (ii) The angular distribution observed in the experiment is more pronounced in structure than the DWBA curve.

Because the monopole component of the interactions commonly used gives a form factor with a strong peak in the nuclear interior, the density dependence should greatly influence monopole transitions. This effect has been investigated quantitatively in the reaction $^{28}Si(p,p')$, Q = -4.97 MeV at E_p = 17 MeV. In a 1p 1h -model the excitation was restricted to the breathing mode $|0> \rightarrow$ $\rightarrow |(1p^{-1}_{1/2} 2p_{1/2})0>$. The peak of the form factor inside the nucleus depends strongly on the force parameter (Fig. 3). Taking into account a density-dependent term one gets a rapidly oscillating angular distribution (Fig. 4). For the parameter set a=−2, b=2 the cross section in forward direction is diminished by two orders of magnitude as compared with the case a=−2, b=0.

In a recent paper, Davies and Satchler [24] reported on calculations with the Skyrme force. They investigated the excitation of the lowest 3^- and 5^- states of ^{40}Ca by inelastic proton scattering of 17 MeV and 30 MeV incident energy. The transition density was constructed from par-

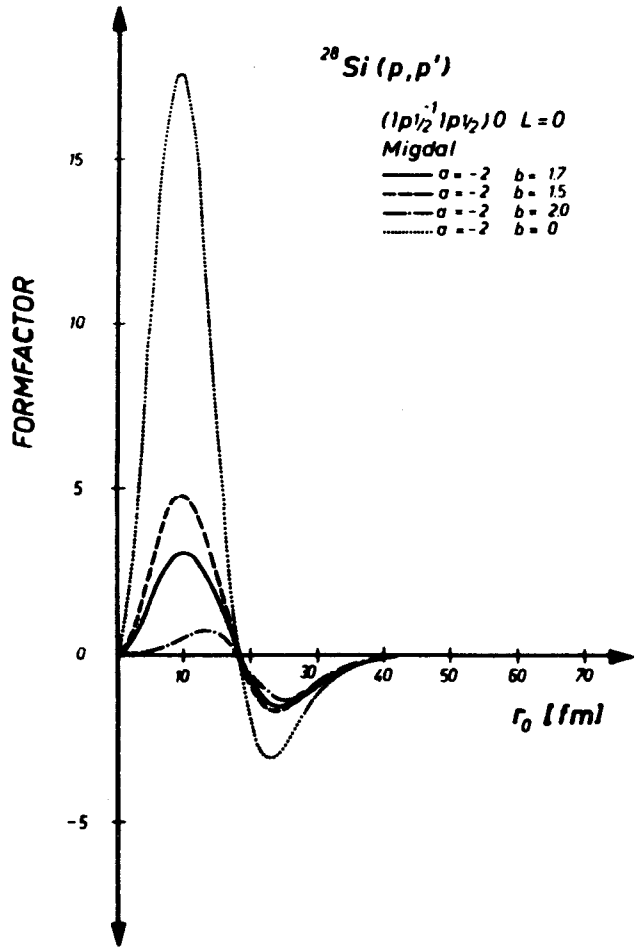

Fig. 3. Form factors for the transition $|0>\rightarrow|(1p_{1/2}^{-1}\,2p_{1/2})0>$ in $^{28}Si(p,p')$ with different parameters (a,b) of the Migdal force.

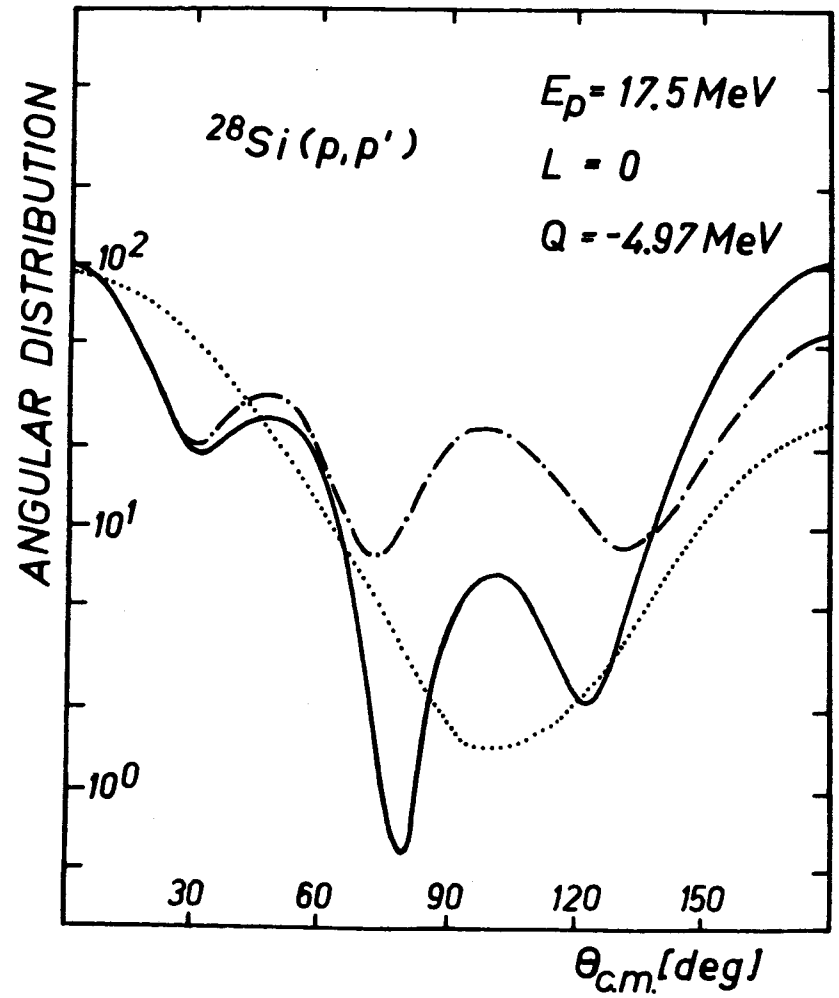

Fig. 4. DWBA angular distributions for the reaction $^{28}Si(p,p')$ calculated with form factors of Fig. 3.

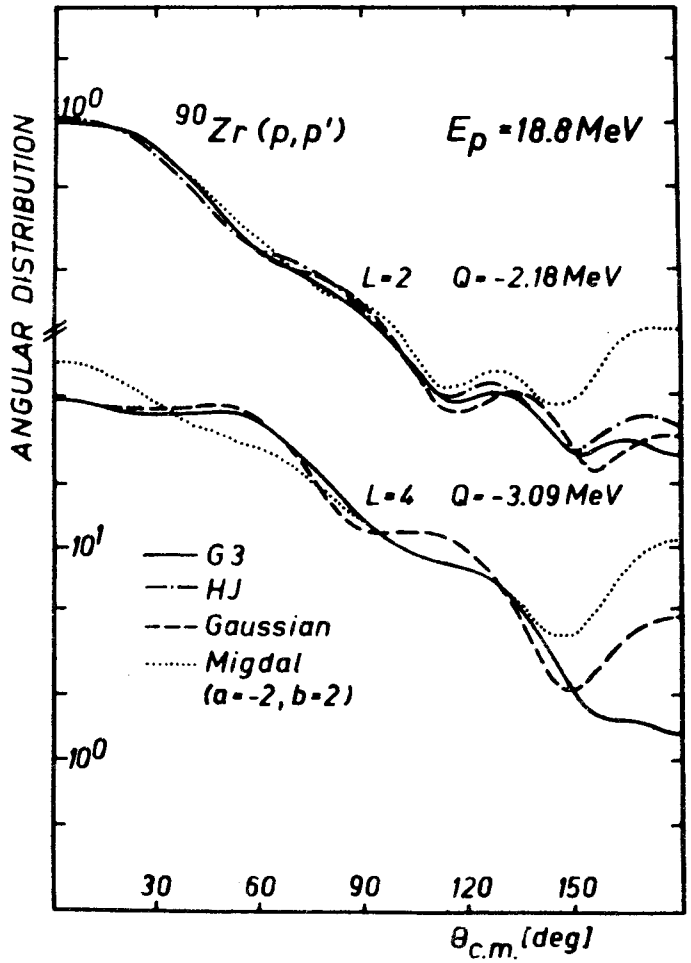

Fig. 5. DWBA angular distribution for the L=2 , Q= -2,28 MeV and L=4 , Q = -3.09 MeV transitions in $^{90}Zr(p,p')$ for different realistic interactions.

ticle-hole wave functions in the random phase approximation renormalized to reproduce experimental B(E2) values. The calculated cross section was too large in the first peak. The theoretical angular distribution is too pronounced in structure and has too large cross section in the backward region.

4. Finite Range Forces

A density-dependent force of finite range has been given by Sprung and Benerjee /14/. The interaction was calculated at four densities ρ/ρ_0 = 0.13, 0.5, 1.0 and 1.96 and expressed as a sum over five Gaussian potentials with different range parameters b. In its radial dependence the force is qualitatively fitted to the Negele interaction, that means to the OPEP at large distances.

$$V_{ST}(r_{i0}) = \sum_{n=1}^{5} c_n^{(ST)}(k_F) e^{-(r_{i0}/b_n)^2}$$

b_n = 0.5, 0.95, 1.70, 2.85 and 5.0 fm

The density dependence is parametrized according to

$$c_n^{(ST)}(k_F) = a_n^{(ST)} + b_n^{(ST)} \cdot k_F^{\lambda}(r_i, r_0)$$

with three different values of λ: λ = 0.5, 1.0, 3.0. The three forces are denoted as G0, Gl and G3. The coefficients a_n, b_n are chosen to reproduce the diagonal G-matrix element in the Fermi-sea using the Reid potential These matrix elements are required to calculate binding energies of nuclear matter. Partial waves up to J=2 have been taken into account exactly.

The G0 force is the most successful one in reproducing the saturation curve. This interaction is also prefered in Hartree-Fock calculations. A good overall description of binding energies, density distributions and electron scattering data was obtained for doubly magic nuclei (O, He) /2/ as well as for even tin isotopes /25/.

In order to make a multipole expansion of the interaction one has to define the dependence of k_F on the position of the interacting pair. For our purpose it is most

convenient to use the geometric mean of the densities at the radial positions r_i and r_0, because k_F is factorized then in the coordinates of the projectile and the target nucleon.

$$k_F^{\lambda}(r_i, r_0) = [(\frac{3}{2}\pi^2 \rho(r_i))^{1/3} (\frac{3}{2}\pi^2 \rho(r_0))^{1/3}]^{\lambda/2} \quad .$$

Other possibilities are to use the arithmetic mean $k_F = \frac{1}{2}(k(i) + k_F(0))$ or to take the density at the centre of mass of the pair.

As an example we have investigated the reaction $^{90}Zr(p,p')$ at $E_p = 18.8$ MeV treating the transition as a recoupling of the two-proton configuration $(1g_{9/2})^2$. Neglecting odd-state forces, the singlet-even component is taken as the effective interaction for inelastic proton scattering from a proton configuration. The calculated differential cross sections are compared with the results for the Hamada-Johnston potential with cut-off (from ref. /9/) as well as for the realistic Gaussian potential (b = 1.78 fm, V_{SE} = 32.5 MeV /26/).

Apart from a minor shift of the peak position near the nuclear radius, the form factor calculated with the Sprung-Benerjee force has a shape very similar to the corresponding curve with the Gaussian potential. Density-dependent effects are smeared out because of the radial integration containing the multipole components of an interaction the range of which is comparable with the range in which the density is varying. Therefore, the results do not depend strongly on the strength of the density dependence given by the exponent λ. One obtaines identical angular distributions with the forces G0, G1 and G3. The magnitude of the cross section calculated with G3 is reduced by about 10% as compared with the cross section for G0. The angular distributions calculated with the Sprung-Benerjee forces are in a good agreement with the curves for the Hamada-Johnston potential or the Gaussian potential (Fig. 4). The Migdal force produces an angular distribution too large in the backward region.

Including exchange effects, Love and Satchler /9/ have found, that the Gaussian potential for different multipole transitions gives the cross section which is larger

by 25-50% than the H-J potential with a cut-off at d = 1.0 fm. The differential cross section in forward direction is by about 16% (for L=2) and 26% (for L=4) larger for the Gaussian potential than for the G0 force. With the Sprung-Benerjee force one obtains very similar absolute transition strength as with the Hamada-Johnston potential: $\sigma(HJ)/\sigma(G3) \approx 0.78$ for L=2 , $\sigma(HJ)/\sigma(G3) \approx 0.94$ for L=4 . To reproduce the absolute transition strength given by the G3 interaction one needs normalization constants c ≈ 300 MeV and c ≈ 140 MeV of the Migdal force for L=2 and L=4, respectively.

Introducing a cut-off at d = 1.25 fm, the cross section for the G3 force is reduced by about 20%.

One may conclude that in comparison with the Migdal force the finite range density-dependent forces give smaller differential cross sections at backward angles and a smaller enhancement of higher multipole transitions over lower multipole orders connected with rising interaction in the outer region.

5. Concluding Remarks

Central density-dependent forces of zero or finite range may be used in inelastic scattering calculations as successfully as other more complicated realistic effective interactions. The main effect of the density dependence consists in the modification of the ratio of inner and surface contributions to the direct reaction. In this way, the diffraction pattern of the angular distribution, the absolute transitions strength and the cross section ratio for different multipole excitations can be changed. Monopole transitions should be altered appreciably.

Some improvements are possible. Sprung and Benerjee renormalized the coefficients of the shortest range Gaussians in the even states so that saturation is achieved for $k_F = 1.35$ fm^{-1}, with B/A = 16.5 MeV per particles. Moreover, a starting energy dependence was introduced represented by a zero-range force which contains a large part of the density dependence. For constructing the

transition density one should use single particle wave functions corresponding to a mean potential which follows the nuclear density distribution consistent with the density dependence of the nucleon-nucleon force.

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