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BASIC EQUATIONS OF THE QUASIPARTICLE-PHONON NUCLEAR MODEL WITH THE EFFECTS DUE TO THE PAULI PRINCIPLE AND THE PHONON GROUND STATE CORRELATIONS

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INTRODUCTION

The experimental and theoretical investigations of the giant multipole resonances (GMR) have been of considerable interest in the last years. An important progress in the description of the integral resonance characteristics has been achieved understanding the key role of the coupling of simple one-phonon (or 1p-1h) excitations to two-phonon (or 2p-2h) configurations. One can gain a thorough knowledge of theoretical and experimental studies of this question in reviews [1-3]. We also note a series of papers, where the 2p-2h configurations are treated in the framework of the nuclear field theory (NFT) [4,5] and the theory of finite Fermi systems (TFFS) [6]. Interesting results are obtained also in refs. [7,8].

In the quasiparticle-phonon nuclear model (QPM) [1] the nuclear excitations are described in terms of the phonon operators. Phonons include all particle-hole states. The nuclear fermion Hamiltonian is transformed then completely into the Hamiltonian described only by the phonon operators. It is clear that in this approach the problem of double-counting arisen, e.g., in the NFT disappears automatically. Since phonons are not ideal bosons one must consider the requirements of the Pauli principle for two-phonon components of wave functions. In the traditional calculations of the QPM [1] the Pauli principle was taken into account approximately with the elimination of Pauli violating components. It was realized by the computing program GIRES [9]. The basic equations of the QPM including the corrections due to the Pauli principle have been derived in ref. [10]. In ref. [11,12] it has been shown these corrections do not lead to a significant change at low-lying energies and have a slight influence on characteristics of the GMR. Equations of ref. [10] have been derived under the assumption that the ground state wave function is the phonon vacuum. In general, because of the interactions between phonons this assumption is not satisfied. The problem to take care of the phonon correlations in the ground states in the QPM has been solved in ref. [13] for deformed nuclei and in ref. [14] for spherical nuclei. In ref. [13] a formalism of Green functions was utilized to obtain a system of equations. In ref. [14] a similar system has been derived through the linearization of the equations of motion.

The purpose of this paper is to obtain the basic equations of the QPM in which the Pauli principle and the phonon correla-

tions in the ground states of spherical nuclei are taken into account. As an example we will demonstrate the role of both effects for low-lying states calculating a simplified scheme. A comparison of these basic equations of the QPM with other theoretical approaches will be carried out.

1. THE MODEL HAMILTONIAN AND THE GENERAL SYSTEM OF BASIC EQUATIONS

The Hamiltonian of the QPM consists of the average field as the Saxon-Woods potential together with the pairing interactions and the effective separable multipole and spin-multipole isoscalar and isovector forces. An explicit derivation of the Hamiltonian in terms of the creation and annihilation quasiparticle a^+, a and phonon Q^+, Q operators can be found in ref. [1].

The terms of the QPM Hamiltonian including the phonon interaction can be written as:

$$H = \sum_{\lambda\mu i} \omega_{\lambda i} Q^+_{\lambda\mu i} Q_{\lambda\mu i} + \frac{1}{2} \sum_{\substack{\lambda_1\mu_1 i_1 \\ \lambda_2\mu_2 i_2 \\ \lambda\mu i}} \langle \lambda_1\mu_1\lambda_2\mu_2 | \lambda -\mu \rangle \times \tag{1}$$

$$\times [U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2} Q_{\lambda -\mu i} +$$

$$+ (-)^{\lambda-\mu} V^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2} Q^+_{\lambda\mu i} + \text{h.c.}].$$

Here $Q^+_{\lambda\mu i}$ ($Q_{\lambda\mu i}$) are the phonon creation (annihilation) operators:

$$Q^+_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} \{ \psi^{\lambda i}_{jj'} [a^+_{jm} a^+_{j'm'}]_{\lambda\mu} - (-)^{\lambda-\mu} \phi^{\lambda i}_{jj'} [a_{j'm'} a_{jm}]_{\lambda-\mu} \} \tag{2}$$

$\psi^{\lambda i}_{jj'}$, $\phi^{\lambda i}_{jj'}$ are the phonon amplitudes; $\omega_{\lambda i}$ are the frequencies determined through the corresponding RPA equations [1]. In general the model Hamiltonian has a complex form, so the expression (1) is correct under the assumption that neglected terms vanish if we take the functions as a superposition of one- and two-phonon components.

The coefficients $U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)$ and $V^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)$ are defined as follows [1,14]:

$$U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) = (-)^{\lambda_1+\lambda_2-\lambda} \sqrt{(2\lambda_1+1)(2\lambda_2+1)} \times$$

$$\times \sum_{j_1 j_2 j_3} [\frac{\langle j_1 \| f_\lambda(r) Y_\lambda \| j_2 \rangle v^{(-)}_{j_1 j_2}}{\sqrt{2 \mathcal{Y}_{\lambda i}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} (\psi^{\lambda_1 i_1}_{j_3 j_1} \phi^{\lambda_2 i_2}_{j_2 j_3} + \psi^{\lambda_2 i_2}_{j_2 j_3} \phi^{\lambda_1 i_1}_{j_3 j_1}) +$$

$$+\frac{\langle j_1\|f_{\lambda_1}(r)Y_{\lambda_1}\|j_2\rangle v^{(-)}_{j_1j_2}}{\sqrt{2\mathcal{Y}_{\lambda_1 i_1}}}\begin{Bmatrix}\lambda_1&\lambda_2&\lambda\\ j_3&j_2&j_1\end{Bmatrix}(\psi^{\lambda_2 i_2}_{j_3j_1}\psi^{\lambda i}_{j_2j_3}+\phi^{\lambda i}_{j_2j_3}\phi^{\lambda_2 i_2}_{j_3j_1})+ \qquad (3)$$

$$+\frac{\langle j_1\|f_{\lambda_2}(r)Y_{\lambda_2}\|j_2\rangle v^{(-)}_{j_1j_2}}{\sqrt{2\mathcal{Y}_{\lambda_2 i_2}}}\begin{Bmatrix}\lambda_1&\lambda_2&\lambda\\ j_1&j_3&j_2\end{Bmatrix}(\psi^{\lambda i}_{j_3j_1}\psi^{\lambda_1 i_1}_{j_2j_3}+\phi^{\lambda_1 i_1}_{j_2j_3}\phi^{\lambda i}_{j_3j_1})],$$

$$V^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) = (-)^{\lambda_1+\lambda_2-\lambda}\sqrt{(2\lambda_1+1)(2\lambda_2+1)}\times \qquad (4)$$

$$+\sum_{j_1j_2j_3}\frac{\langle j_1\|f_\lambda(r)Y_\lambda\|j_2\rangle v^{(-)}_{j_1j_2}}{\sqrt{2\mathcal{Y}_{\lambda i}}}\begin{Bmatrix}\lambda_1&\lambda_2&\lambda\\ j_2&j_1&j_3\end{Bmatrix}(\psi^{\lambda_1 i_1}_{j_3j_1}\phi^{\lambda_2 i_2}_{j_2j_3}+\psi^{\lambda_2 i_2}_{j_2j_3}\phi^{\lambda_1 i_1}_{j_3j_1}),$$

where $\langle j_1\|f_\lambda(r)Y_\lambda\|j_2\rangle$ are the reduced matrix elements of the single-particle operators, which generate the phonon excitations; $v^{(-)}_{j_1j_2}=u_{j_1}u_{j_2}-v_{j_1}v_{j_2}$ is a combination of Bogolubov transformation coefficients; $\mathcal{Y}_{\lambda i}$ are normalization coefficients. These expressions for U and V correspond to the case, when only phonons of the natural parity are included.

The graphs associated with matrix elements U and V are presented in fig.1.

We define the wave functions of excited states for even-even spherical nuclei as in ref. /14/:

$$\Psi_r(JM) = \Theta^+_{JMr}\Psi_0' , \qquad (5)$$

where

$$\Theta^+_{JMr} \equiv \sum_\nu [\xi^{Jr}_\nu\Omega^+_{JM\nu}-(-)^{J-M}\zeta^{Jr}_\nu\Omega_{J-M\nu}], \qquad (6)$$

$$\Omega^+_{JM\nu} \equiv \sum_i R^\nu_i(J)Q^+_{JMi}+\sum_{\lambda_1i_1,\lambda_2i_2}P^{\lambda_2i_2}_{\lambda_1i_1}(J\nu)[Q^+_{\lambda_1\mu_1i_1}Q^+_{\lambda_2\mu_2i_2}]_{JM}. \qquad (7)$$

The nuclear ground state is defined as the vacuum Ψ_0':

$$\Theta_{JMr}\Psi_0' = \Psi_0'^{*}\Theta^+_{JMr}=0. \qquad (8)$$

Thus, in comparison with traditional expressions of the QPM in which the wave functions of excited states $\Psi_\nu(JM)=\Omega^+_{JM\nu}\Psi_0$, where $Q_{\lambda\mu i}\Psi_0=0$, expression (5) includes back going amplitudes $\sim\Omega_{JM\nu}$ acting on the modified vacuum (8). We want to allow here for the Pauli principle, so we shall take the exact commutators between operators Q and Q^+. These commutation relations are given by ref. /10/. From these relations and the orthonormalization condi-

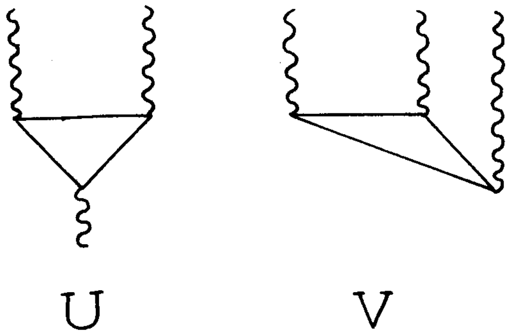

Fig.1. The vertices U and V.

tions of the wave functions of ground and excited states, one can derive the equations coupling amplitudes ξ, ζ, R, P in (6,7) as follows:

$$\sum_\nu\{[\xi^{Jr}_\nu]^2-[\zeta^{Jr}_\nu]^2\}\{\sum_i[R^\nu_i(J)]^2+2\sum_{\lambda_1i_1,\lambda_2i_2}[P^{\lambda_2i_2}_{\lambda_1i_1}(J\nu)]^2+ \qquad (9)$$

$$+\sum_{\lambda_1i_1\lambda_2i_2,\,\lambda_1'i_1'\lambda_2'i_2'}P^{\lambda_2i_2}_{\lambda_1i_1}(J\nu)P^{\lambda_2'i_2'}_{\lambda_1'i_1'}(J\nu)K^J(\lambda_2'i_2',\lambda_1'i_1'|\lambda_1i_1,\lambda_2i_2)\}=1,$$

In (9) the terms containing $K^J(\lambda_2'i_2',\lambda_1'i_1'|\lambda_1i_1,\lambda_2i_2)$ are caused by the Pauli principle. Analytic expressions for K^J in the general case and in the diagonal approximation are given in ref. /10/.

From the point of view of violating the Pauli principle the most dangerous terms are the components corresponding to the diagonal part of K^J. As has been shown in refs. /11,12/ the corrections of the Pauli principle are not significant at low-lying energies and influence slightly the GMR. In the following expressions we will consider only the diagonal terms of K^J.

Coefficients K^J associate with graphs in fig.2, where graph 2a corresponds to the diagonal approximation. Graph 2b is necessary for calculating nondiagonal components in K^J.

Using the linearization method for the equations of motion:

$$[H,\Theta^+_{JMr}]=\eta_{Jr}\Theta^+_{JMr},$$

$$[H,\Theta_{JMr}]=-\eta_{Jr}\Theta_{JMr},$$

or the variational principle under the assumption that the number of phonons in the ground states is small: $\langle\Psi_0'|Q^+Q|\Psi_0'\rangle\approx 0$ /14/, we obtain a system of equations to determine the coefficients ξ^{Jr}_ν, ζ^{Jr}_ν, $R^\nu_i(J)$, and $P^{\lambda_2i_2}_{\lambda_1i_1}(J\nu)$ in the diagonal approximation for K^J:

$$\sum_\nu\{\xi^{Jr}_\nu[(\omega_{Ji}-\eta)R^\nu_i(J)+\sum_{\lambda_1i_1,\lambda_2i_2}U^{\lambda_2i_2}_{\lambda_1i_1}(Ji)P^{\lambda_2i_2}_{\lambda_1i_1}(J\nu)(1+\frac{1}{2}K^J(\lambda_2i_2,\lambda_1i_1|\lambda_1i_1,\lambda_2i_2))]+$$

$$+\, 3\zeta_\nu^{J\tau} \sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right)\} = 0\,,$$

$$\sum_\nu \{\xi_\nu^{J\tau}[(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta)\, P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu) + \frac{1}{2}\sum_i U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, R_i^\nu(J)] +$$

$$+\, \frac{3}{2}\zeta_\nu^{J\tau}\sum_i V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, R_i^\nu(J)\} = 0\,,$$

$$\sum_\nu\{\zeta_\nu^{J\tau}[(\omega_{Ji}+\eta)R_i^\nu(J) + \sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right)] + \qquad (10)$$

$$+\, 3\xi_\nu^{J\tau}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right)\} = 0\,,$$

$$\sum_\nu\{\zeta_\nu^{J\tau}[(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)+\eta)\, P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu) + \frac{1}{2}\sum_i U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, R_i^\nu(J)] +$$

$$+\, \frac{3}{2}\xi_\nu^{J\nu}\sum_i V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)\, R_i^\nu(J)\} = 0\,,$$

$i = 1,2,\dots,n_1$; $\tau, \nu = 1,2,\dots, n_1+n_2$; n_1 and n_2 are the full number of one and two-phonon components in (5). Expressing all variables of system (10) through

$$\mathcal{P}_i^{J\tau} = \sum_\nu \xi_\nu^{J\tau} R_i^\nu(J); \quad \mathcal{R}_i^{J\tau} = \sum_\nu \zeta_\nu^{J\tau} R_i^\nu(J)$$

we infer a simplified expression from (10):

$$M.\vec{L} = \left(\begin{array}{c|c} \text{I} & \text{II} \\ \hline & M_{kk'} \\ \text{III} & \text{IV} \end{array}\right) \cdot \begin{pmatrix} \mathcal{P}_1 \\ \vdots \\ \mathcal{P}_{n_1} \\ \mathcal{R}_1 \\ \vdots \\ \mathcal{R}_{n_1} \end{pmatrix} = 0\,. \qquad (11)$$

Rank of matrix M is $2n_1$. The matrix elements $M_{kk'}$ in different quadrants (11) are:

I. $1 \le k,\ k' \le n_1$:

$$M_{kk'} = (\omega_{Jk}-\eta)\delta_{kk'} - \frac{1}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta} + \right.$$

$$\left. + 9\frac{V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)+\eta}\right]\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right).$$

II. $1 \le k \le n_1$; $n_1+1 \le k' \le 2n_1$:

$$M_{kk'} = -\frac{3}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta} + \right.$$

$$\left. + \frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)+\eta}\right]\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right).$$

(12)

III. $n_1+1 \le k \le 2n_1$; $1 \le k' \le n_1$:

$$M_{kk'} = -\frac{3}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)+\eta} + \right.$$

$$\left. + \frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta}\right]\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right)$$

IV. $n_1+1 \le k,\ k' \le 2n_1$:

$$M_{kk'} = (\omega_{Jk}+\eta)\delta_{kk'} - \frac{1}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)+\eta} + \right.$$

$$\left. + 9\frac{V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)\, V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta}\right]\left(1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right).$$

From (12) one can find the eigenenergies $\eta_{J\tau}$ by solving the secular equation:

$$\det\|M(\eta)\| = 0\,. \qquad (13)$$

System (10) is more general as compared with basic equations utilized so far in the QPM. From system (10) all equations of the QPM described, for example, in ref.[1,10,14] can be derived. Thus, if we neglect the corrections of the Pauli principle, $\mathcal{K}^J$ and $\Delta\omega$ become zero and system (10) is transformed completely into the system of equations of the QPM including

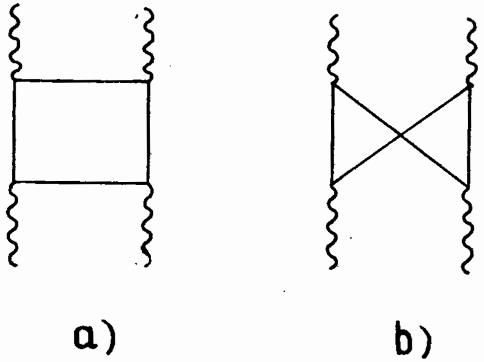

Fig.2. Pauli principle corrections K^J.

only the phonon interactions in the ground state [14]. If we deal with our old phonon vacuum Ψ_0 in which phonon correlations in the ground state are neglected, we shall derive basic equations with effects due to the Pauli principle from (10). In the case if K^J and V are equal to zero, the system (10) is changed into the traditional QPM equations [1] corresponding to the well-know secular equation:

$$\det\left|\left|(\omega_{Ji} - \eta_\nu)\delta_{ii'} - \frac{1}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} \frac{U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)\, U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i')}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_\nu}\right|\right| = 0 .$$

2. DIAGRAMS OF THE QPM. COMPARISON WITH OTHER MODELS

All types of diagrams summed in the QPM are represented in fig.3. Diagrams in fig.3 a-c correspond to neglecting the Pauli principle. Diagrams in fig.3d-f contain the corrections of the Pauli principle. We also give in fig.3 the expressions of the matrix elements $M_{kk'}(12)$ associated with these diagrams. Therefore, fig.3 can be a diagrammatic mapping for the QPM equations.

Let us now consider how to rewrite these diagrams into the diagrams of other approaches including 2p–2h configurations, namely the NFT [3-5] and the TFFS [6].

As has been shown in ref. [4,12] the diagrams taken into account in the NFT can be obtained if one of the intermediate phonons in the pole diagrams given by fig.3a is noncollective, i.e., it can be replaced by two-quasiparticle states. Moreover, in the calculations of ref. [4] only diagrams describing the coupling between the TDA phonons were included. It means that from diagrams in fig.4 arising by the above-mentioned transition only diagrams given by fig. 4a)-b) are included. Diagrams in fig. 4c)-d) associate with the back going RPA-amplitudes ϕ. Analytic expressions for the contribution to the self-energy of a vibration associated with these diagrams can be infered

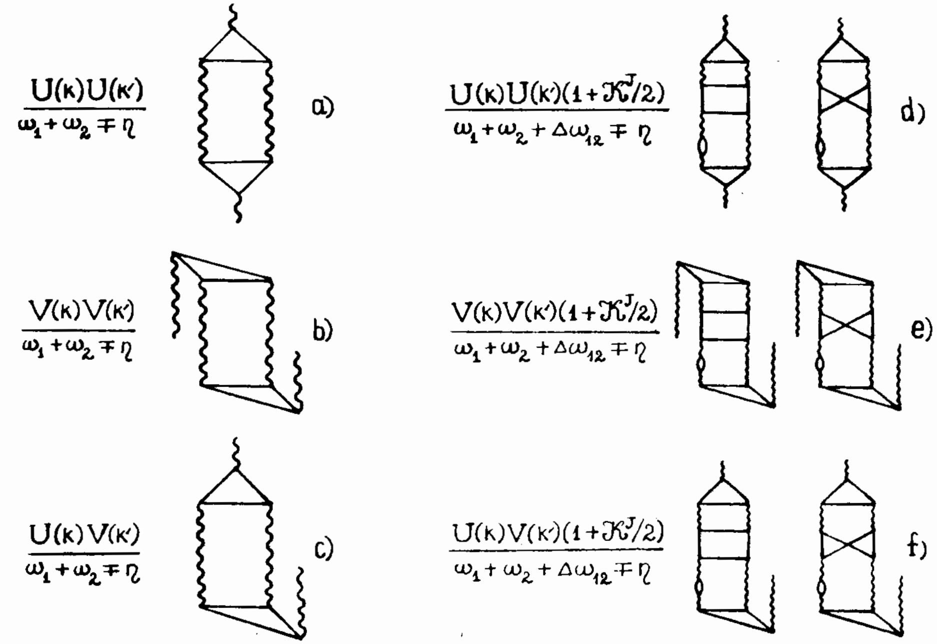

Fig.3. Correspondence between the diagrams included in the QPM with the matrix elements $M_{kk'}(12)$ a,b,c) if the corrections of the Pauli principle are neglected. d,e,f) if the corrections of the Pauli principle are taken into account.

easily from the QPM formula in the transition, when one of intermediate phonons is changed by two quasiparticles. Indeed as has been shown in ref. [15,16] the transition $Q^+_{\lambda\mu i} \to [a^+_{j_1m_1}, a^+_{j_2m_2}]_{\lambda\mu}$ is equivalent to a substitution into all expressions:

$$\phi^{\lambda i}_{s_1 s_2} = 0;\ \psi^{\lambda i}_{s_1 s_2} = \delta_{s_1 j_1}\delta_{s_2 j_2} + (-)^{j_1 - j_2 + \lambda}\delta_{s_1 j_2}\,\delta_{s_2 j_1};\ \omega_{\lambda i} = \epsilon_{j_1} + \epsilon_{j_2},$$

where ϵ_{j_1} and ϵ_{j_2} are the quasiparticle energies. Therefore (3) for $U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)$ becomes:

$$U^{\lambda_2 i_2}_{ss'\lambda_1}(\lambda i) = (-)^{\lambda_1 + \lambda_2 - \lambda}\sqrt{\frac{(2\lambda+1)(2\lambda_2+1)}{2\mathcal{Y}_{\lambda i}}}\ \times$$

$$\times \sum_{j_2}\{f^{\lambda}_{s'j_2}\, v^{(-)}_{s'j_2}\left\{\begin{matrix}\lambda_1 & \lambda_2 & \lambda\\ j_2 & s' & s\end{matrix}\right\}(\psi^{\lambda_2 i_2}_{j_2 s} + \sqrt{\frac{(2\lambda_1+1)}{(2\lambda+1)}}\,\phi^{\lambda_2 i_2}_{j_2 s}) +$$

$$+ f^{\lambda}_{sj_2} v^{(-)}_{sj_2} \left\{ \begin{matrix}\lambda_1 & \lambda_2 & \lambda \\ j_2 & s & s'\end{matrix} \right\} (-)^{s-s'+\lambda_1} \left(\psi^{\lambda_2 i_2}_{j_2 s'} + \sqrt{\frac{(2\lambda_1+1)}{(2\lambda+1)}}\, \phi^{\lambda_2 i_2}_{j_2 s'}\right)] .$$

If we use now the quasiparticle vacuum lacking amplitudes $\phi^{\lambda_2 i_2}_{j_2 s(s')}$, the contributions to the self-energies $\Sigma^{(a)}, \Sigma^{(b)}$ given by ref./3,4/ arise at once from the sum:

$$\sum_{\substack{\lambda_2 i_2 \\ ss'\lambda_1}} \frac{U^{\lambda_2 i_2}_{ss'\lambda_1}(\lambda i)\, U^{\lambda_2 i_2}_{ss'\lambda_1}(\lambda i')}{\epsilon_s + \epsilon_{s'} + \omega_{\lambda_2 i_2} - \eta} .$$

Using the phonon vacuum which includes back going amplitudes $\phi^{\lambda_2 i_2}_{j_2 s(s')}$ we can receive from this sum $\Sigma^{(c)}$ and $\Sigma^{(d)}$ together with

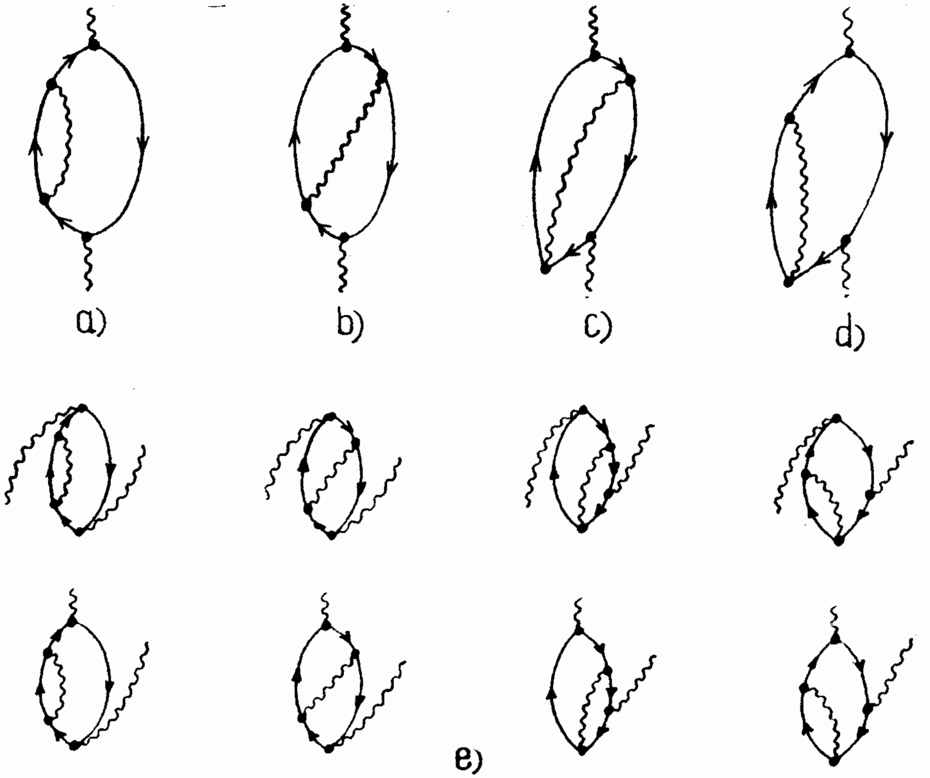

Fig.4. The diagrams describing the couplings used in the NFT a,b,c,d) - without the Pauli principle corrections and the phonon ground state correlations, e) - with the phonon ground state correlations only.

$\Sigma^{(a)}$ and $\Sigma^{(b)}$. The contributions $\Sigma^{(c)}$ and $\Sigma^{(d)}$ arise by the diagrams 4c)-d) which were omitted in calculations in ref./3-5/.

The role of collective and noncollective intermediate two-phonon states in the description of the GMR has been investigated in the QPM in ref./12/. It has been shown that although intermediate states with a strongly collectivized and a noncollective (practically a two-quasiparticle state) phonons contribute significantly to the total width of the damping, the contribution of other components are vital. Therefore, it is desirable to take into account all two-phonon components.

The ground state phonon correlations give rise to unpole diagrams in fig.3a) and diagrams represented by fig.3b)-c). The last diagrams are changed into the NFT diagrams given by fig.4e) through the transition $Q^+ \to \alpha^+\alpha^+$. The Pauli principle produces a wide class of topological unequivalent diagrams in transition from QPM to NFT. In our paper there is no need to represent all these diagrams. As an illustration we show by fig. 5b) an example of this transition for the diagram given in the left side of fig. 3d).

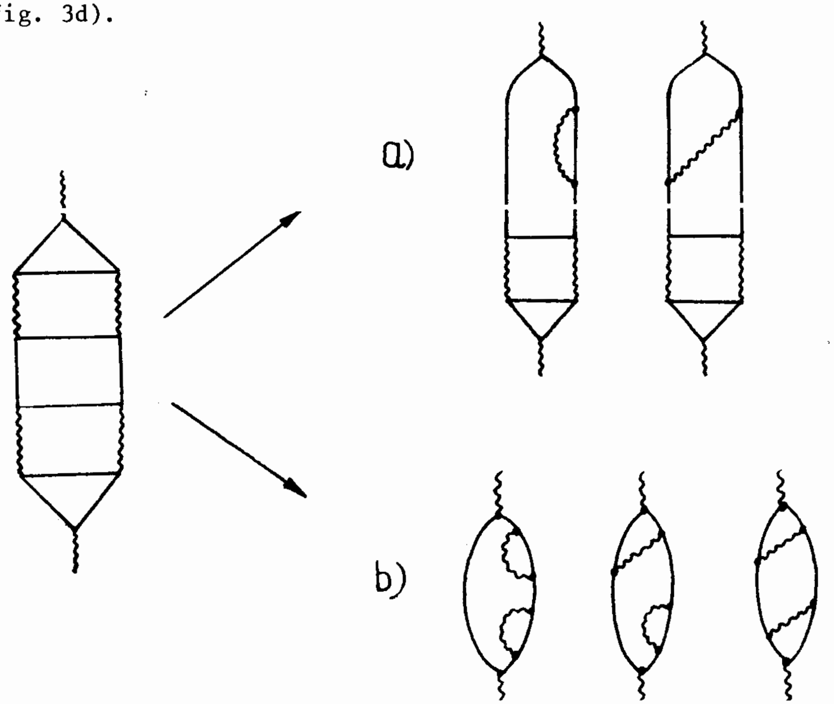

Fig.5. Relation between the QPM diagrams allowing for the effects due to the Pauli principle with the diagrams included in the TFFS /6/ and the NFT /3/ a) Diagrams included in the QPM and the TFFS /6/ , b) Diagrams included in the QPM, the TFFS /6/ and the NFT /3/.

An attempt to allow for 2p – 2h configurations by a generalization of the RPA with the Green function method has been made in ref.[6]. The authors of these works modified the TFFS Green functions by "invisible" effective quasiparticles and obtained equations for the vertices of the "1p1h + phonon" configurations. The diagrammatic correspondence between this approach and the QPM in the case of separable interaction is given by fig.5. A variant for including intermediate two-phonon states as 2p – 2h configurations has been suggested also in refs.[6] and seems to be equivalent to the diagram in fig.3a).

3. MODEL CALCULATIONS

Solving the secular equation (13) turns out to be a complicated task even in the case when the exact corrections of the Pauli principle are not taken into account. To evaluate the influence of the effects due to the Pauli principle and phonon ground state correlations, we perform our calculations with a simple model of the two-level scheme utilized in ref.[14]. In this model the wave function (5) includes only the lowest 2^+ - phonon (with energy $\omega = \omega_{2^+_1}$) and one two-phonon component (with energy $2\omega_{2^+_1}$).

We calculate the eigenenergies $\eta^{(U)}_{1,2}$ (without the phonon correlations) and $\eta^{(U,V)}_{1,2}$ (with the phonon correlations) in two cases when the effects due to the Pauli principle are taken into account approximately[1,9] and exactly[10,11]. Calculations have been done with the modefied computing program GIRES[9,11]. The RPA-phonons are used as 2^+_1-phonons, the structure of which is computed to reproduce the values of the reduced probabilities $B(E2;\, 2^+_1 \to 0^+)$ for nuclei represented in the Table, where the results of our calculations are collected. From this table one can see that the phonon ground state correlations and the corrections of the Pauli principle lead to opposite effects. While the interaction between phonons in the ground state tends to decrease the energies of 2^+_1-states, the Pauli principle leads to an increase in these energies and the influence of the corrections due to the Pauli principle turns out to be stronger. In semimagic nuclei both investigated effects are negligible. In nonmagic nuclei these effects are substantially stronger and affect considerably the energies. Analogous results are obtained by using the experimental values of 2^+_1-energies as the phonon energies $\omega_{2^+_1}$ though in this case these effects would be more prominent because of a stronger interaction. We deal here, of course, with a simplified scheme, so our estimations have a qualitative character.

Table

Amplitudes U and V and the energies of states described by the wave functions $\Theta^+_{JMr}\Psi'_0$ and $\Omega^+_{JM\nu}\Psi_0$. I) The calculations containing the approximate account of the Pauli principle are represented; II) The calculations containing the exact account of the Pauli principle are represented. All the values are given in MeV.

	^{118}Sn		^{124}Te		^{144}Sm		^{148}Sm	
	I	II	I	II	I	II	I	II
U	-0.460	-0.480	-0.016	-0.086	0.162	0.185	0.872	0.553
V	-0.089	-0.117	0.027	-0.005	0.009	0.025	0.264	0.188
ω	1.40		0.670		2.15		0.99	
$\eta^{(U)}_1$	1.328	1.335	0.6698	0.667	2.1439	2.1442	0.694	0.906
$\eta^{(U,V)}_1$	1.319	1.401	0.6682	0.966	2.1438	2.2262	0.484	1.180
2ω	2.80		1.340		4.30		1.98	
$\eta^{(U)}_2$	2.872	3.015	1.340	1.803	4.3061	4.4639	2.276	2.556
$\eta^{(U,V)}_2$	2.864	3.003	1.339	1.783	4.3060	4.4627	2.190	2.509

CONCLUSION

We have shown that the obtained system of equations of the QPM is fairly general and includes all particular cases derived in the framework of the QPM in earlier papers. This system can be utilized for calculating the energy spectra in spherical nuclei. Based on calculations with a simple scheme we come to a conclusion that in nonmagic nuclei one should allow for both effects due to the Pauli principle and to the phonon ground state correlations to describe correctly the characteristics of low-lying levels. For the GMR, as it has been shown in ref.[12], the Pauli principle is not so significant, and the phonon correlations would influence slightly the damping because the coefficients V (5) are small at these energies. The comparison of the QPM diagrams with the diagrams of the NFT[3] and TFFS[6] shows that main diagrams included in calculations by these approaches are the same. Moreover, in numerical calculations of the NFT only particular cases of the QPM diagrams are taken into account.

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Основные уравнения квазичастично-фононной модели ядра с учетом принципа Паули и взаимодействия фононов в основном состоянии

В рамках квазичастично-фононной модели получена система уравнений, позволяющая вычислять энергию и структуру возбужденных состояний, описываемых волновой функцией, содержащей одно- и двухфононные компоненты. При этом учтены требования принципа Паули для двухфононных компонент и фононные корреляции в основном состоянии ядра. На модельном примере даны количественные оценки этих эффектов. Обсуждена взаимосвязь полученных уравнений с результатами других теоретических подходов.

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Basic Equations of the Quasiparticle-Phonon Nuclear Model with the Effects Due to the Pauli Principle and the Phonon Ground State Correlations

A system of basic equations of the quasiparticle-phonon nuclear model is obtained for calculating energies and a structure of excited states described by the wave functions containing one- and two-phonon components. The effects due to the Pauli principle for two-phonon components and the phonon ground state correlations are taken here into account. The quantitative estimations of these effects are given by a simplified scheme. The relation between these equations with the results from other theoretical approaches is discussed.

The investigation has been performed at the Laboratory of the Theoretical Physics, JINR.

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