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SOFTNESS OF Gd, Dy, Er, AND Yb NUCLEI TO NONAXIAL DEFORMATION

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1. INTRODUCTION

Low-lying states of nuclei have been intensively studied both experimentally and theoretically in last decades. It has been well established that the low-lying states of spherical nuclei have a vibrational character while the deformed nuclei are good rotators. In the transitional nuclei an intermediate picture is observed: the vibrational and rotational degrees of freedom are coupled. It is usually assumed that the nuclei in the ground state possess at least one symmetry axis. However, recently a vast experimetal material obtained by Davidson et al./1/. for ^{168}Er indicates, according to the analysis performed by Bohr and Mottelson/2/. (see also the paper of Dumitrescu and Hamamoto/3/), a possibility of the existence of nonaxial deformation in the ground state and in the neighbouring states.

In ^{168}Er a collective 4_1^+ state is observed which is assumed to have a two-phonon γ-vibrational stucture /2,3/. The ratio of the energy of this two-phonon state to the corresponding energy of a one-phonon states is

$$\frac{E_{4,4_1^+}}{E_{2,2_1^+}} \approx 2.5 \tag{1}$$

that indicates sizeable unharmonic effects in this nucleus. However, the additional data obtained recently/4/ allow one to ascribe this 4_1^+ state (2031 KeV) to a rotational band with $K = 0_4^+$ with the bandhead in 1833.5 keV that probably is a two-phonon state. Therefore the analysis of refs./2,3/ is relevant rather to the $I = 4^+$ state with $E = 2.055$ MeV, also seen in the data.

The existence or nonexistence of the collective two-phonon states in the low-lying energy region in deformed nuclei has been also discussed in/9/, where the conclusion about nonexistence of the two-phonon low-lying states in the deformed nuclei has been made.

One can see that the question of existence of the low-lying collective two-phonon states in deformed nuclei is connected with the unharmonicity effects caused by the small nonaxial γ-deformation in these states. Therefore the main aim of this paper is to investigate the softness of a deformed nuclear field with respect to nonaxial deformations.

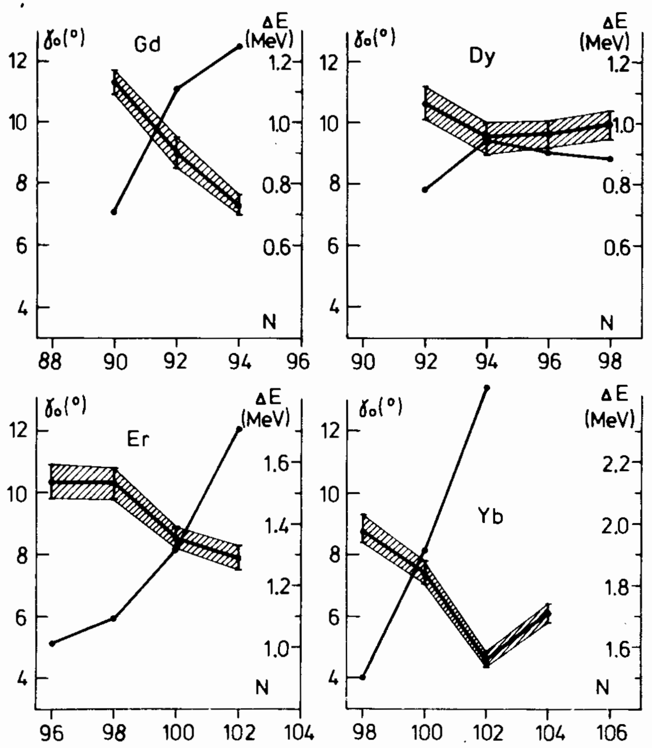

Fig. 1. The dependence of the zero-γ vibration amplitude and the total energy difference E_{tot} (γ = 20°) - E_{tot} (γ = 0°) on the neutron number for isotopes of Gd, Dy, Er and Yb. The zero amplitudes γ_0 have been obtained from B(E2, gs → 2^+_γ) values /22-27/ and are given with their exp. errors.

The analysis of the experimental data on B(E2) reduced probabilities of transitions from the ground state to a low-lying 2^+_γ state performed in this work indicates quite large γ_0 amplitudes ($\gamma_0 \sim 10°$) for some well deformed nuclei in the vicinity of Dy and Er (see fig. 1). The large values of these amplitudes manifest the susceptibility of these nuclei to nonaxial deformations. It should be remarked that calculations performed in 1968 by Arseniev et al./5/ for some nuclei in this region have already shown such an effect which, however, remains unnoticed. Similar calculations have been done by Kumar et al./6/ , Pomorski et al./7/ and Gotz /8/. In refs./5,6,7/ the Nilsson potential has been used, whereas in ref./8/ the Wood-Saxon potential was employed. In all these papers the hexadecapole deformation wasn't taken into account in treating of nonaxial degrees of freedom. In the present work we include the nonaxial hexadecapole deformation.

2. DESCRIPTION OF THE METHOD

In our calculations we apply the shell-correction method developed by Strutinsky /10,11,12/ . As this method is widely described in the literature, we restrict ourselves to a brief presentation of its basic assumptions.

Let us write the deformation energy in the usual form:

$$E = E_{LD}(\hat{\beta}) + \delta E(\hat{\beta}), \tag{2}$$

where $E_{LD}(\hat{\beta})$ is a liquid-drop component and $\delta E(\hat{\beta})$ is a shell correction. E_{LD}, which is a smooth function of the deformation and particle numbers, N and Z, can be written in the droplet model /13-15/ as:

$$E_{LD}(\hat{\beta}) = E_s^{sph}\,[(B_s(\hat{\beta}) - 1) + 2x(B_c(\hat{\beta}) - 1)], \tag{3}$$

where $B_c(\hat{\beta})$ and $B_s(\hat{\beta})$ describe the Coulomb and nuclear-surface energy, respectively, and are normalized so that they become unity for spherical nuclei

$$E_s^{sph} = 17.9439\,[1 - k(\frac{N-Z}{A})^2]\,A^{2/3}\ \mathrm{MeV},$$

$$x = (\frac{Z^2}{A})\,/\,[50.88(1 - k(\frac{N-Z}{A})^2)], \tag{4}$$

$$k = 1.7826.$$

In the above formulae $\hat{\beta}$ is a set of the parameters characterizing the nuclear shape. The nuclear surface for moderate deformations can be described by using the multiple expansion

$$R(\Omega) = C(\hat{\beta})R_0\,[1 + \sum_{L=2}^{4}\sum_{M}\beta_{LM}Y^*_{LM}(\Omega)], \tag{5}$$

where $\beta^*_{LM} = (-1)^M\beta_{L-M}$, Ω stands for the set of the polar angles (ϕ, θ) and $R_0 = r_0 A^{1/3}$ is the radius of the corresponding spherical nucleus. The function $C(\hat{\beta})$ secures the conservation of the nuclear volume with changes of the nuclear surface.

One can choose a coordinate system in which

$$\beta_{2\pm1} = 0, \quad \beta_{22} = \beta_{2-2} \tag{6}$$

and introduce Bohr parameters β_2 and γ

$$\begin{aligned} \beta_{20} &= \beta_2 \cos\gamma, \\ \beta_{22} &= \beta_{2-2} = \frac{\beta_2}{\sqrt{2}} \sin\gamma. \end{aligned} \tag{7}$$

In such a parametrization of the quadrupole degrees of freedom, axially symmetric nuclear surfaces are described by $\gamma = k\frac{\pi}{3}$, (k = 0, ±1, ±2, ±3). To extend this property to hexadecapole degrees of freedom, we have used the Cayley-Hamilton theorem to write the spherical, rank-four tensor, β_{4M} as [21]:

$$\begin{aligned} \beta_{40} &= \frac{\beta_4}{6}(5\cos^2\gamma + 1), \\ \beta_{42} &= \beta_{4-2} = \frac{\beta_4}{6}\sin 2\gamma, \\ \beta_{44} &= \beta_{4-4} = \frac{\beta_4}{6}\sqrt{\frac{35}{2}}\sin^2\gamma, \\ \beta_{4\pm1} &= \beta_{4\pm3} = 0. \end{aligned} \tag{8}$$

A more general parametrization of hexadecapole degrees of freedom has been suggested recently in ref. [16]. Relations (8) leave us with a set of three independent deformation parameters.

The shell corrections in formula (2), $\delta E(\hat\beta)$, have been calculated in the usual way [13-15] by means of a correction polynomial of the sixth order, using the single particle spectrum of the Saxon-Woods potential. The latter has been taken in the form [17,18]

$$V(\vec r, \hat\beta) = V_0/[1 + \exp(\ell(\vec r, \hat\beta)/a)], \tag{9}$$

where V_0 is the depth of the potential well and a is the diffuseness of the nuclear surface. The function $\ell(\vec r, \hat\beta)$, describing the distance between a given point $\vec r$ and the nuclear surface, has been determined numerically and taken negative for points inside the nucleus. For spherical nuclei $\ell(\vec r, \hat\beta = 0) = r - R_0$, where $R_0 = r_0 A^{1/3}$, is the radius of the corresponding spherical nucleus. The usual form of the spin orbit interactin has been assumed:

Table 1

Parameters of the Wood-Saxon average field potential for ^{162}Dy

	Central potential			Spin-orbital potential		
	r_0 [fm]	a [fm]	V_0 [fm]	$(r_0)_{so}$ [fm]	$(a)_{so}$ [fm]	λ
Prot.	1.275	0.7	-57.5	0.901	0.7	18.51
Neut.	1.347	0.7	-41.7	1.221	0.7	32.82

$$V_{so}(\vec r, \hat\beta) = -\lambda\left(\frac{h^2}{2Mc}\right)^2 (\vec\nabla V \times \vec p)\cdot \vec s\,\Big|_{r_0 \to (r_0)_{s0}} . \tag{10}$$

where $\vec p$ and $\vec s$ are nucleon momentum and spin operators, respectively, and V is given by eq. (9) with $(r_0)_{s0}$ being the corresponding spin-orbit interaction radius given in table 1. The Coulomb potential for protons has been determined as a classical electrostatic potential of a uniformly charged nucleus with a nuclear shape given by eq. (5), and with Coulomb radius $(r_0)_c$, equal to the radius of the central part: $(r_0)_c$ = 1.275 fm. All calculations presented in this paper have been carried out by using the level spectrum of ^{162}Dy nucleus. The values of parameters used in the calculations taken from ref. [18] are listed in table 1. The pairing strength given in ref. [19] equals

$$G_{n,p} = [G_0 \mp G_1(N - Z)]/A,$$

where

G_0 = 18.95

for neutrons

G_1 = 0.078

$$G_0 = 17.90 \qquad \text{for protons.} \qquad (11)$$
$$G_1 = 0.176$$

3. RESULTS

The calculation was performed for nuclei in the rare-earth region: $^{154-170}$Gd, $^{156-170}$Dy, $^{158-170}$Er, and $^{162-172}$Yb. For these nuclei energies were calculated at deformation points $(\beta_2, \beta_4, \gamma)$ in limits:

$$\beta_2 \in [\,0.21\ (0.04)\ \ 0.33\,],$$
$$\beta_4 \in [-0.02\ (0.03)\ \ 0.07\,], \qquad (12)$$
$$\gamma \ \in [\,0°\ (4°)\ \ 20°\,].$$

The results are analysed in the form of contour-maps showing the energies in the (β_2, γ) and (β_4, γ) planes. At each point (β_2, γ) and (β_4, γ) the minimization was performed with respect to β_4 and β_2, respectively.

Figures 2 and 3 show examples of such contour maps for Gd, Dy, Er, and Yb, and for N = 96, 98 and 100. It is seen from the figures that for N = 96 and 98 the nuclei of Dy, Er, and Gd are quite susceptible to nonaxial deformations. This effect is most pronounced for ^{164}Dy, where $\gamma = 14°$ corresponds to an energy increase from the minimum only by about 200 keV. One should notice for comparison that the nuclei of Yb are rather rigid with respect to nonaxial deformations. It is also seen that the trajectory of the minimum energy of fixed γ corresponds to an approximately constant $\beta_2(\beta_4)$ and vice versa. (see also Figs. 7 and 8).

It is interesting to observe that this result of our calculations coincides with the assumption made by Bohr and Mottelson in their analysis /2/. Figure 4 shows the dependence of components of the total energy of ^{164}Dy on the parameter γ. (It should be noticed that the total energy was minimized with respect to β_2 and β_4 at each point γ of fig. 4. Such points form the so-called trajectory in γ-direction in the $(\beta_2, \beta_4, \gamma)$-space). It is seen that the liquid-drop component E_{LD} changes rather slowly, while the pairing δE^{tot}_{PAIR} and shell corrections δE^{tot}_{shell} are more sensitive to γ. However, their effect tends to channel, and the resulting total energy V has a flat minimum around $\gamma = 0$. An analogous dependence is shown in fig. 5 for single-particle levels.

The particular softness observed for N = 96, 98, in figs. 2 and 3 is reflected here by an energy gap which is approximately constant with increasing γ.

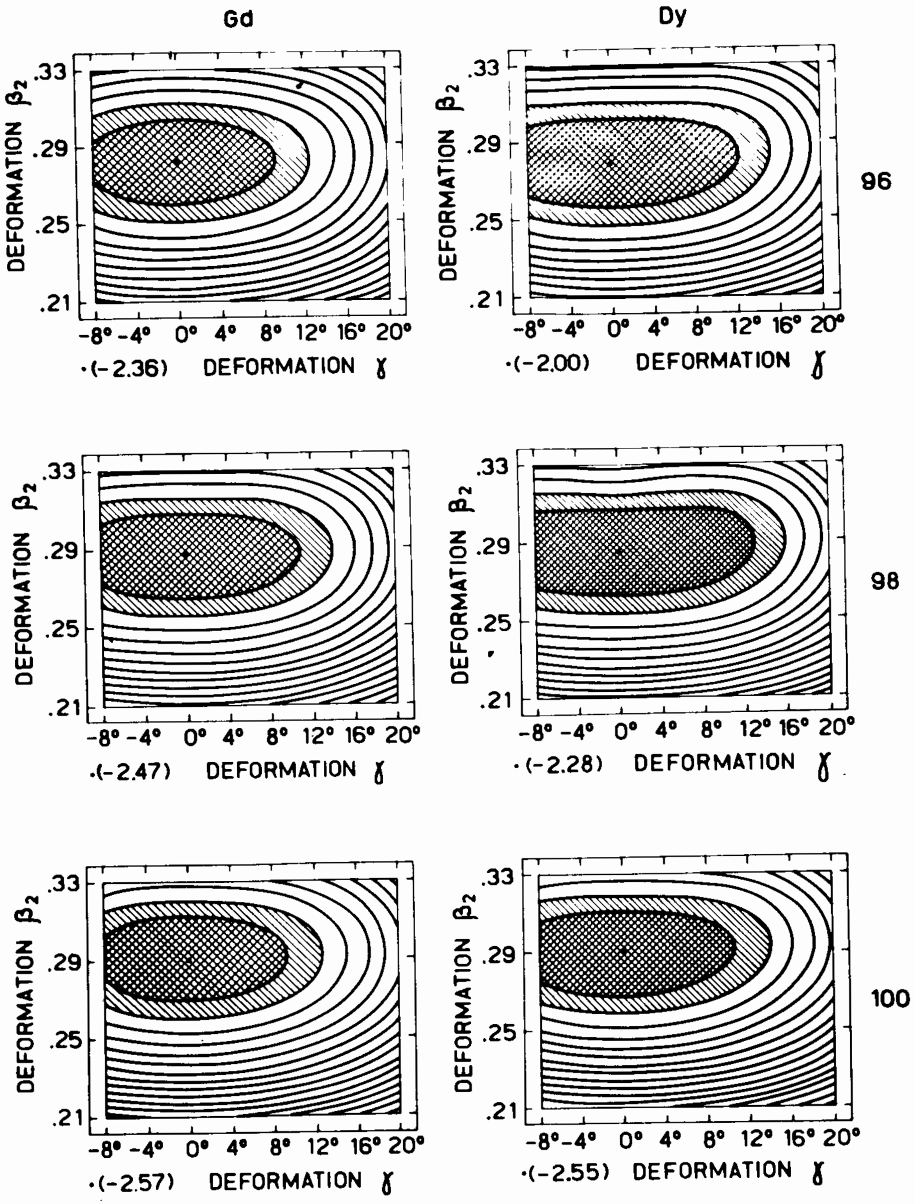

Fig. 2. The contour map of the total nucleus energy in (β_2, γ) plane for Gd and Dy and for N = 96, 98, 100. At each point (β_2, γ) the minimization with respect to β_4 was performed. Energy intervals separating the contour-lines are 100 KeV. Energy values at the minima are given at the bottom of the figures.

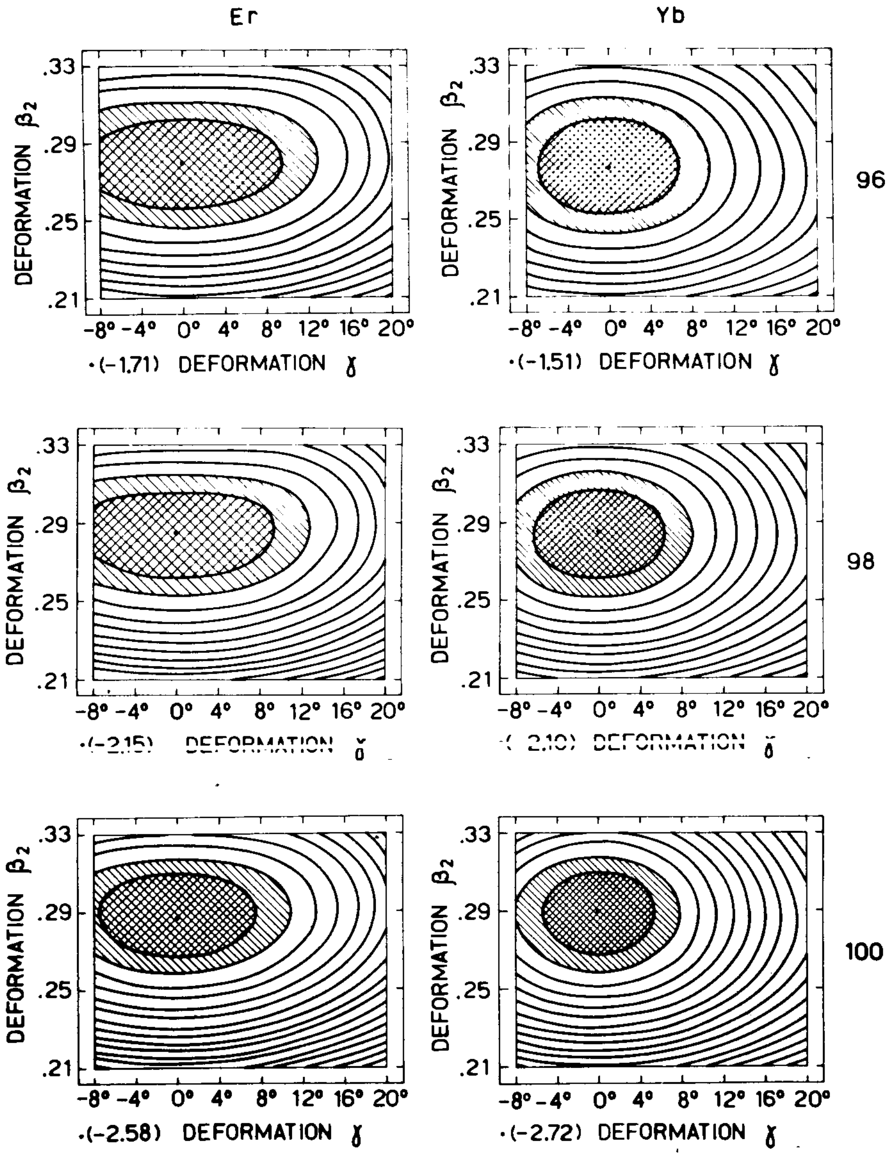

Fig. 3. The same as in fig. 2 for **Er** and **Yb** isotopes.

In general, one can expect the nuclei to be susceptible to nonaxial deformations in two quite opposite situations;

a) An energy gap at $\gamma = 0$ which persists in some region of γ. This corresponds to a large shell-correction component (N = 92, 96, 98, 104).

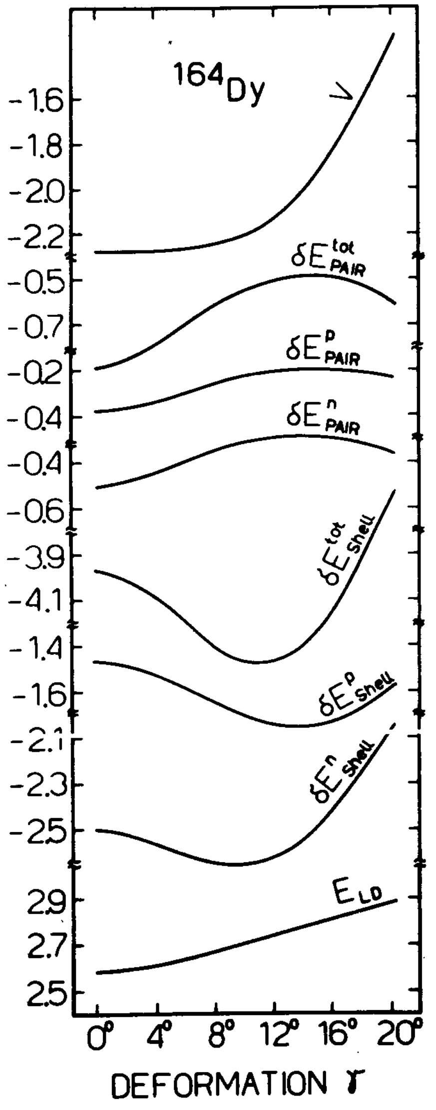

Fig. 4. The total nucleus energy E and its components E_{LD}, δE^{tot}_{shell}, δE^{tot}_{PAIR}, δE^{n}_{shell}, δE^{p}_{shell}, δE^{n}_{PAIR}, δE^{p}_{PAIR} ($V = E = E_{LD} + \delta E^{tot}_{shell} + \delta E^{tot}_{PAIR}$) for ^{164}Dy as function of the nonaxial deformation γ.

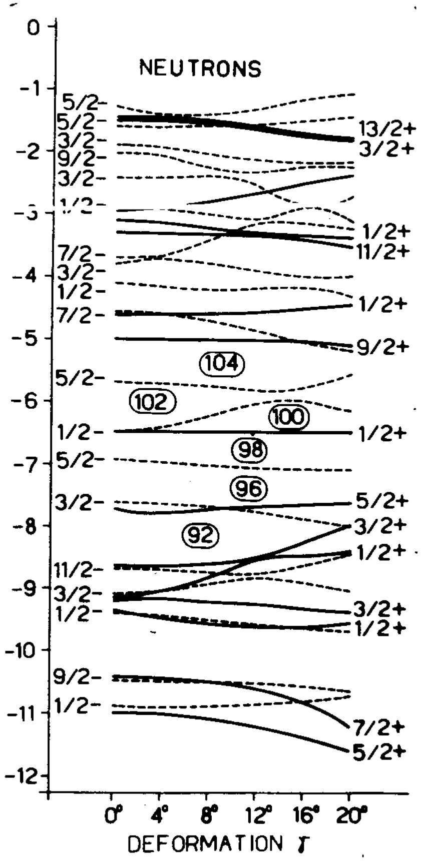

Fig. 5. The dependence of the neutron single-particle levels on the γ-deformation. The parameters of the Saxon-Woods single-particle potential are given in table 1.

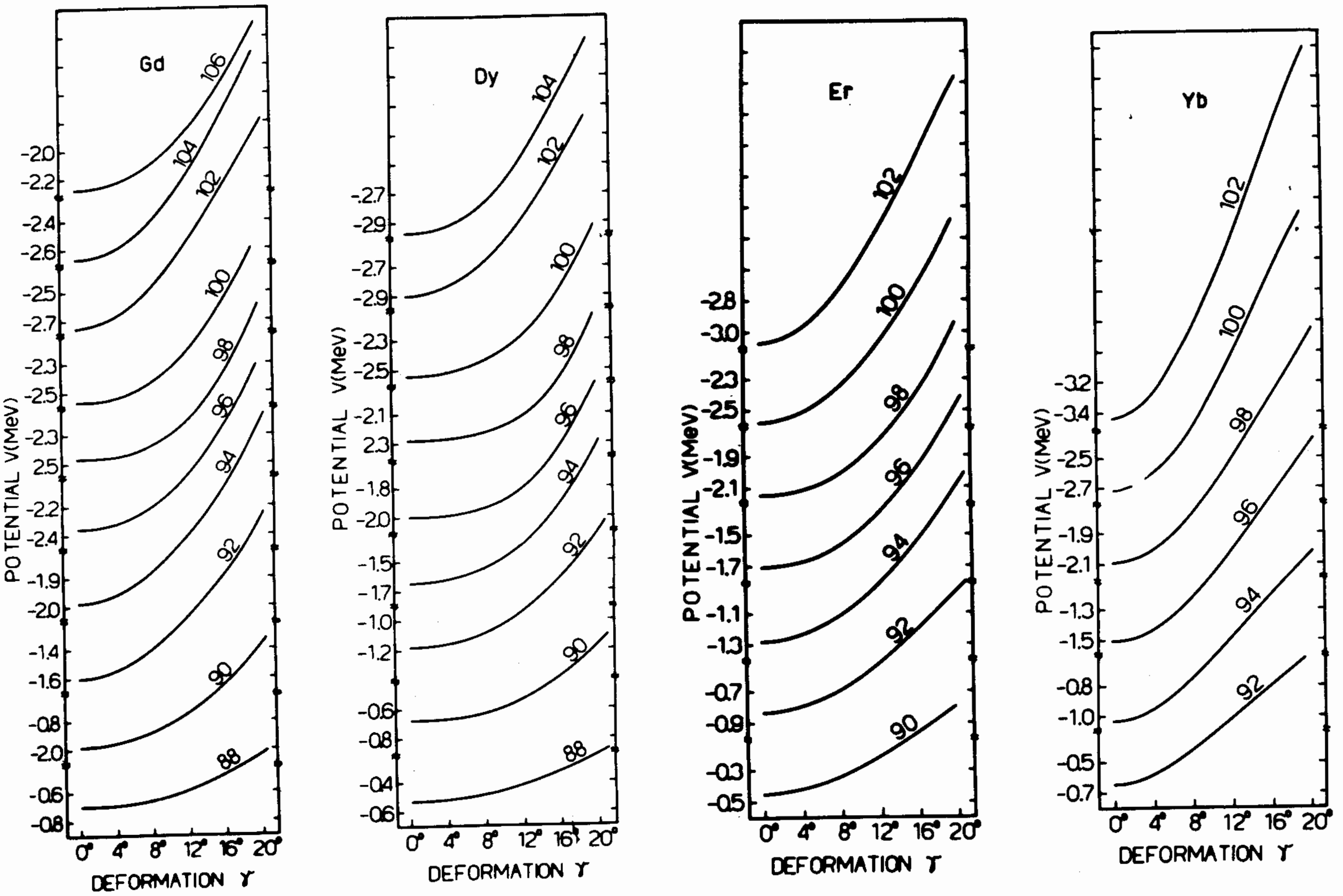

Fig. 6. The dependence of the total nucleus energy on the non-axial deformation γ along the trajectory (minimum of total energy in (β_2, β_4) space for each γ) in γ-direction a) for isotopes of Gd, b) for Dy isotopes, c) for Er isotopes, d) for Yb isotopes.

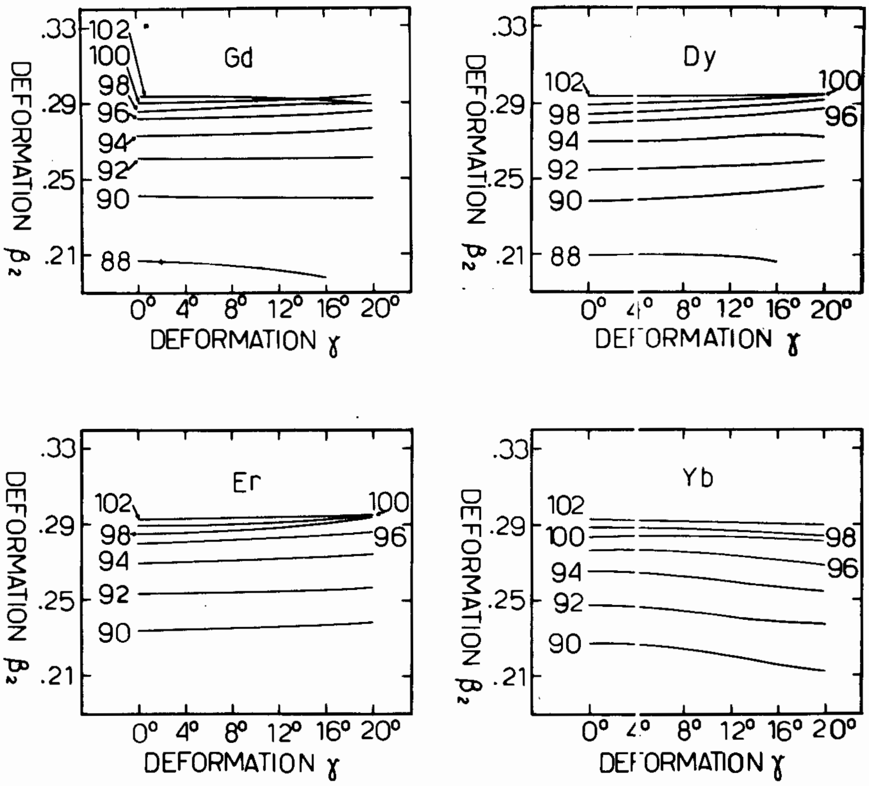

Fig. 7. The dependence of β_2 on the γ deformation along the trajectory in γ-direction for Gd, Dy, Er and Yb isotopes.

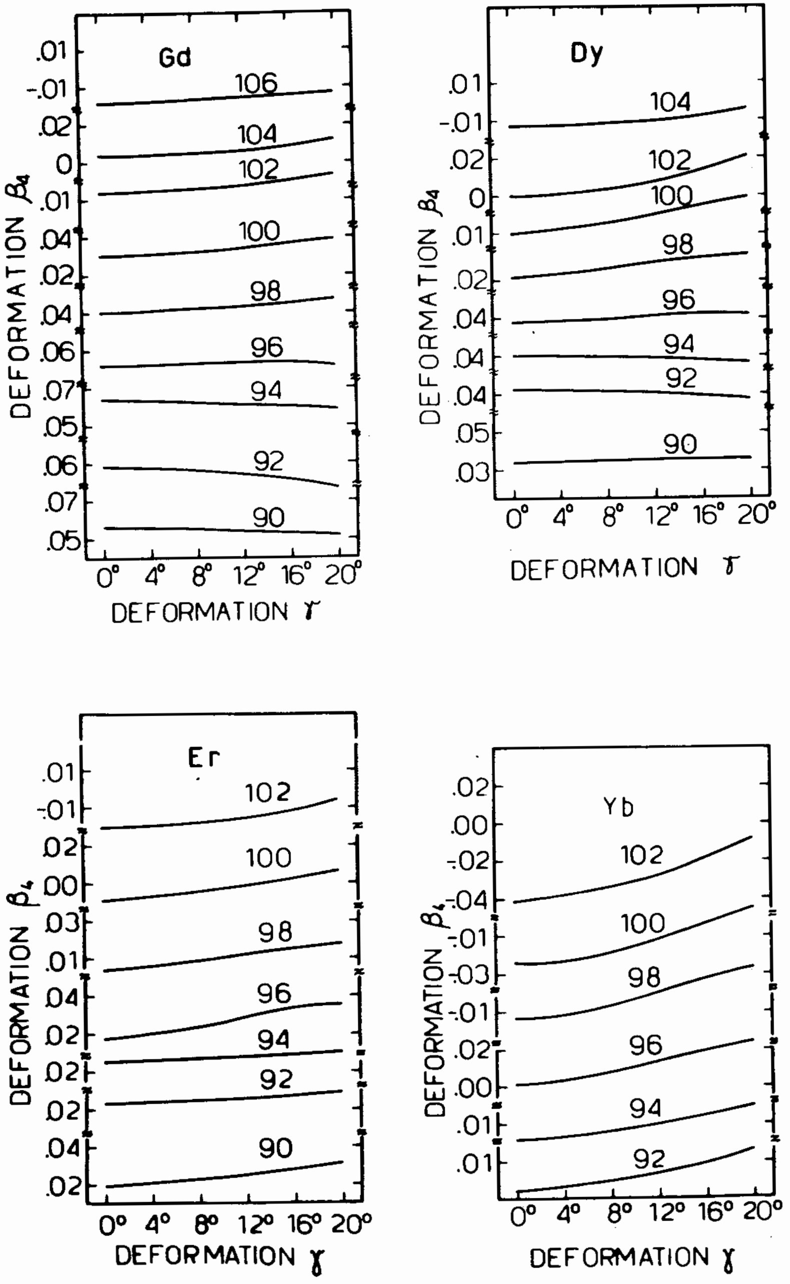

Fig. 8. The dependence of β_4 on the γ deformation along the trajectory in γ-direction for **Gd**, **Dy**, **Er**, and **Yb** isotopes.

Table 2

The first two components of the expansion of the total potential energy in powers of γ:

$$V(\gamma) = V(0) = \frac{1}{2} C_2\gamma^2 + C_4\gamma^4 + \dots$$

N	Gd		Dy		Er		Yb	
	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4
96	15.0	42.1	3.45	113.7	13.7	1.54	28.3	48.4
98	5.02	189.6	-0.60	104.9	-10.0	166.4	33.1	16.2
100	14.1	14.0	7.98	85.2	23.0	-11.17	47.9	-174.8

b) A condensation of levels in a narrow interval between two energy gaps at $\gamma = 0$ which also persists in some region of γ. This corresponds to a small shell-correction (N = 88).

Figures 6 a-d) show the dependence of the total energy on the nonaxial deformation γ along the trajectory in the γ-direction, for different nuclei. A striking feature is the flatness of the minimum for ^{164}Dy (N = 98) and neighbouring nuclei. A similar, though less pronounced affect is also seen for Gd and Er at the same neutron number.

Let us also turn one's attention to the flatness of the total enenrgy of Gd and Dy for N = 88 which corresponds to the situation (b). It is obvious that for such nuclei one can expect significant unharmonic effects in γ-vibrational modes. To illustrate the magnitude of these effects, in table 2 we present the first two coefficients of the expansion of the total energy in powers of γ^2. Softness of the nucleus (that means a fast or slow increase of the total energy with the γ-deformation along γ-trajectory) is correlated with the amplitude γ_0 of γ-zero vibration (γ_0 is the square root of γ dispersion in the ground state) as is shown in fig. 1, where the dependence of the total-energy difference (E_{tot} ($\gamma = 20°$) - E_{tot} ($\gamma = 0°$)) on the neutron number is given together with the γ_0 dependence on the neutron number. The dependence of β_2 and β_4 on γ along the above-mentioned trajectory is presented in figs.7 and 8, respectively. It is seen that β_2 and β_4 are approximately constant. The values of β_2 and β_4 at the minimum of the total energy of the considered nuclei are plotted in fig.9, as functions of the neutron number.

Table 3

The total energies at $\gamma = 0°(E_{pr})$ and $\gamma = 60°(E_{0b})$ and the difference $E_{0b}-E_{pr}$ for some isotopes of Gd, Dy, Er, and Yb

N	Gd		Dy		Er		Yb	
N	E_{ob} E_{pr}	$E_{ob}-E_{pr}$	E_{ob} E_{pr}	$E_{ob}-E_{pr}$	E_{ob} E_{pr}	$E_{ob}-E_{pr}$	E_{ob} E_{pr}	$E_{ob}-E_{pr}$
	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
88	.92 -.70	1.42 2.0*	.89 -.52	1.41 1.5*	.64			
90	1.76 -.98	2.74 3.0*	1.75 -.68	1.43 2.5*	1.51 -.46	1.97 2.0*	1.04 -.43	1.47
92	2.12 -1.61	3.73 3.5*	2.17 -1.18	3.35 3.5*	1.98 -.83	2,81 2.5*	1.54 -.66	2.20
94	2.44 -2.07	4.51 3.5*	2.52 -1.65	4.17 3.5*	2.37 -1.29	3.66 3.5*	1.96 -1.04	3.00
96	2.69 -2.36	5.05 4.5*	2.79 -2.00	4.79 4.0*	2.64 -1.71	4.35 4.0*	2.23 -1.52	3.75 2.5*
98	2.82 -2.47	5.29	2.89 -2.28	5.17 5.0*	2.71 -2.15	4.86 4.5*	2.29 -2.10	4.39 3.5*
100	2.93 -2.57	5.50	2.94 -2.55	5.49 5.0*	2.72 -2.58	5.30 5.0*	2.27 -2.70	4.97 4.0*
102	2.82 -2.76	5.58	2.77 -2.91	5.68	2.51 -3.08	5.59 4.5*	2.03 -3.30	5.33 4.5*
104	2.61 -2.67	5.28	2.51 -2.99	5.50	2.21			

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*Results from Ref./8/.

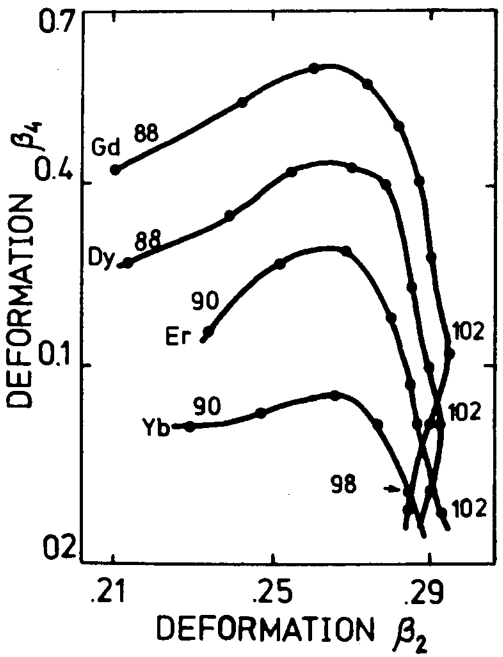

Fig. 9. The values of β_2 and β_4 deformations at the minimum of the total energy (i.e. $\gamma = 0°$) as a function of the neutron number for Gd, Dy, Er, and Yb isotopes.

Table 3 presents the values of the total energy of the considered nuclei.

i) at the minimum - for prolate (ground state) nuclei and

ii) at the saddle-point for the oblate ones.

iii) the difference between the two energies (prolate and oblate).

The results obtained in ref./8/ are also presented. A good agreement with our values is observed. The calculations of the prolate and oblate minimum energy made by Libert et al./20/ for ^{168}Er with Skyrme forces are very analogous to our values.

4. SUMMARY

We briefly summarize the main conclusions of our analysis:

i) There is no stable non-axial deformation in ground states of deformed nuclei. Nevertheless some deformed nuclei possess the softness with respect to γ-deformation (especially Er and Dy with N = 96, 98 and 100).

ii) From the point of view of the potential energy the vibrations conserving axial symmetry (β-vibrations) separate from the vibrations violating axial symmetry (γ-vibrations).

iii) The correlation between the γ-dispersion in ground states (obtained from experimental B(E2) values) and the softness of nucleus with respect to γ-deformation is observed.

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REFERENCES

1. Davidson W.F. et al. J. of Phys. G, 1981, 47, p.455.
2. Bohr A., Mottelson B.R. Phys.Scripta, 1982, 25, p.28.

3. Dumitrescu T.S., Hamamoto I. Nucl.Phys., 1982, A383, p.205.
4. Davidson W.F. et al. Proc. Intern. Conf. on Nucl.Phys., Florence, 1983.
5. Arseniev D.A., et al. Izv. AN USSR, ser. fiz., 1968, 32, p.866.
6. Kumar K., Baranger M. Nucl.Phys., 1967, A92, p.608.
7. Pomorski K., et al. Nucl.Phys., 1967, A283, p.394.
8. Gotz V. et al. Nucl. Phys., 1972, A192, p.1.
9. Soloviev V.G., Shirikova N.Yu., Yad.Fiz., 1982, 36, p.1376; Zeit. f. Phys., 1981, A301, p.263.
10. Strutinsky V.M. Nucl.Phys., 1967, A95, p.470.
11. Strutinsky V.M. Nucl.Phys., 1968, A122, p.1.
12. Brack M., Rev.Mod.Phys. 1972, 44, p.320.
13. Myers W.D., Swiatecki W.J. Arkiv Fysik, 1967, 36, p.343.
14. Myers W.D., Swiatecki W.J. Ann. Phys., 1969, 55, p.395.
15. Myers W.D., Droplet Model of Atomic Nuclei, IFI-Plenum, New York, 1977.
16. Rohozinski S.G., Sobszewski A. Acta Phys. Polonica, 1981, B12, No.10.
17. Dubek J., Werner T. J.Phys. 1978, 64, p.1543.
18. Dudek J. et al. J.Phys., 1979, G5, p.1359.
19. Dudek J., Majhofer A., Skalski J. J.Phys., 1980, 66, p.447.
20. Libert J., Quentin P. ORSAY-comm., 91406, 1983.
21. Rohozinski S.G. Report No. 1520/VII, PH/B, Inst. of Nuclear Research, Warszaw.
22. Grotdal T. et al. Nucl.Phys., 1968, A110, p.385.
23. Tjøm P. et al. Nucl.Phys., 1968, A107, p.385.
24. Burke D. et al. K.Dan.Vid.Selsk. Mat.Fys. Med., 1967, 36, No.6.
25. McGowan F.K. et al. Phys.Rev., C, 1981, vol. 23, p.1926.
26. Ronningen R.M. et al. Phys.Rev.C, 1982, vol.26, p.97.
27. Andrejtscheff W., Schilling K.D. Atomic Data and Nucl.Data Tables 1975, 16, p.515.

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Цвиок С., Лоевский З., Квасил Я. E4-83-647
Мягкость ядер Gd, Dy, Er и Yb относительно γ-деформации

Исследуется мягкость полной энергии ядер Gd, Dy, Er и Yb относительно неаксиальной деформации γ. Методом Струтинского, основанном на деформированном потенциале Саксона-Вудса, вычислены энергетические поверхности вышеприведенных ядер в зависимости от β_2, β_4 и γ-деформаций. Зависимость полной энергии ядра от деформаций приводится в виде карт потенциальной поверхности. Результаты подтверждают аксиальную форму ядер в основном состоянии, тем не менее некоторые ядра редкоземельной области обладают большой мягкостью относительно γ-деформации. Обнаружена корреляция между γ-дисперсией в основном состоянии /полученной из экспериментальных значений B/E2/ вероятностей/ и мягкостью на γ-деформацию. С точки зрения потенциальной энергии колебания формы ядра, сохраняющие аксиальную форму /β - колебания/, сепарируются от колебаний нарушающих аксиальную симметрию /γ - колебания/. Результаты сравниваются с другими работами с такой же проблематикой.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Cwiok S., Lojewski Z., Kvasil J. E4-83-647
Softness of Gd, Dy, Er, and Yb Nuclei to Nonaxial Deformation

The paper is devoted to the investigation of the softness of the nuclear average field with respect to γ-deformation. The method of Strutinsky based on the deformed Wood-Saxon potential is used for calculation of energy surface of some Gd, Dy, Er, and Yb nuclei. The dependence of total nuclear energy on deformations β_2, β_4, and γ is presented in the form of contour maps. Results confirm the axial symmetry of nucleus in the ground state. Nevertheless some deformed nuclei possess the softness with respect to γ-deformation. From the point of view of the potential energy the vibrations conserving axial symmetry (β-vibrations) separate from the vibrations violating axial symmetry (γ-vibrations). The correlation between the γ-dispersion in ground state (obtained from experimental B(E2) values) and the softness of nucleus with respect to γ-deformation is observed. Results are compared with the another papers concerning the same problem.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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