27/II-84

объединенный
институт
ядерных
исследований
дубна

1164/84

E4-83-858

D.N.Poenaru*, M.Ivascu*, A.Sandulescu

COMPARISON OF SEMIEMPIRICAL FORMULAE FOR ALPHA DECAY HALF-LIVES

Submitted to ЯФ

* Institute for Physics and Nuclear Engineering, Bucharest

1983

1. INTRODUCTION

By using semiempirical relationships [1-6], the alpha decay partial half-lives, T, can be estimated if the kinetic energy of the emitted particle, E_{α}, is known. An up-to-date version of these formulae has been obtained [7,8] by changing their additive parameters $\{C_k\}$ in such a way that the absolute error $(1/n)\sum_{i=1}^{n}\log(T_i/T_{i\,exp})$ - vanishes in each group of even-even (e-e), odd-even (o-e), even-odd (e-o), and odd-odd (o-o) nuclides.

In order to improve the description of data in the neighbourhood of the magic numbers, a new formula with six parameters $\{B_k\}$ has been derived [7-9]. The computer programme described in ref. [8] allows us to improve automatically the parameters $\{C_k\}$ and $\{B_k\}$, every time a better set of experimental data is available.

The purpose of this paper is to compare our calculations and different other estimations with the measured half-lives (see ref. [10] and the literature quoted there).

2. SEMIEMPIRICAL RELATIONSHIPS

As the range of life-times of different nuclides extends over many orders of magnitude, it is more practical to use the decimal logarithm log T. The experimental values of T will be denoted by T_{exp}. The formula given by Fröman [1]:

$$\log T = [139.8 + 1.83\,(Z-90) + 0.012\,(Z-90)^2] / \sqrt{Q} - 0.3\,(Z-90) - 0.001\,(Z-90)^2 + C_F \qquad (1)$$

is limited to the region of even-even nuclei with $Z \geq 84$. Q-values are expressed in MeV; and T, in seconds throughout this work.

Almost all parameters $\{C_k\}$ are negative. Hence the values $-C_k$ are given in Table 1. These are the "new" parameters obtained from the condition that the mean value of the absolute error vanishes. All diagrams presented in this paper (except Fig. 6a) are computed by using these new values.

In Fig. 1a the half-lives of the e-e nuclei calculated with eq. (1) are compared with the experimental ones. To aid the

Table 1

Improved values of the parameters $(-C_k)$

"k"	e-e	o-e	e-o	o-o
F	51.699	51.317	51.299	50.705
W	52.400	52.026	51.940	51.377
T	20.789	20.470	20.346	19.758
K	20.226	20.643	20.383	20.571
V	0.043	- 0.196	- 0.399	- 0.962
H	20.347	20.051	19.922	19.355

eye the consecutive isotopes of a given element are connected with a segment of line; a dashed line is used if one or more isotopes of a sequence are missing. From N = 60 to 82 there is a gap of stable nuclides toward alpha decay or emitters undiscovered yet. Up to now only a few components of the new island of alpha activity close to the double magic ^{100}Sn have been found.

One can see a very good agreement at $N \geq 128$, but for the new region of nuclei produced in heavy ion reactions, the errors as high as 5 orders of magnitude are obtained because eq. (1) was not designed for lighter nuclei.

A better overall result (Fig.1b), though the dispersion for heavy nuclei is larger, is given by a very simple relationship of Wapstra et al. /2/:

$$\log T = (1.2\,Z + 34.9)/\sqrt{Q} + C_w \qquad (2)$$

also valid for e-e nuclei with $Z \geq 85$. This time the maximum error affects Z = 60 nucleus, not Z = 52 as in Fig. 1a.

In the range of N from 88 to 126 and Z from 74 to 90, the mean value of these errors tends to have, in Fig. 1a, a negative slope, and in Fig. 1b it is almost constant. After a steep rise from N = 126 to N = 128 this trend is changed: for $N \geq 128$ it is constant (Fig. 1a) and rises slowly (Fig. 1b).

As it can be seen from Figs. 4-6, the formula presented by Taagepera and Nurmia /3/

$$\log T = 1.61\,(Z_d/\sqrt{E_\alpha} - Z_d^{2/3}) + C_T, \qquad (3)$$

where $Z_d = Z-2$ is the atomic number of the daughter nucleus, and C_T was allowed to vary in different groups of nuclei, remains one of the best; it is practically exceeded only by a new variant (Keller and Münzel /4/ see figs. 2b, 3b, 4b):

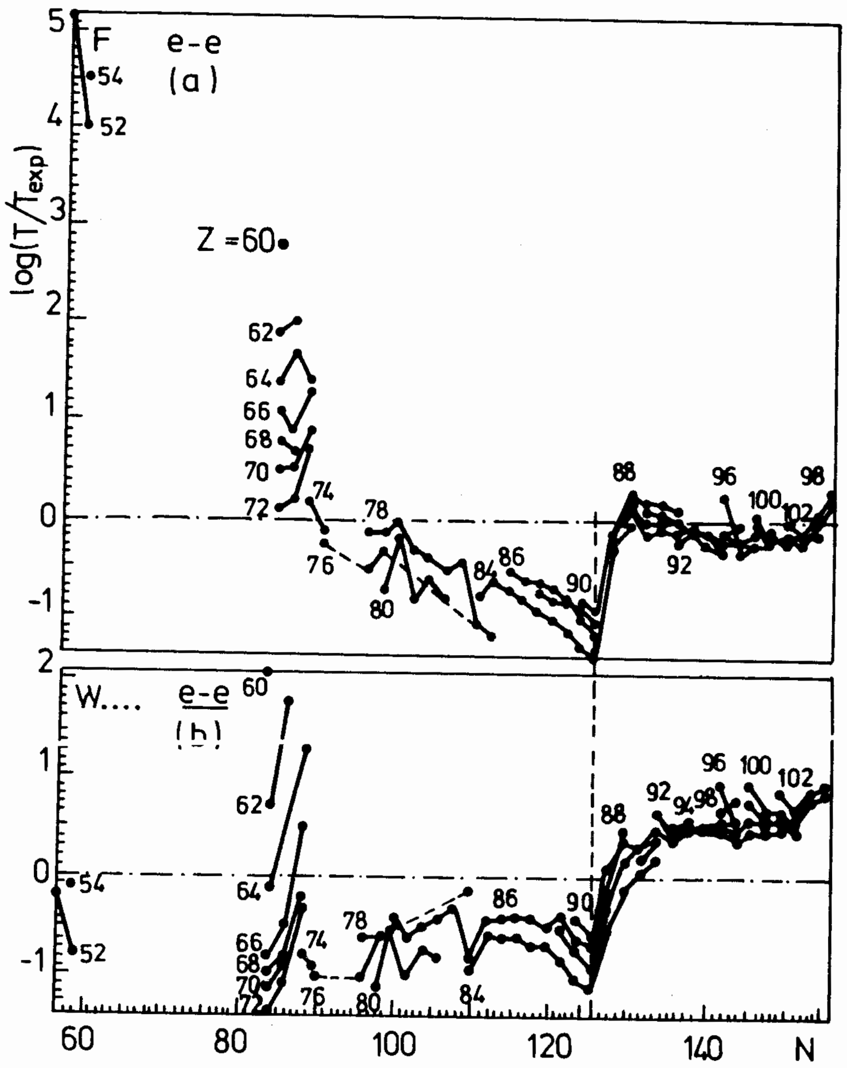

Fig.1. The error of life-time predictions with Fröman's (a) and Wapstra's et al. (b) formula for even-even nuclei.

$$\log T = H_k(Z_d/\sqrt{Q} - Z_d^{2/3}) + C_k\,, \qquad (4)$$

where H_k = 1.61 for e-e, 1.65 for e-o, 1.66 for o-e, and 1.77 for o-o nuclei.

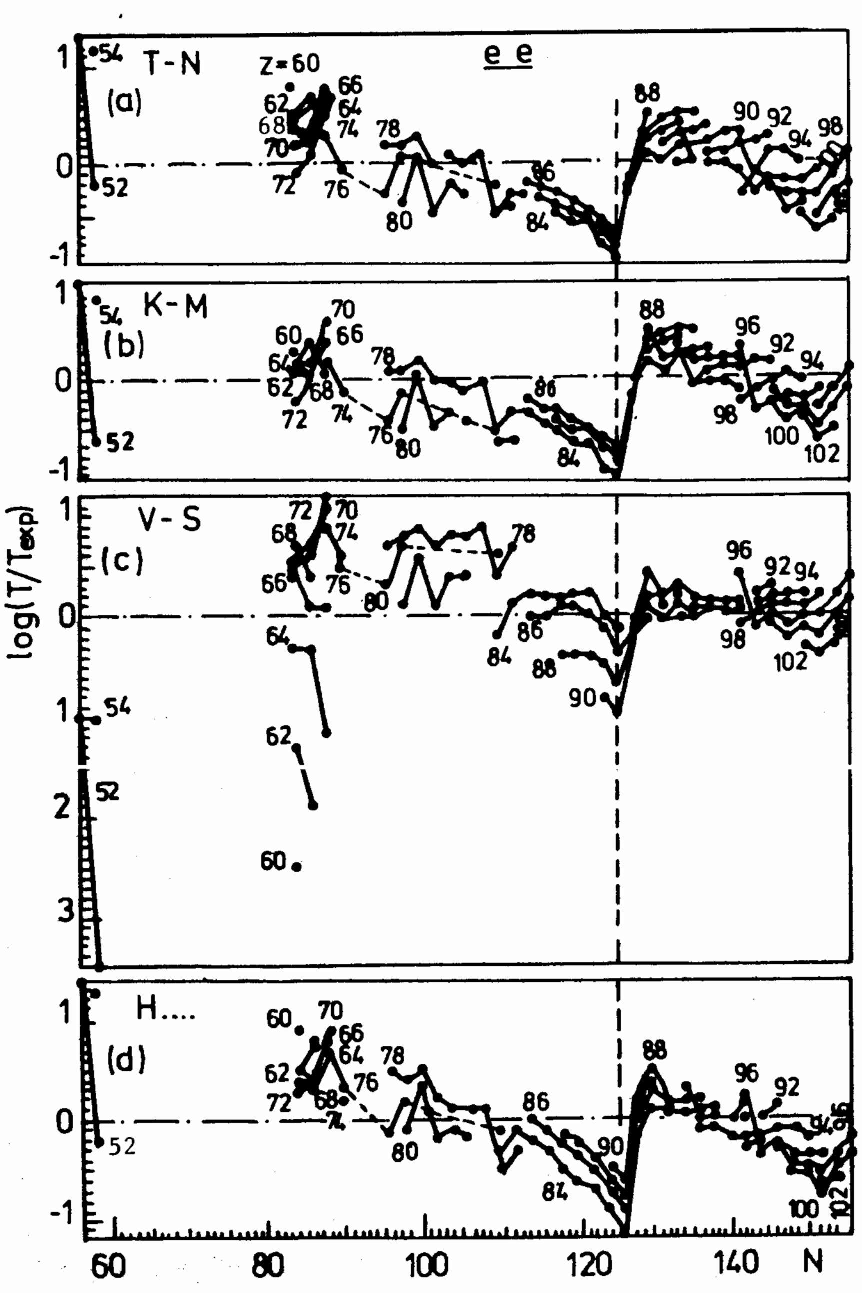

Fig.2. The errors of life-time predictions with Taagepera-Nurmis's (a), Keller-Münzel's (b), Viola-Seaborg's (c) and Hornshøj's et al. (d) formula for even-even nuclei.

The equation presented by Viola and Seaborg [5] (Figs. 2c, 3c, 4c) is of the form

$$\log T = (a_1 Z - a_2)/\sqrt{Q} - b_1 Z - b_2 + C_V , \qquad (5)$$

where $a_1 = 2.42151$; $a_2 = 62.3848$; $b_1 = 0.59015$; $b_2 = 4.2109$ for $N < 126$ and $a_1 = 2.11329$; $a_2 = 48.9879$; $b_1 = 0.39004$; $b_2 = 16.9543$ for $N > 126$, $Z > 82$. It gives excellent agreement in the region of actinides with $N > 128$, but as it can be seen from Fig.2c for e-e nuclei, it underestimates the lifetime of lighter nuclei in contrast with the overestimation of Eq. (1).

Hornshøj et al. [6] have proposed (Figs. 2d, 3d, 4d) the formula

$$\log T = 0.80307 \left(\frac{A_d^{4/3} Z_d}{A} \right)^{1/2} \left(\frac{\arccos\sqrt{x}}{\sqrt{x}} - \sqrt{1-x} \right) + C_H \qquad (6)$$

in which $x = 0.538243\, Q A_d^{1/3}/Z_d$ and C_H is not changed in various groups of nuclei, like C_F of eq. (1) and C_w of eq. (2).

In spite of the strong influence of the neutron shell effects in eqs. (1)-(6), mainly the Z dependence was stressed. From Figs. 1 and 2 (except the region of low Z up to 72 in Figs 1a, 1b, 2c), one can see that for e-e nuclei in all equations a good enough dependence on Z was chosen, because at a given N the spread of the results for various Z is not very large. This fact allows us to see, as it was discussed above in connection with Fig.1, the trend of variation with N of the mean value or errors. It is similar with that of Fig.1a for $N \leq 126$. The jump of almost one order of magnitude at N = 128 is followed again by a constant value in Fig. 2c.

The dispersion of results for o-e (Fig.3), e-o (Fig. 1 in ref. [7]) and o-o (Fig. 4) nuclei is larger than that for e-e nuclei. In Fig.4 there are very pronounced negative errors (-5.6, -5.2, -6.4, and -5.8 orders of magnitude) for Z = 83, N = 127. The fact that the neighbourhood of the magic number of nucleons is very badly described by all these formulae is attested by the presence of the negative peaks in Figs. 1-4.

Our formula [7-9] is written as

$$\log T = \chi K_s / \ln 10 - 20.446 , \qquad (7)$$

where

$$K_s = 2.52956\, Z_d (A_d / A Q)^{1/2} \left(\arccos\sqrt{x} - \sqrt{x(1-x)}\right) ,$$

$$x = 0.4253\, Q (1.5874 + A_d^{1/3}) / Z_d . \qquad (8)$$

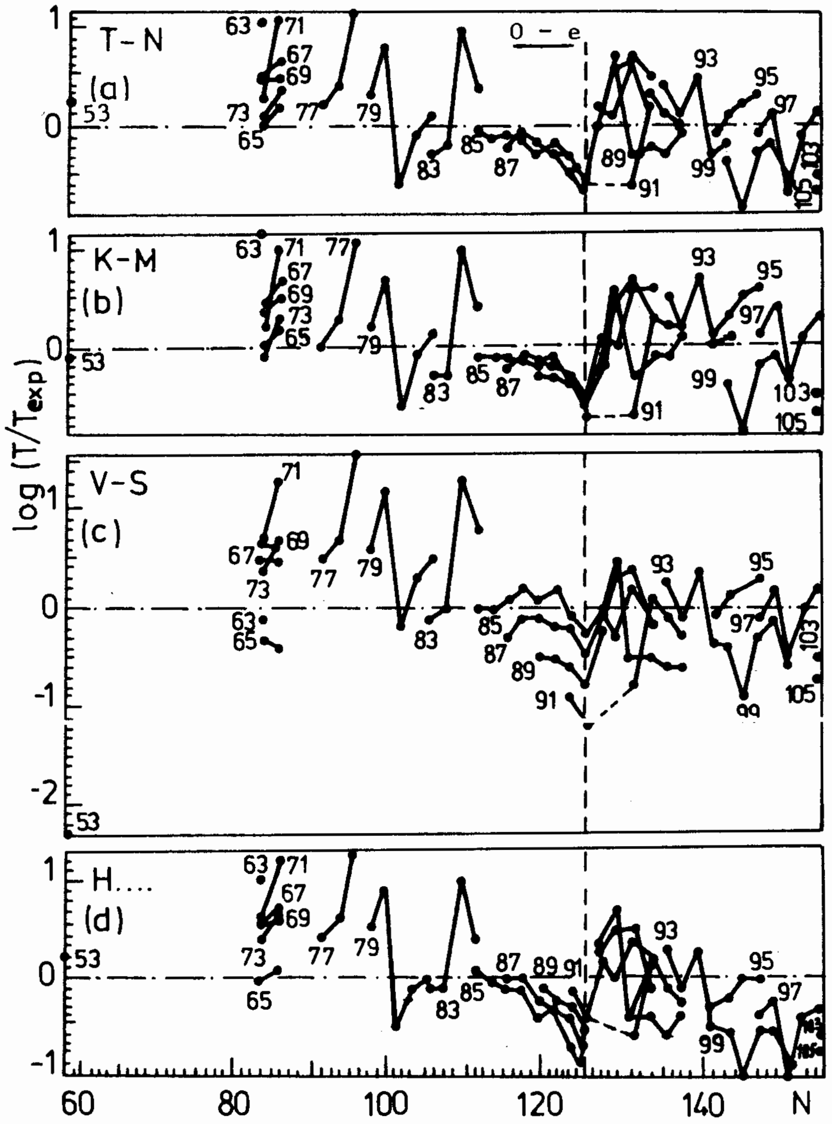

Fig.3. The same as Fig. 2 for odd-even nuclei.

The function

$$\chi = B_1 + B_2 y + B_3 z + B_4 y^2 + B_5 zy + B_6 z^2 \qquad (9)$$

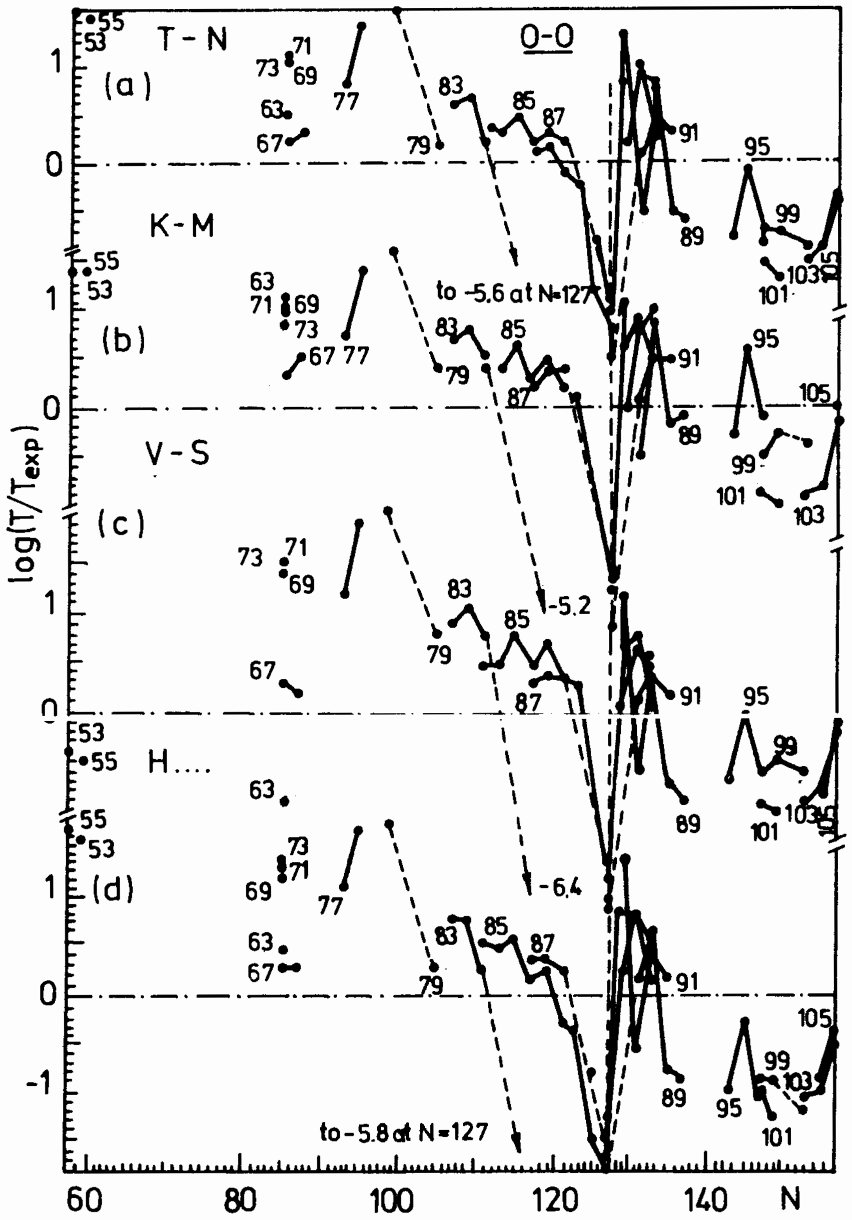

Fig.4. The same as in Fig.2 for odd-odd nuclei.

has the reduced variables y and z, expressing the distance from the closest magic-plus-one number N_i (or Z_i):

$$y = (N - N_i')/(N_{i+1}' - N_i);\ N_i < N \leq N_{i+1}',$$

$$N = \ldots\ldots, 51, 83, 127, 185, \ldots\ldots \quad (10)$$

$$z = (Z - Z_i)/(Z_{i+1}' - Z_i);\ Z_i < Z \leq Z_{i+1}',$$

$$Z = \ldots\ldots, 29, 51, 83, 115, \ldots\ldots \quad (11)$$

The parameters $\{B_i\}$ given in Table 2 are obtained from the fit with our set /10/ of experimental data.

Table 2

Parameters B_k

Group of nuclei	B_1	B_2	B_3
e-e	0.987 722	0.021 227	0.017 407
o-e	1.003 660	0.021 626	0.034 805
e-o	1.014 620	-0.119 094	0.030 606
o-o	1.008 070	-0.184 468	0.259 041
	B_4	B_5	B_6
e-e	0.023 342	0.001 546	-0.016 509
o-e	0.000 870	0.039 190	-0.066 147
e-o	0.156 507	0.228 848	-0.109 130
o-o	0.231 988	0.326 171	-0.406 200

We have tried also /7/ a first order polynomial or a constant approximation for χ.

The capacity of our formula to describe the experimental data can be appreciated from Fig.5a, b, c, d (see also ref. /11/). The increased error in the vicinity of the magic number of neutrons N = 126 which is present for all known formulae (see Fig.1-4) is practically smoothed out (Fig.5a). This performance is only partly achieved for o-e (Fig.5b), e-o (Fig.5c) and o-o (Fig.5d) nuclei. In any case a comparison with Figs. 1-4 demonstrates the advantage of using our formula. Even the very large errors of 5-6 orders of magnitude obtained for Z = 83, N = 127 in Fig.4 are greatly reduced below 0.4 order.

An overall estimation of how well various formulae can describe experimental data could be quantitatively obtained by introducing the standard rms deviation of log T values:

$$\sigma = \{ \sum_{i=1}^{n} \log(T_i / T_{i\ exp})^2 / (n-1) \}^{1/2}. \quad (12)$$

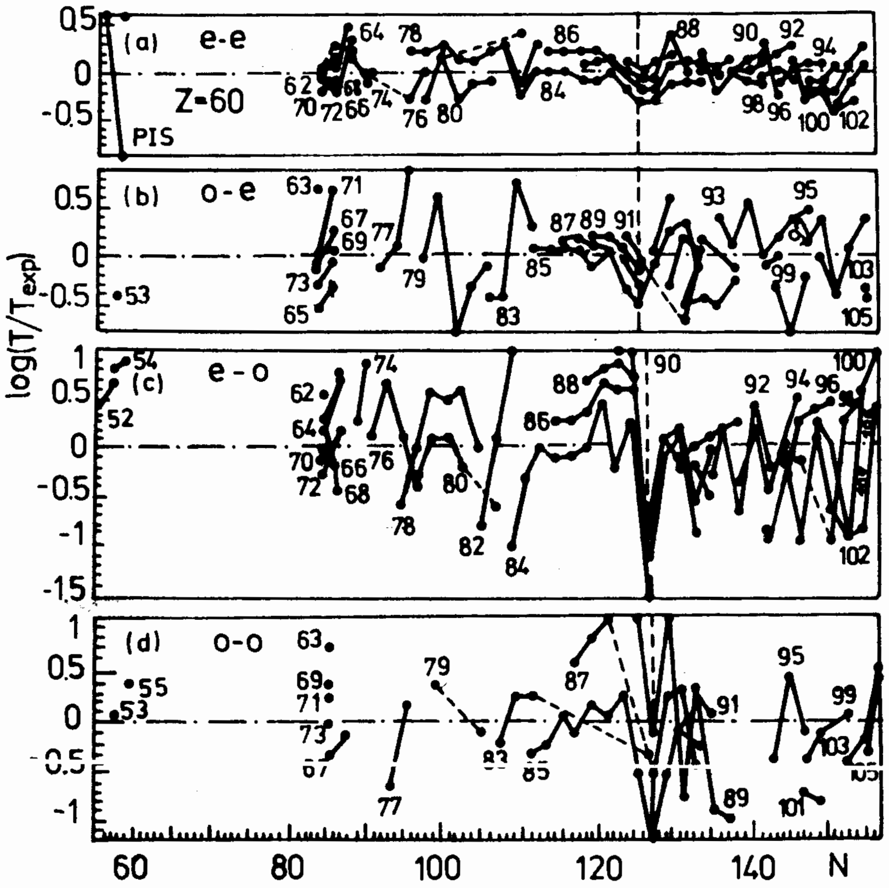

Fig.5. The errors of life-time prediction with our formula when χ is approximated with a second order polynomial of two varialbles for even-even (a), odd-even (b), even-odd (c), and odd-odd nuclei (d).

This quantity was displayed in Fig.6a for the original and in Fig.6b for the improved additive coefficients in each of the four groups of nuclei. Only for some particular cases (hatched area at the top of the column): the Viola-Seaborg and Hornshøj et al. formulae for e-e nuclei, Viola-Seaborg formula for o-o nuclei, and Keller-Münzel equation for all group of nuclei, the reduction of σ is not larger than 0.02. This is a property of the experimental data used by various authors in comparison with our set of data.

Even the constant χ approximation of our formula has σ somewhat lower than the best of the known relationships /4/. Of course

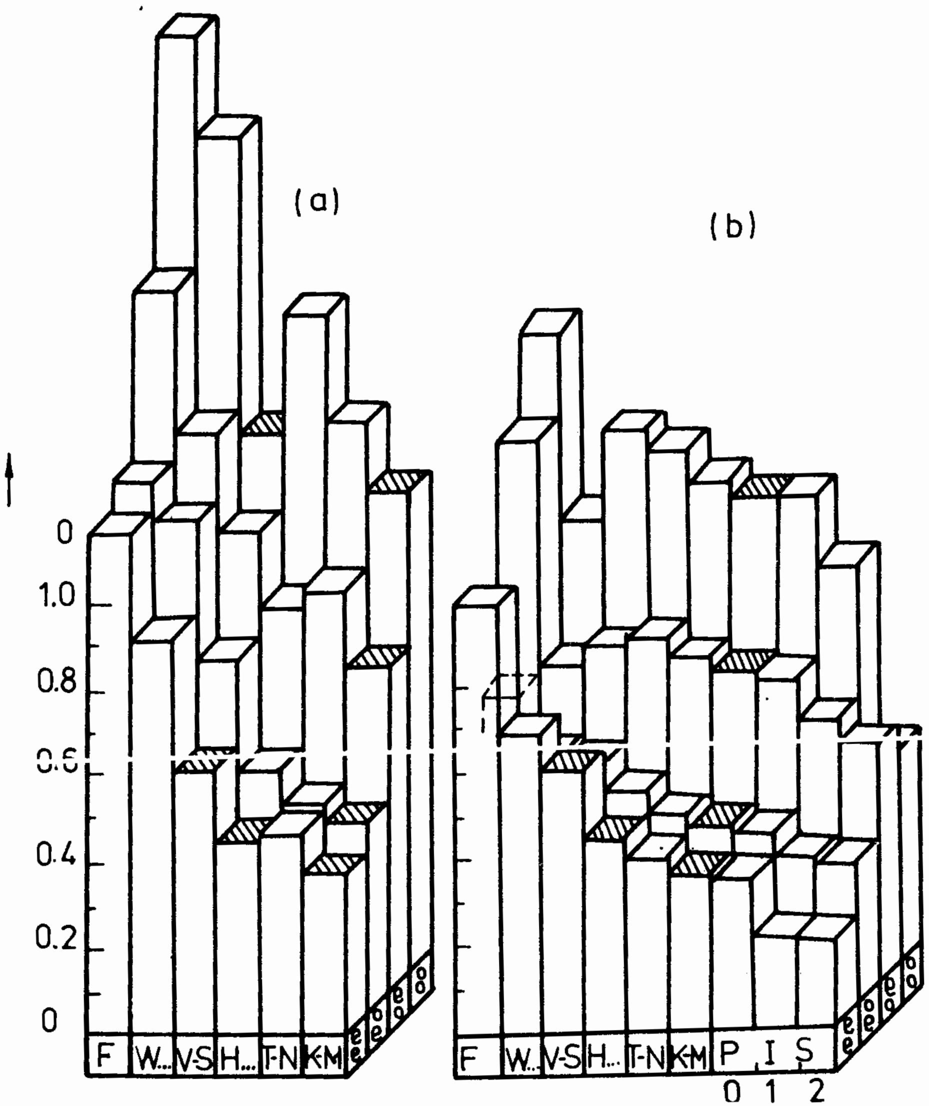

Fig.6. The standard deviation of various formulae in each group of nuclei for both old (a) and new (b) values of the additive parameters.

the second order polynomial approximation leads to smaller standard deviations.

CONCLUSIONS

A relatively reliable estimate of alpha decay partial half-lives can be made by using the semiempirical relationships. After changing the additive parameters of the examined formulae in such a way that the mean value of the errors $(1/n)\sum_{i=1}^{n}\log(T_i/T_{i\,exp})$ vanishes in each group of the nuclei, the standard deviation, σ, of log T values is usually reduced. The reduction is not significant in the following cases: the Viola-Seaborg formula for e-e and o-o nuclei, Hornshøj et al. for e-e nuclei and Keller-Münzel for all groups of nuclei, showing that our data have similar properties with those of the cited authors.

For three of these equations (Hornshøj et al., Taagepera-Nurmia and Keller-Münzel)the lowest errors are obtained in case of e-e nuclei; then follow o-e, e-o and o-o nuclei. Deviations from this rule are encountered with Fröman's and Wapstra's et al. formulae giving a lower σ for o-o nuclei. Our second order polynomial approximation for χ leads also to a reduced σ for o-o nuclei in comparison with that of e-o nuclei.

Keller-Münzel's relationship is the best of all formulae, but even this has an increased error in the vicinity of the magic number of neutrons N = 126.

This is practically smoothed by our formula for e-e nuclei due to the explicit consideration of the dependence on the reduced variables expressing the relative distance of neutron and proton from the closest magic-plus-one numbers. In this way, one can obtain the smallest errors in all groups of nuclei.

In the future when a better set of experimental data (more accurate or more complete) will be available, the parameters $\{B_k\}$ and $\{C_k\}$ could be automatically improved by a computer program/8/.

REFERENCES

1. Fröman P.O. Mat.Fys. Sk. Dan. Vid.Selsk., 1957, 1, p.3.
2. Wapstra A.H., Nijgh C.J., Van Lieshout R. In: Nuclear Spectroscopy Tables, Amsterdam, North-Holland, 1959.
3. Taagepera R., Nurmia M. Ann.Acad,Sci, Fennicae Ser. A Physica Nr. 78, 1961.
4. Keller K.A., Münzel H. Z.Physik, 1972, 255, p. 419.
5. Viola V.E., Seaborg G.T. J.Inorg.Chem., 1966, 28, p. 741.
6. Hornshøj P. et al. Nucl.Phys., 1974, A230, p. 365.
7. Poenaru D.N., Ivascu M. J.Physique, 1983, 44, p. 791.
8. Poenaru D.N., Ivascu M., Mazilu D. Comput.Phys.Communication, 1982, 25, p. 297.

9. Poenaru D.N., Ivascu M., Mazilu D. J.Physique Lett., 1980, 41, L-590.
10. Poenaru D.N., Ivascu M. Rev.Roum.Phys., 1983, 28, p. 309.
11. Rurarz E. Inst. of Nucl.Res. Warszawa, report INR 1950/IA/PL/A, 1983.

Received by Publishing Department
on December 16,1983.

Поэнару Д.Н., Ивашку М., Сэндулеску А. E4-83-858
Сравнение полуэмпирических формул для периодов альфа-распада

Сравниваются полуэмпирические соотношения, данные Фрёманом, Вапстрой и др., Хорншойей и др., Таагепарой и Нумрией, Келлером и Мюнцелем, Виолой и Сиборгом для периодов альфа-распада, с экспериментальными данными и новыми формулами, выведенными авторами из теории давления применительно к альфа-распаду для четно-четных, нечетно-четных, четно-нечетных и нечетно-нечетных ядер. Принимая во внимание оболочечные эффекты, можно с помощью новых формул получить наилучшее согласие с экспериментальными данными даже вблизи магических чисел.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Poenaru D.N., Ivascu M., Sandulescu A. E4-83-858
Comparison of Semiempirical Formulae for Alpha Decay Half-Lives

The semiempirical relationships given by Fröman, Wapstra et al., Viola and Seaborg, Hornshøj et al., Taagepera and Nurmia, Keller and Münzel for alpha decay half-lives are compared with experimental results and with a new formula derived by the authors from the fission theory of alpha decay in even-even, odd-even, even-odd and odd-odd nuclei. By taking into consideration the shell effects, the new formula allows one to obtain a better agreement with experimental data, even in the neighbourhood of the magic numbers.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983