ОБЪЕДИНЕННЫЙ
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Экз. чит. зала

E4 - 8350

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MUON CAPTURE PHENOMENOLOGY (SPIN ZERO TARGETS)

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 8350

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MUON CAPTURE PHENOMENOLOGY (SPIN ZERO TARGETS)

Submitted to Nuclear Physics

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Научно-техническая
библиотека
ОИЯИ

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Феноменология μ -захвата (мишени с нулевым спином)

Дан вывод ограничений, которые следуют из экспериментов по измерению гамма-нейтрино корреляции (Миллер и др.) и поляризации ядер отдачи (Поссоз и др.) в парциальном мю-захвате. Приведена максимальная информация, которая может быть заключена из измерений такого рода.

Препринт Объединенного института ядерных исследований.

Дубна, 1974

Oziewicz Z. E4 - 8350

Muon Capture Phenomenology (Spin Zero Targets)

The restrictions on the effective muon-nucleus interaction which follow from gamma-neutrino angular distributions (Miller et al.) and recoil nuclear polarization (Possoz et al.) data, reported for the first time for partial muon capture, are considered. Some inconsistencies are pointed out. The maximal information which can be deduced from such experiments (limit on T-violation, two-component theory confirmation, etc.) is studied.

Preprint. Joint Institute for Nuclear Research.

Dubna, 1974

1. The Effective Weak Nuclear Currents

In the lepton spin space the muon capture effective Hamiltonian has the form

$$\mathcal{H} = u_\nu^+ (1 - \vec{\alpha}_L \cdot \vec{\nu}) \{ J_V + \vec{\alpha}_L \cdot \vec{B} \} u_\mu , \tag{1}$$

where $\vec{\nu}$ is a unit vector in the direction of the neutrino linear momentum. The current $\vec{B}$ can further be splitted into a nonrotational and solenoidal parts

$$\vec{B} = \vec{\nu} J_A - \vec{\nu} \quad \vec{J} . \tag{2}$$

In (1)-(2) we introduced the effective weak nuclear currents.

The interaction with longitudinal lepton field is described in terms of J_V and J_A currents, whereas $\vec{j}$ current takes part in interaction with transverse lepton field. The important observation is that by means of the angular and polarization distribution data or time dependence of the capture rate we may get information on each of these currents separately. For instance it is well known that for transitions $J_i = 0 \xrightarrow{\mu^-} J_f = 0$ the transverse part does not contribute, at all. Therefore it would be interesting to make a nuclear-model-dependent analysis of these currents separately, not only for normal muon capture, but also for processes with neutron emission.

The Primakoff /1/ formulae in the impulse approximation are

$$J_V = G_V \int 1 - \frac{g_V}{M} \vec{\nu} \cdot \int \vec{p} + \ldots$$

$$J_A = (G_A - G_P) \vec{\nu} \cdot \int \vec{\sigma} - \frac{g_A}{M} \int \vec{\sigma} \cdot \vec{p} + \ldots$$

$$\vec{J} = G_A \vec{\nu} \int \vec{\sigma} + \frac{g_V}{M} \int i \vec{p} + \dots \qquad (3)$$

The currents J_V and J_A have no analogs in the electromagnetic radiation, however, $\vec{J}$ current is (up to g_B term) just the isovector part of the electromagnetic current rotated in isospin space. Evidently, up to the relativistic corrections to muon wave function /2/, the contributions from nucleon vector and axial currents are separated in longitudinal part of the Hamiltonian.

The capture rate in the case of zero spin targets, in terms of currents defined in (1)-(2), has the form

$$\Lambda = S \sum_{spins} \int \frac{d\vec{\nu}}{4\pi} \{ |J_V|^2 + |J_A|^2 + (\vec{\nu}\ \vec{J})(\vec{\nu}\ \vec{J}^*) \}.$$

Here S is a phase space vector. The weak multipoles related to above affective nuclear currents are introduced in the next section by means the multipole expansion of the muon capture matrix element.

II. Multipoles

In order to write down the multipole expansion it is convenient to describe the muon in the 1s state by the component μ of the angular momentum in the direction of ν rather than z'-component μ':

$$|\vec{\nu}[1s]\mu> := \sum_{\mu'} D^{|h|}_{\mu'\mu}(\vec{\nu})\,|[1s]\mu'> . \qquad (4)$$

The matrix element for muon capture $J_i \xrightarrow{\mu^-} J_f$ with emission of a neutrino in a sharp helicity state $|\nu\vec{\nu}[h]>$ (ν - is a fixed neutrino energy) can be written in the form (cf /3,4/)

$$<\nu\vec{\nu}[h]; -\nu\vec{\nu}[J_f]\mu_f|\ S - 1\,|[J_i]\mu_i ; \vec{\nu}[1s]\mu> =$$

$$\sum_L C^{J_f \mu_f}_{J_i \mu_i L M}\ \hat{L}\, T^{\eta}_L\, D^L_{M\eta}(\vec{\nu}) . \qquad (5)$$

This partial wave decomposition can be treated as a definition of weak multipoles T^{η}_L which are just complex numbers, where $\eta = \mu - h$ and $\hat{L} = (2L+1)^{1/2}$. For given L and neutrino helicity h, the capture from 1s state is described by the n independent multipole amplitudes, where

$$n = \begin{cases} \frac{3}{2} - |h| + L & \text{if } L < \frac{1}{2} + |h| \\ 2 & \text{if } L \geq \frac{1}{2} + |h| . \end{cases}$$

The transitions $0 \xrightarrow{\mu^-} I \neq 0$ with neutrino emission $|h| = 1/2$ in two component theory are determined by two amplitudes. Only relative phases could be measured. The neutrino emission $|h| = 3/2$ is forbidden for $0 \xrightarrow{\mu^-} 1$ transitions from 1s state. For $0 \xrightarrow{\mu^-} 1$ transitions the angular and polarization distributions are determined from kinematics in this case. In what follows, only neutrino emission is considered.

The multipoles (5) with $\eta \neq 0$ describe the transverse lepton field interaction, whereas with $\eta = 0$ are for the case of the longitudinal lepton field.

Depending on the change of the parities of the nuclear levels $\Delta\pi$ we define in table I, electric E_L, magnetic M_L, vector V_L and axial A_L weak multipoles*.

The weak E_L and M_L multipoles are built up from exactly the same nuclear operators as the usual EL and magnetic ML multipoles (in the impulse approximation). T^{long}_L has no analogy in the radiation processes. From (3) in the Morita and Fujii /6/ notation we get (cf /2,4,5/):

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*Here we again change the previous notation /2,4,5/. A_L (previously) is electric E_L here, and P_L (pseudoscalar previously) is denoted now as the axial A_L. In /3/ the interchange $E_L \longrightarrow M_L$ in all formulae should be introduced.

$$M_L = G_A L^{1/2} \left([\,1L-1L\,] - \left(\frac{L}{L+1}\right)^{1/2} [\,1L+1L\,] \right) - g_V \hat{L} \left(\frac{L}{L+1}\right)^{1/2} [\,1LL \frac{\vec{P}}{M}]$$

$$A_L = (G_A - G_P)\left(L^{1/2} [\,1L-1L\,] + (L+1)^{1/2} [\,1L+1L\,] \right) + g_A \sqrt{3}\hat{L} [\,0LL \frac{\vec{\sigma}\cdot\vec{P}}{M}]$$

$$E_L = (-)^{L+1} \left\{ G_A \hat{L} \left(\frac{L}{L+1}\right)^{1/2} [\,1LL\,] + g_V L^{1/2} \left([\,1L-1L \frac{\vec{P}}{M}] - \left(\frac{L}{L+1}\right)^{1/2} [\,1L + 1L \frac{\vec{P}}{M}] \right) \right\}$$

$$V_L = (-)^{L} \left\{ G_V \hat{L} \sqrt{3} [\,0LL\,] - g_V \left(L^{1/2} [\,1L-1L \frac{\vec{P}}{M}] + (L+1)^{1/2} [\,1L + 1L \frac{\vec{P}}{M}] \right) \right\}. \tag{6}$$

Table I

	$\Delta\pi = (-)^{L}$	$\Delta\pi = (-)^{L+1}$	Remarks
T_L^{trans}	$\left(\frac{L+1}{L}\right)^{1/2} E_L$	$-i2h\left(\frac{L+1}{L}\right)^{1/2} M_L$	$\lvert\eta\rvert = 1$; $L \neq 0$
T_L^{long}	V_L	$-i2h\, A_L$	$\eta = 0$

Denoting

$$x \exp(i\phi) = \begin{cases} \dfrac{A_L}{M_L} & \text{for } L = 1^+, 2^-, 3^+, \ldots \text{(magnetic = unique transition)} \\[2ex] \dfrac{V_L}{E_L} & \text{for } L = 1^-, 2^+, 3^-, \ldots \text{(electric = non-unique transitions)} \end{cases} \tag{7}$$

and moreover

$$N = S\,(\,|E_L|^2 + |M_L|^2\,)\ , \tag{8}$$

where S is a statistical factor, the normal capture rate $0 \xrightarrow{\mu^-} L \neq 0$ can be written as follows

$$W = N\left(\frac{L+1}{L} + x^2 \right). \tag{9}$$

The angular and polarization distributions (e.g., gamma-neutrino /7,5,2,4/, recoil nuclear polarization /8,9,10/, etc.) for partial muon capture by spinless nuclei are described by the ratio (7) alone. The relative phase ϕ is equal to 0 or π if the interaction responsible for the muon capture is invariant under time reversal. The only independent dynamical quantities describing a weak process that can be deduced from the muon capture experimental data are N, x and ϕ. The variety of conclusions about the induced pseudoscalar coupling weak magnetism form factor, inferred from observables (i.e., from N, x) depends strongly on nuclear models. However data on the capture rate, angular correlations and/or polarizations taken together imply more restrictions on the nuclear model and coupling constants. At the same time these data provide the possibility of confirming the two-component theory, helicity neutrino, T-conservation, nucleon vector current contributions, independently on nuclear structure. Some additional nuclear structure information on nuclear spins and on mixing ratio could also be extracted and elementary particle approach can be tested.

III. Angular Distributions: Grenacs-Deutsch-Lipnik-Macq Method [11]

Let us consider the nuclear cascade process

$$\mu^- + {}^{A}Z(J_i = 0) \longrightarrow \nu(h) + {}^{A}(Z-1)^* (J_f \equiv I \neq 0) \quad \llcorner\!\!\to \gamma + {}^{A}(Z-1)(j) , \qquad (10)$$

where h is now a neutrino polarization ($|h| = 1$ for two-component theory) and denote by $\vec{p}$ the muon spin rest polarization on the K-shell at the instant of capture (not unit vector) and by $\vec{k}$ the unit vector in gamma-ray direction. Grenacs, Deutsch, Lipnik and Macq [11] have proposed a method for observing angular correlations between the emitted neutrino, de-excitation gamma-ray and initial mu-mesic atom polarization, in terms of Doppler broadening of the transition gamma ray. In this method the angle between the plans $\vec{k}$, ν and $\vec{k}$, $\vec{p}$ is unobservable. Integrating over this angle we get from (5) the following formulae for angular distribution in the process (10)

$$W = \sum_{S=0}^{2I} B_S \{ a_S P_S (\vec{k}\cdot\vec{\nu}) \qquad (11)$$

$$+ \vec{p}\cdot\vec{k} [(S\beta_S - a_S) k\cdot\nu P_S (\vec{k}\cdot\vec{\nu}) - S\beta_S P_{S-1}(\vec{k}\cdot\vec{\nu})] \}.$$

Here B_S is determined by electromagnetic $I \xrightarrow{\gamma} j$ transition and looks like

$$B_S := (1+\delta^2)^{-1} \{ R_S(L,L) + 2\eta\delta R_S(L,L+1) + \delta^2 R_S(L+1,L+1) \},$$

where

$$R_S(L,L') := (-)^{L-\eta} \hat{L}\hat{L}'\hat{I}\, C^{S0}_{L\eta\ L-\eta}\, W(jLISIL') . \qquad (12)$$

Here L is the gamma-ray multipolarity, $\eta = \pm 1$ for right and left polarized nuclear radiation, respectively, and, finally, δ is a mixing ratio as defined by Rose and Brink [13]. If the circular polarization of nuclear gamma-rays is not observed then S in (13) takes even values only.

We would like to notice that formulae for gamma-neutrino distribution were first derived by Popov [7] (see also [2,4,5]), however, the integration over the unobservable angle, appropriate for the Grenacs-Deutsch-Lipnik-Macq method was performed only in Miller thesis [12, Appendix E].

The formula (11) is valid for an arbitrary neutrino polarization h, i.e., it could be used as a test of a two-component theory of neutrino. This theory could, in principle be deviated in muon capture dut to, e.g., the induced tensor currents which are absent in beta decay because of small momentum trasnfer or due to neutrino mass. The deviations from the two-component theory are described by two parameters: the neutrino polarization h and the parameter ϵ. If we denote by N_R and N_L the transverse lepton field describing factor (8) and similarly by $x_R e^{i\phi_R}$ and $x_L e^{i\phi_L}$ the longitudinal multipoles (7) for right and left handed neutrino emission, respectively, then

$$\epsilon := \frac{N_R - N_L}{N_R + N_L} ,$$

$$x := \left\{ \frac{1+\epsilon}{2} x_R^2 + \frac{1-\epsilon}{2} x_L^2 \right\}^{1/2} ,$$

$$x e^{i\phi} := \frac{1+\epsilon}{2} x_R e^{i\phi_R} - \frac{1-\epsilon}{2} x_L e^{i\phi_L} . \qquad (13)$$

For neutrino polarization h we get

$$h = \epsilon + \frac{(1-\epsilon^2)}{2} \left(\frac{x_R^2 - x_L^2}{\frac{I+1}{I} + x^2} \right) . \qquad (14)$$

From furmula (14) it follows that $|\epsilon|=1$ $|h|=1$. The weak interaction coefficients for S-even in formula (11) have the following form (independently of the nuclear parity change)

$$a_S = S\,\hat{C}^{\,10}_{10\,S\,0}\left(1 - \frac{S(S+1)}{2I(I+1+I\,x^2)}\right),$$

$$\alpha_S = \hat{S}\,C^{\,10}_{10\,S\,0}\left(\frac{4I(I+1) - S(S+1)}{2I(I+1+I\,x^2)}\,\epsilon - h\right), \qquad (15)$$

$$\beta_S = 2\hat{S}\left[\frac{I(I+1)}{S(S+1)}\right]^{1/2} C^{\,10}_{I-1\,S\,1}\,\frac{x\cos\phi}{I+1+I\,x^2}.$$

Supposing that B_S electromagnetic factors are already known the gamma-neutrino angular distributions (11) by means of formula (15) are expressed in terms of the four independent parameters $x \geq 0$, $|\epsilon| \leq 1$, $|h| \leq 1$ and $\cos\phi$. The formulae (11)-(15) follow from kinematics of the nuclear cascade process (10). All information on nuclear structure is absorbed in electromagnetic mixing ratio δ (in B'_S s) and x ratio. The capture rate or other observable quantities, which are independent of the phase ϕ give twofold values of the coupling (e.g., pseudoscalar one), so for this reason the experimental determination of the phase ϕ is important.

IV. Kinematics of $0 \xrightarrow{\mu^-} 1 \xrightarrow{\gamma} j$ Transitions

For such spin sequences (with arbitrary j) the formula (11) reduces to the form given in Miller's thesis

$$W = 1 + A\,P_2(\vec{k}\cdot\vec{\nu}) + \vec{p}\cdot\vec{k}\;\vec{k}\cdot\vec{\nu}\,[\,B + C\,P_2(\vec{k}\cdot\vec{\nu})\,]. \qquad (16)$$

For angular correlation coefficients in (16) we get

$$A = \frac{3F}{2+x^2} - F$$

$$B + C = h(1-F) - \epsilon\,\frac{4-F}{2+x^2} \qquad (17)$$

$$C = F\left\{-h + \frac{\epsilon + 2x\cos\phi}{2+x^2}\right\},$$

where $F = F_j(\delta) \equiv \sqrt{2}\,B_S$, should be calculated from (12). Particularly for $1 \xrightarrow{\gamma} 0$ transitions we have $F = 1$, and for $1 \xrightarrow{\gamma} 2$ the relevant formula, neglecting $L=3$ radiation is

$$F(1 \xrightarrow{\gamma} 2) = \frac{1}{10}\left(\frac{1 - 6\sqrt{5}\,\delta + 5\,\delta^2}{1+\delta^2}\right). \qquad (18)$$

From (18) the following kinematical bounds follow at once

$$-0.4 \leq F \leq +1.$$

From $0 \xrightarrow{\mu^-} 1 \xrightarrow{\gamma} 0$ cascade the confirmation of the two-component theory can be deduced, as we have

$$B + C = -\epsilon\,(1 + A) \qquad (\text{if } F = 1). \qquad (19)$$

Generally, however, the information on neutrino polarization (if $|\epsilon| < 1$) and on $\cos\phi$ from $0 \to 1 \to 0$ data alone can not be obtained. The angular distribution data for the same transition $0 \xrightarrow{\mu^-} 1$ but with de-excitation to spin $j = 2$ level (if $F \neq 1$) together with $0 \to 1 \to 0$ data are sufficient for the determination of all four parameters and in addition can give the F value.

In the case of $0 \to 1 \to 0$ cascade we have $F = 1$ and from (17) the following bounds follow

$$-1 \leq A \leq +0.5. \qquad (20)$$

$$|B + C| \leq \frac{3}{2}.$$

In the two-component theory $\epsilon = h$, therefore we have moreover

$$[\,(1-2A)^{1/2} - (1+A)^{1/2}\,]^2 \leq 3|C| \leq [\,(1-2A)^{1/2} + (1+A)^{1/2}\,]^2. \qquad (21)$$

If $|\cos\phi| = 1$ (and $|h| = F = 1$), then

$$3C = -h\{(1-2A)^{1/2} - (1+A)^{1/2}\cos\phi\}^2. \qquad (22)$$

From (13) we know that in the two-component limit $\cos\phi \to h\cos\phi$ therefore $h = -(\mathrm{Sign}\,C)(\mathrm{Sign}\,F)$ (if $C \neq 0$).

The kinematical constraints for $0 \to 1 \to 0$ processes are shown on figure 1. The allowed region for two-compo-

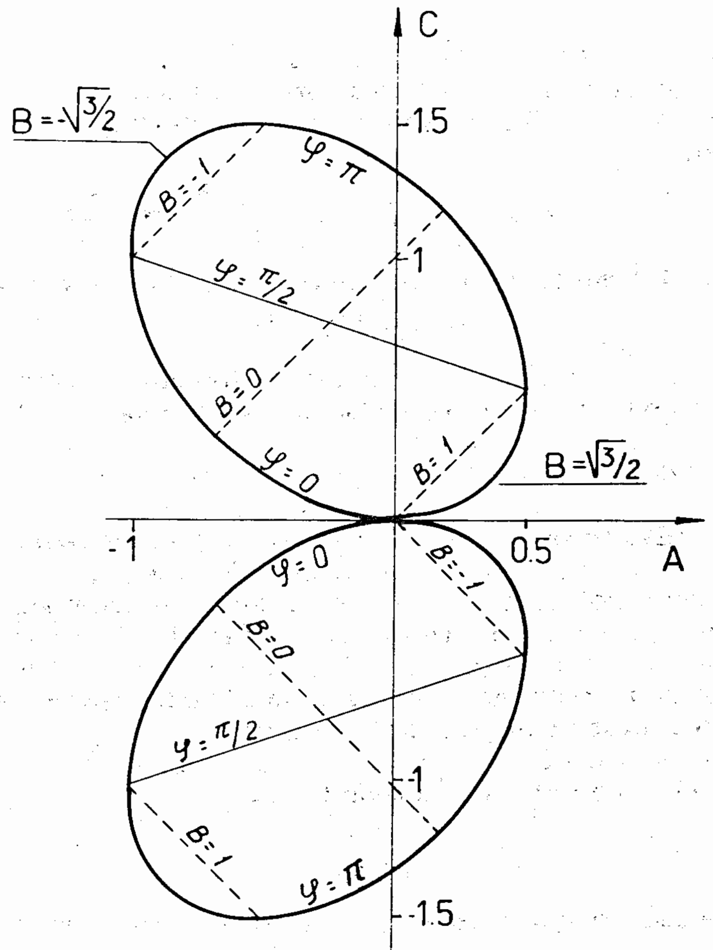

Fig. 1. The kinematical constraints for $0 \xrightarrow{\mu} 1 \xrightarrow{\gamma} 0$ cascade.

nent theory is the interior (21) of the two ellipses (including the boundary curves). The ellipses (22) correspond to the T-parity conservation. The exteriors of the ellipses, but within ranges (20), should be interpreted as a deviation from two-component theory. The straight lines which correspond to the maximal T-violation $\cos\phi = 0$ with $|h| = 1$ are also shown on figure 1, as well as the dotted straight lines each of which corresponds to a fixed value of the B coefficient in (19) with $|\epsilon| = 1$.

From A value in expression (16) for the $0 \xrightarrow{\mu^-} 1 \xrightarrow{\gamma} 2$ cascade one can find on the F and $(1-x^2)/(2+x^2)$ -plot (a hyperbola) restrictions on both dynamical parameters x and F.

V. Miller-Eckhause-Kane-Martin-Welsh Angular Distribution Experiment

The Doppler-broadened gamma-ray transitions have been observed for the first time in muon capture in ^{28}Si by Miller et al. /14/ (for details see also /12/). The two targets were used: $^{28}SiO_2$ when the rest polarization of muons vanishes $\vec{p} \approx 0$, and the metallic natural Si traget with nonvanishing muon polarization.

The three different allowed cascade processes $0 \xrightarrow{\mu^-} 1 \xrightarrow{\gamma} j = 0$ or 2 were studied. The results of the least-square fit of the experimental data on reaction

$$^{28}Si\,(\mathrm{g.s.}) \xrightarrow{\mu^-} {}^{28}Al^*(1^+,\ 2202\ \mathrm{keV}) \xrightarrow{M1} 0^+$$

to formula (16) as reported by Miller et al. /14/, are as follows

$$A = +0.20 \pm 0.30 \ \text{(cf. with (20))} \qquad (23)$$

($^{28}SiO_2$ data)
and

$$A = +0.15 \pm 0.25$$
$$B = +1.12 \pm 0.10 \qquad (24)$$
$$C = +0.02 \pm 0.03 \ \text{(natural Si data)}$$

The results (24) assume [14,12], however, the left handed neutrino emission $h = \epsilon = -1$ and T-conservation in formulas (17). Therefore, they cannot be used as a test (19) of two-component theory. Instead, we expect that the following two relations (from formulas (19), (22) and figure 1) should be strictly valid

$$B + C = 1 + A \qquad (25)$$
$$3C = (\sqrt{1-2A} - \sqrt{1+A})^2 .$$

From fig. 2 it is clear that constraints (25) are satisfied only approximately. The overlaping for the data (23) and (24) is

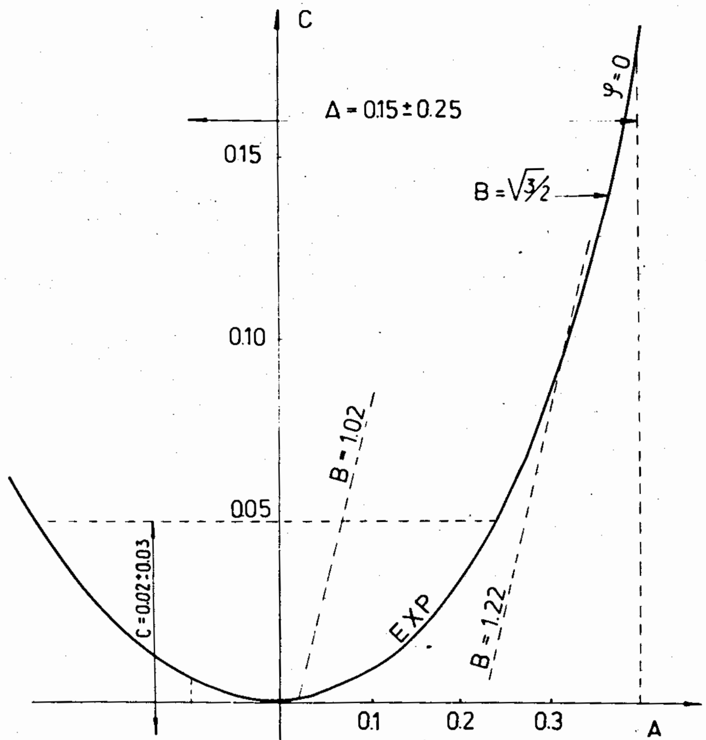

Fig. 2. The least-square fit for $^{28}Si \xrightarrow{\mu} {}^{28}Al\,(1^+,\ 2202\ keV) \longrightarrow 0^+$ cascade as reported by Miller et al[14].

$$A \in [+0.02,\ +0.24]$$
or $\qquad\qquad$ (26)
$$x \in [\ 0.65,\ 0.96\ ]$$

with $\cos\phi = +1$ (from figure 1).

The result (26) could be interpreted in terms of the pseudoscalar coupling. The comprehensive analysis using Wildenthal, McGrory and De Voigt nuclear wave functions[15] was recently performed by Ciechanowicz[16]. The range (26) can be satisfied[16] by $-4.6 < g_P/g_A < +1.0$. The second solution is excluded because of the phase (26).

Another allowed cascade processes studied are [14]

$$^{28}Si\ (g.s.) \xrightarrow{\mu^-} {}^{28}Al^*(1^+,\ 2202\,keV) \xrightarrow{M1 + E2} 2^+, \qquad (27)$$

$$^{28}Si\ (g.s.) \xrightarrow{\mu^-} {}^{28}Al^*(1^+,\ 1372.6\ keV) \xrightarrow{M1 + E2} 2^+. \qquad (28)$$

The correlation coefficient A in (16) was fitted for the above reactions to the following (not necessarily correct) formula

$$A = \frac{1}{10}\ \frac{1 - a^2}{2 + a^2}\ , \qquad (29)$$

where the real fitting parameter a was chosen actually as a linear function of the induced pseudoscalar coupling[12]. In fact, the parameter a can be expressed in terms of F and x^2 in formula (17). The $F = 0.1$ is not the only solution. The fitting formula (29) was chosen under the supposition of pure emissions in (27-28), however, the (29) is not equivalent to $\delta = 0$ in expression (18). The only essential fact is that formula (29) gives the following very strong restrictions on the A coefficient (17):

$$-0.1 \le F\left(\frac{3}{2 + x^2} - 1\right) \le +0.05 . \qquad (30)$$

The supposed bounds (30) are open to objection:

It is not clear why the fitting formula (29) which is equivalent to bounds (30), could give the following reported data [12,14]

(a) $A = -0.17 \pm 0.08$ }
(b) $A = -0.37 \pm 0.10$ } for reaction (27)

(c) $A = -0.55 \pm 0.20$ }
(d) $A = -0.29 \pm 0.15$ } for reaction (28).

Here (a)-(c) refer to $^{28}SiO_2$ data, (b)-(d) to natural $^{28}SiO_2$ data. The (b)-(c)-(d) data are inconsistent with (29)

VI. Forbidden Transitions $0 \xrightarrow{\mu^-} (2 \text{ or } 3)$

In what follows we neglect M3 and E3 radiations. Then we conclude from (11)-(12) that the angular distributions for $0 \xrightarrow{\mu} 2 \xrightarrow{\gamma} j$ and for $0 \xrightarrow{\mu} 3 \xrightarrow{\gamma} j$ reactions have exactly the same form

$$W = 1 + B_2 a_2 P_2(\vec{k}\cdot\vec{\nu}) + B_4 a_4 P_4(\vec{k}\cdot\vec{\nu})$$

$$-\vec{p}\cdot\vec{k}\{(a_0 + 2B_2\beta_2)\,\vec{k}\cdot\vec{\nu} + B_2(a_2 - 2\beta_2)\,\vec{k}\cdot\vec{\nu}\,P_2(\vec{k}\cdot\vec{\nu})$$

$$+ 4B_4\beta_4 P_3(\vec{k}\cdot\vec{\nu}) + B_4(a_4 - 4\beta_4)\,\vec{k}\cdot\vec{\nu}\,P_4(\vec{k}\cdot\vec{\nu})\}. \qquad (31)$$

If a gamma radiation is a pure M1 or E1 we have $\delta = 0$ $B_4 = 0$ .In this case formula (31) reduces to the allowed form (16) derived in Miller thesis /12, p. 102/. The conclusion of Miller et al. /14/ about the spin 2^+ of the 2138.5 keV level in ^{28}Al is based on the allowed formula (16). However the pure emission should be confirmed for (2138.5 keV) $\xrightarrow{\gamma}$ (30.6 keV or g.s.) gamma transition in ^{28}Al .

In Table II the angular correlation coefficients in formula (31) are listed. The relevant expressions for B_S coefficients are given in Table III.

Table II
Correlation coefficients in formula (31)

	$0 \xrightarrow{\mu} 2$	$0 \xrightarrow{\mu} 3$
a_2	$-\sqrt{\frac{5}{14}}\ \frac{3+4x^2}{3+2x^2}$	$-2\sqrt{3}\ \frac{1+x^2}{4+3x^2}$
a_4	$-6\sqrt{\frac{2}{7}}\ \frac{1-x^2}{3+2x^2}$	$\sqrt{\frac{2}{11}}\ \frac{2+9x^2}{4+3x^2}$
α_0	$\frac{6}{3+2x^2}\epsilon - h$	$\frac{8}{4+3x^2}\epsilon - h$
α_2	$-\sqrt{\frac{5}{14}}\left(\frac{9}{3+2x^2}\epsilon - 2h\right)$	$-\frac{2}{\sqrt{3}}\left(\frac{7}{4+3x^2}\epsilon - h\right)$
α_4	$3\sqrt{\frac{2}{7}}\left(\frac{1}{3+2x^2}\epsilon - h\right)$	$\sqrt{\frac{2}{11}}\left(\frac{14}{4+3x^2}\epsilon - 3h\right)$
β_2	$\sqrt{\frac{10}{7}}\ \frac{x\cos\phi}{3+2x^2}$	$\frac{2}{3}\ \frac{x\cos\phi}{4+3x^2}$
β_4	$-3\sqrt{\frac{2}{7}}\ \frac{x\cos\phi}{3+2x^2}$	$-3\sqrt{\frac{2}{11}}\ \frac{x\cos\phi}{4+3x^2}$

Finally in Table IV the angular correlation coefficients are listed (two-component theory is assumed) in the limit of pure L=1 gamma radiation. They follow directly from the results of Tables II and III in the limit $\delta \to 0$.

The two different forbidden cascade processes $0 \xrightarrow{\mu} (2 \text{ or } 3) \xrightarrow{\gamma} (2 \text{ or } 3)$ were studied /12,14/. However the least-square fits were performed to the allowed formula

Table III

Some B_S electromagnetic coefficients defined by formula (12)

	$(1+\delta^2)\,B_2$	$(1+\delta^2)\,B_4$
$2 \xrightarrow{\gamma} 2$	$-\frac{1}{10}\sqrt{\frac{35}{2}}\left(1-2\sqrt{\frac{15}{7}}\,\delta+\frac{45}{49}\,\delta^2\right)$	$-\frac{4}{7}\sqrt{\frac{2}{7}}\;\delta^2$
$2 \xrightarrow{\gamma} 3$	$\frac{1}{\sqrt{70}}\left(1-2\sqrt{30}\;\delta+\frac{20}{7}\,\delta^2\right)$	$\frac{1}{7}\sqrt{\frac{2}{7}}\;\delta^2$
$3 \xrightarrow{\gamma} 2$	$\frac{1}{5}\sqrt{3}\left(1+\delta\sqrt{30}-\frac{5}{14}\,\delta^2\right)$	$\frac{1}{7}\sqrt{22}\;\delta^2$
$3 \xrightarrow{\gamma} 3$	$-\frac{1}{4}\sqrt{3}\left(1-2\delta-\frac{11}{21}\,\delta^2\right)$	$-\frac{2}{21}\sqrt{22}\;\delta^2$

(16) rather than to formula (31). In addition to this assumption the formulae valid for $h=-1$ with T-conservation were used [5, 12].

It seems however that some inconsistencies exist. The examples are listed below.

(a) For $0 \xrightarrow{\mu^-} 2 \xrightarrow{\gamma} 2$ transitions

$$\frac{B+C}{4-11A} = \begin{cases} +1 & \text{-should be strictly valid (from Table IV)} \\ +1/2 & \text{Miller et al. data [14]} \end{cases}$$

(b) For $0 \xrightarrow{\mu} 2 \xrightarrow{\gamma} 3$ transitions

$$7\,\frac{B+C}{19+175A} = \begin{cases} +1, & \text{should be strictly valid (Table IV)} \\ \approx -(0.50 \pm 0.03), & \text{Miller et al. data [14].} \end{cases}$$

(The minus sign for right handed neutrino).

Table IV

Correlation coefficients for forbidden transitions in formula (31) in the limit $\delta \to 0$.

	$0 \xrightarrow{\mu} 2 \xrightarrow{\gamma}$	$0 \xrightarrow{\mu} 3 \xrightarrow{\gamma}$	
A	$\frac{1}{4}\,\frac{3+4x^2}{3+2x^2}$	$-\frac{6}{5}\,\frac{1+x^2}{4+3x^2}$	
B + C	$-\frac{3}{4}h\,\frac{5-4x^2}{3+2x^2}$	$-\frac{1}{5}h\,\frac{14-9x^2}{4+3x^2}$	$\xrightarrow{\gamma} 2$
C	$-\frac{1}{4}h\,\frac{3-4x^2+4x\cos\phi}{3+2x^2}$	$\frac{2}{5}h\,\frac{3-3x^2+2x\cos\phi}{4+3x^2}$	
A	$-\frac{1}{14}\,\frac{3+4x}{3+2x}$	$\frac{3}{2}\,\frac{1+x}{4+3x}$	
B + C	$-\frac{1}{14}h\,\frac{39-24x}{3+2x}$	$-\frac{1}{2}h\,\frac{11-9x}{4+3x}$	$\xrightarrow{\gamma} 3$
C	$+\frac{1}{14}h\,\frac{3-4x^2+4x\cos\phi}{3+2x^2}$	$-\frac{1}{2}h\,\frac{3-3x^2+2x\cos\phi}{4+3x^2}$	

(c) From Table IV we conclude that only the following bounds should be allowed in fits

$$+0.25 \le A \le +0.50 \quad (\text{for } 2 \xrightarrow{\gamma} 2)$$
$$-0.15 \le A \le -0.07 \quad (\text{for } 2 \xrightarrow{\gamma} 3)\,.$$

However, the Miller et al. [14] values are

$$A = +0.06 \pm 0.01 \text{ (for } 2 \xrightarrow{\gamma} 2 \text{)}$$
$$A h \; -0.41 \pm 0.20 \text{ (for } 2 \xrightarrow{\gamma} 3 \text{)}.$$

VII. Recoil Nuclear Polarization. Information from Louvain Group Experiment

The thoretical estimations for recoil nuclear polarization were performed as early as in 1957-1959 [8]. However, the most general form was derived by Korenman and Eramzhyan [9]. Particularly, in two-component theory, the average vector polarization of the spin J final nuclear state in terms of the muon rest polarization p on the K-shell and of the quantities defined in Sec. III has the form

$$\vec{P}_N = \frac{1}{3}(J+1)\,\frac{1+2Jx\cos\phi}{J+1+Jx^2}\,\vec{p}\;. \qquad (32)$$

The nuclear wave function given by Hirooka et al. [17] for the muon capture $^{12}C \longrightarrow {}^{12}B(g.s.)$, were recently applied to Korenman-Eramzhyan formula (32) by Devanathan et al. [10].

The most general functional dependence of x on the induced pseudoscalar coupling (cf. (6) and (7)) is

$$x\cos\phi = a\,\frac{g_A - b\,g}{g_A - d\,g}\;. \qquad (33)$$

Here among the three nuclear-model-dependent quantities a, b and d the last one, d, vanishes if we neglect the relativistic corrections to muon wave function [2].

Recently the measurements of the recoil nuclear polarization in muon capture reaction $^{12}C \longrightarrow {}^{12}B(g.s.)$ were reported by Possiz et al. [18]. Their result is

$$P_N = (0.43 \pm 0.10)\;p\;. \qquad (34)$$

The ranges for x and $\cos\phi$ allowed by this result are shown on fig. 3. The data are compatible with the fact that the longitudinal contribution is absent in this process.

It could be interesting to see on the N and $(2+x^2)$ plot (hyperbola (9)) the restrictions which follow from both the Possoz et al. experiment and the data on partial capture rate.

From (34), assuming $|\cos\phi| = 1$, we get

$$x\cos\phi \in (-0.01,\; +0.35)$$
$$\text{or } x \in (+2.2,\; +4.0 \quad \text{(with } \cos\phi = +1) \qquad (35)$$

It is interesting to note that Possoz et al. (34) and Miller et al. (26) data exclude the Fujii-Primakoff approximation, which gives the same x value for all magnetic transitions (7):

$$x \approx 1 - \frac{G_P}{G_A} \approx 0.87\left(1 - \frac{1}{21}\,\frac{g_P}{g_A}\right)\;.$$

However the results (26) and (35) have no overlaping. Some theoretical estimations for process $^{12}C \xrightarrow{\mu^-} {}^{12}B(g.s.)$ are listed below.

a) Morita and Fujii [6], in the j-j model give

$$x = 0.74\left(1 - \frac{1}{19.5}\,\frac{g_P}{g_A}\right)\;.$$

b) Balashov and Eramzhyan [9] with Boyarkina nuclear wave functions [19] predict

$$x = 0.62\left(1 - \frac{1}{16.0}\,\frac{g_P}{g_A}\right)$$

c) Mukhopadhyay [20] with the Cohen and Kurath intermediate coupling, wave functions [21] obtain

$$x = 0.57\left(1 - \frac{1}{18.2}\,\frac{g_P}{g_A}\right)\;.$$

d) Devanathan et al. [10] calculations with the Hirooka et al. nuclear model wave functions lead to

$$x = 0.46\left(1 - \frac{1}{14.7}\,\frac{g_P}{g_A}\right)\;.$$

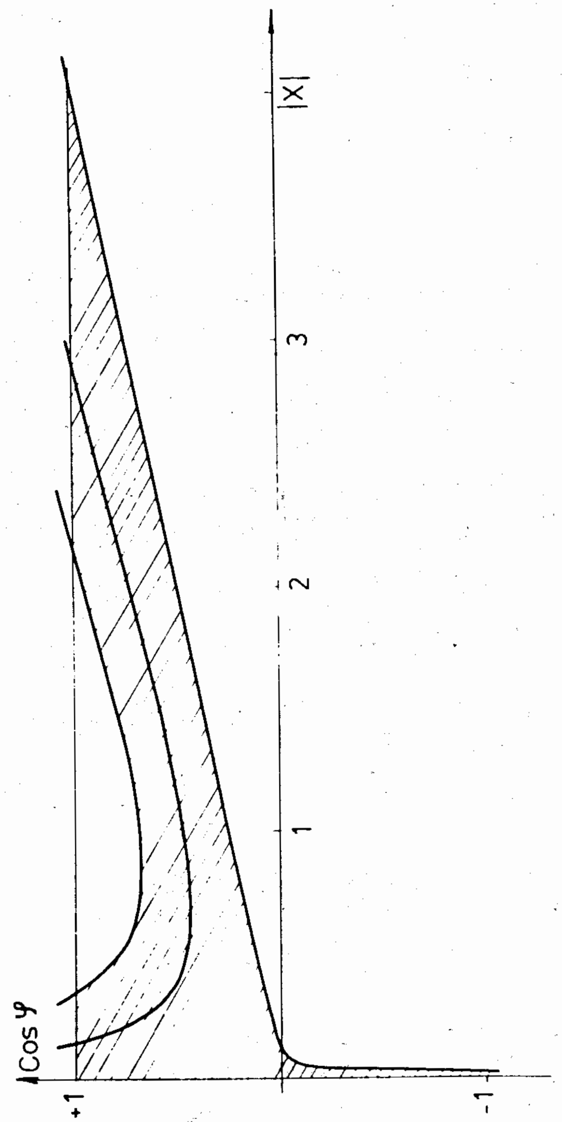

Fig. 3. The allowed ranges, which follow from recoil nuclear polarization in $^{12}C \longrightarrow {}^{12}B\,(g.s.)$ measurement according, to Possoz et al /18/

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Received by Publishing Department
on October 29, 1974.