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BASIC EQUATIONS OF THE QUASIPARTICLE-PHONON NUCLEAR MODEL

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1. INTRODUCTION

Fragmentation (distribution of strength) of quasiparticle and vibrational (phonon) states leads to the complication of the nuclear state structure with increasing excitation energy. It has been mentioned more than ten years ago [1-3] that the fragmentation is caused to a great extent by the quasiparticle-phonon interaction. The fragmentation of one-quasiparticle states has been first calculated in refs. [4-5] and of one-phonon states in ref. [6]. The formulation of the quasiparticle-phonon nuclear model [7-9] enabled a consistent calculation of the fragmentation of one-quasiparticle, one-phonon, and quasiparticle plus phonon states in a wide excitation energy interval in spherical and deformed nuclei [10-13]. Although the decisive role of the quasiparticle-phonon interaction was not recognized in the first years after the formulation of the quasiparticle-phonon nuclear model, the investigations of the fragmentation taking into account the quasiparticle-phonon interaction are widely used in recent years [14-17].

As a result of calculations within the quasiparticle-phonon nuclear model many properties of medium and heavy nuclei are explained and some predictions are made. Usually, approximate equations have been used in the calculations, and the total system of equations is not yet published. In the present paper a system of basic model equations is derived in the general form for even-even spherical nuclei, and the transition to an approximate system of equations used in the numerical calculations is realized. It is shown that there is a possibility to sum a wide class of diagrams and how this class is reduced in approximate calculations.

2. THE MODEL HAMILTONIAN

The Hamiltonian of the quasiparticle-phonon nuclear model includes the average field as the Saxon-Woods potential, the pairing interactions, the multipole-multipole and spin-multipole-spin-multipole isoscalar and isovector including charge-exchange interactions. The one-phonon states calculated in the RPA are used as basis states. The multipole forces are used to generate phonons with $J^{\pi} = 1^-, 2^+, 3^-, \ldots, 7^-$, and the spin-multipole to generate phonons with $J^{\pi} = 1^+, 2^+, 3^+, \ldots, 7^+$.

The phonon creation operator is

$$Q^{+}_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}A^{+}(jj';\lambda\mu)-(-)^{\lambda-\mu}\phi^{\lambda i}_{jj'}A(jj';\lambda-\mu)\},\qquad(1)$$

where

$$A^{+}(jj';\lambda\mu)=\sum_{mm'}<jmj'm'|\lambda\mu>\alpha^{+}_{jm}\alpha^{+}_{j'm'}$$

and α^{+}_{jm} is the quasiparticle creation operator. The phonon operators satisfy the commutation relations

$$[Q_{\lambda\mu i},Q^{+}_{\lambda'\mu' i'}]=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'}-\phi^{\lambda i}_{jj'}\phi^{\lambda i'}_{jj'}\}-$$

$$-\sum_{\substack{jj'j_2\\ mm'm_2}}\alpha^{+}_{jm}\alpha_{j'm'}\{<j'm'j_2m_2|\lambda\mu><jmj_2m_2|\lambda'\mu'>\psi^{\lambda i}_{j'j_2}\psi^{\lambda' i'}_{jj_2}-\qquad(2)$$

$$-(-)^{\lambda+\lambda'-\mu-\mu'}<jmj_2m_2|\lambda-\mu><j'm'j_2m_2|\lambda'-\mu'>\phi^{\lambda i}_{jj_2}\phi^{\lambda' i'}_{j'j_2}\}.$$

The RPA equations for finding the energies $\omega_{\lambda i}$ of multipole phonons have the following form

$$(\kappa^{(\lambda)}_0+\kappa^{(\lambda)}_1)\{X^{\lambda i}_M(n)+X^{\lambda i}_M(p)\}-4\kappa^{(\lambda)}_0\kappa^{(\lambda)}_1X^{\lambda i}_M(n)X^{\lambda i}_M(p)=1,\qquad(3)$$

$$X^{\lambda i}_M(\tau)=\frac{1}{2\lambda+1}\sum^{\tau}_{jj'}\frac{f^{\lambda}(jj')u^{(+)}_{jj'}\epsilon_{jj'}}{\epsilon^2_{jj'}-\omega^2_{\lambda i}},\quad f^{\lambda}(jj')=<j||R_\lambda(r)i^{\lambda}Y_{\lambda\mu}||j'>$$

$$\mathcal{Y}^{\lambda i}_{\tau}=Y^{\lambda i}_{\tau}+Y^{\lambda i}_{-\tau}\left\{\frac{1-(\kappa^{(\lambda)}_0+\kappa^{(\lambda)}_1)X^{\lambda i}_M(\tau)}{(\kappa^{(\lambda)}_0-\kappa^{(\lambda)}_1)X^{\lambda i}_M(-\tau)}\right\}^2,\quad Y^{\lambda i}_{\tau}=\frac{1}{2}\frac{\partial}{\partial\omega}X^{\lambda i}_M(\tau)|_{\omega=\omega_{\lambda i}}$$

$$\epsilon_{jj'}=\epsilon_j+\epsilon_{j'},\ u^{(\pm)}_{jj'}=u_jv_{j'}\pm u_{j'}v_j,\ v^{(\pm)}_{jj'}=u_ju_{j'}\pm v_jv_{j'}.\qquad(4)$$

Here $\kappa^{(\lambda)}_0$ and $\kappa^{(\lambda)}_1$ are the isoscalar and isovector constants of multipolarity λ, $\tau=n,p$, ϵ_j is the quasiparticle energy, and u_j and v_j are the Bogolubov transformation coefficients. Equations for the spin-multipole forces are given in ref. /18/. The change of τ by $-\tau$ means $n\to p$.

With the use of the multipole-multipole interaction only, the model Hamiltonian, taking into account eq. (3), can be written as

$$H_M=\sum_{jm}\epsilon_j\alpha^{+}_{jm}\alpha_{jm}+H_{Mv}+H_{Mvq},\qquad(5)$$

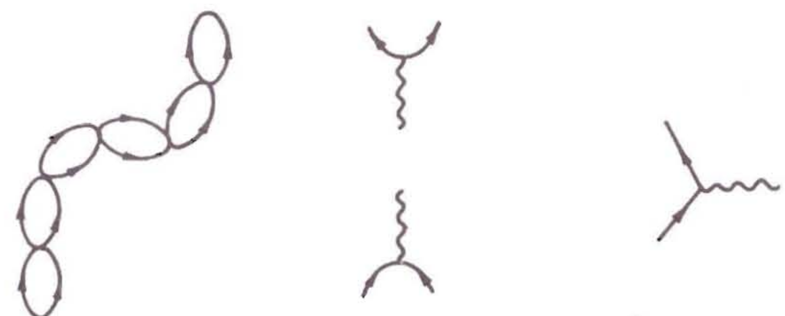
$$H_{Mv}=-\frac{1}{4}\sum_{\lambda\mu ii'\tau}\frac{X^{\lambda i}_M(\tau)+X^{\lambda i'}_M(\tau)}{\sqrt{\mathcal{Y}^{\lambda i}_{\tau}\mathcal{Y}^{\lambda i'}_{\tau}}}Q^{+}_{\lambda\mu i}Q_{\lambda\mu i'},\qquad(6)$$

$$H_{Mvq}=-\frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\{((-)^{\lambda-\mu}Q^{+}_{\lambda\mu i}+Q_{\lambda-\mu i})\sum_{jj'\tau}\frac{f^{\lambda}(jj')v^{(-)}_{jj'}}{\sqrt{\mathcal{Y}^{\lambda i}_{\tau}}}B(j,j';\lambda-\mu)+h.c.\},\qquad(7)$$

where

$$B(jj';\lambda\mu)=\sum_{mm'}(-)^{j'+m'}<jmj'm'|\lambda\mu>\alpha^{+}_{jm}\alpha_{j'-m'}.$$

Our Hamiltonian is constructed so that all the operators $A(jj';\lambda\mu)$ and $A^{+}(jj';\lambda\mu)$ are expressed through the phonon operators, and the quasiparticle operators enter only in the form $\alpha^{+}_{jm}\alpha_{jm}$. When solving eq. (3) the diagrams given in fig.1a have been taken into account. The diagrams of type in fig.1b are taken into account in the term H_{Mv}. The quasiparticle-phonon interaction H_{Mvq} takes into account only the diagrams in fig.1c. Such a construction of the Hamiltonian overcomes the difficulties with the double counting, which arise in the nuclear field theory /19/. It should be mentioned that while constructing the Hamiltonian (5) we neglected the terms $\sim B(jj';\lambda\mu)B(j_2j_2';\lambda\mu)$, which do not lead to coherent effects. Nevertheless, their role is to be studied.



a) b) c)

Fig.1. Diagrams taken into account in the RPA and for the quasiparticle-phonon interaction.

We now give the formulae which will be used in the following section. Taking into account (2), we calculate

$$\sum_{\substack{\mu\mu_2\\ \mu'\mu'_2}} \langle\lambda'\mu'\lambda_2\mu_2|JM\rangle\langle\lambda\mu\lambda_2\mu_2|JM\rangle \times$$
$$\times \langle Q_{\lambda'_2\mu'_2 i'_2} Q_{\lambda'\mu' i'} Q^+_{\lambda\mu i} Q^+_{\lambda_2\mu_2 i_2}\rangle = \tag{8}$$
$$= \delta_{\lambda\lambda'}\delta_{ii'}\delta_{\lambda_2\lambda'_2}\delta_{i_2 i'_2} + \delta_{\lambda\lambda'_2}\delta_{ii'_2}\delta_{\lambda_2\lambda'}\delta_{i_2 i'} +$$
$$+ K^J(\lambda'_2 i'_2, \lambda' i'|\lambda i, \lambda_2 i_2),$$

where

$$K^J(\lambda'_2 i'_2, \lambda' i'|\lambda i, \lambda_2 i_2) = \sum_{\substack{\mu\mu_2\\ \mu'\mu'_2}} \langle\lambda'\mu'\lambda_2\mu_2|JM\rangle\langle\lambda\mu\lambda_2\mu_2|JM\rangle \times \tag{9}$$
$$\times\ K(\lambda'_2\mu'_2 i_2, \lambda'\mu' i'|\lambda\mu i, \lambda_2\mu_2 i_2),$$

$$K(\lambda'_2\mu'_2 i'_2, \lambda'\mu' i'|\lambda\mu i, \lambda_2\mu_2 i_2) =$$
$$= -\sum_{\substack{j_1 j_2 j_3 j_4\\ m_1 m_2 m_3 m_4}} \{\psi^{\lambda' i'}_{j_3 j_4}\psi^{\lambda u}_{j_1 j_4}\langle j_3 m_3 j_4 m_4|\lambda'\mu'\rangle\langle j_1 m_1 j_4 m_4|\lambda\mu\rangle -$$
$$- (-)^{\lambda+\lambda'-\mu-\mu'}\phi^{\lambda' i'}_{j_1 j_4}\phi^{\lambda i}_{j_3 j_4}\langle j_1 m_1 j_4 m_4|\lambda' -\mu'\rangle\langle j_3 m_3 j_4 m_4|\lambda -\mu\rangle\} \times \tag{10}$$
$$\times\{\psi^{\lambda_2 i_2}_{j_3 j_2}\psi^{\lambda'_2 i'_2}_{j_1 j_2}\langle j_3 m_3 j_2 m_2|\lambda_2\mu_2\rangle\langle j_1 m_1 j_2 m_2|\lambda'_2\mu'_2\rangle +$$
$$+ (-)^{\lambda_2+\lambda'_2-\mu_2-\mu'_2}\phi^{\lambda_2 i_2}_{j_1 j_2}\phi^{\lambda'_2 i'_2}_{j_3 j_2}\langle j_1 m_1 j_2 m_2|\lambda_2 -\mu_2\rangle \times$$
$$\times\langle j_3 m_3 j_2 m_2|\lambda'_2 -\mu'_2\rangle\}.$$

In the diagonal approximation, which will be used in deriving approximate equations, K^J has the form

$$K^J(\lambda_2 i_2, \lambda i|\lambda i, \lambda_2 i_2) = \sum_{j_1 j_2 j_3 j_4} (-)^{j_2+j_4-J}(2\lambda+1)(2\lambda_2+1)\begin{Bmatrix} j_1 & j_2 & \lambda_2\\ j_4 & j_3 & \lambda\\ \lambda & \lambda_2 & J\end{Bmatrix}\times$$
$$\times[\psi^{\lambda i}_{j_3 j_4}\psi^{\lambda i}_{j_1 j_4}\psi^{\lambda_2 i_2}_{j_3 j_2}\psi^{\lambda_2 i_2}_{j_1 j_2} - \phi^{\lambda i}_{j_3 j_4}\phi^{\lambda i}_{j_1 j_4}\phi^{\lambda_2 i_2}_{j_3 j_2}\phi^{\lambda_2 i_2}_{j_1 j_2}]. \tag{10'}$$

3. GENERAL FORM OF THE MODEL EQUATIONS

The excited state wave function of a doubly even spherical nucleus is

$$\Psi_\nu(JM) = \{\sum_i R_i(J\nu)Q^+_{JMi} + \sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\sum_{\mu_1\mu_2}\langle\lambda_1\mu_1\lambda_2\mu_2|JM\rangle\times \tag{11}$$
$$\times\ Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}\}\Psi_0 .$$

where Ψ_0 is the ground state wave function of a doubly even nucleus, $Q_{\lambda\mu i}\Psi_0 = 0$. We have used in (11) only the two-phonon terms. In this case one can calculate the fragmentation of one-phonon and two-quasiparticle states and the strength functions determined by this fragmentation. Further the three-phonon terms can be included into the wave function (11). The normalization condition (11) has the form

$$\sum_i (R_i(J\nu))^2 + 2\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 + \tag{12}$$
$$+ \sum_{\substack{\lambda_1 i_1\lambda_2 i_2\\ \lambda'_1 i'_1\lambda'_2 i'_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)P^{\lambda'_1 i'_1}_{\lambda'_2 i'_2}(J\nu)K^J(\lambda'_2 i'_2, \lambda'_1 i'_1|\lambda_1 i_1, \lambda_2 i_2) = 1.$$

Now we calculate the average value of H_M according to (11), and as a result we get

$$
\begin{aligned}
(\Psi_\nu^*(JM)H_M\Psi_\nu(JM)) &= \sum_i \omega_{Ji} R_i^2(J\nu) + \\
&+ \sum_{\substack{\lambda_1 i_1\lambda_2 i_2 \\ \lambda_1' i_1'\lambda_2' i_2'}} P^{\lambda_1' i_1'}_{\lambda_2' i_2'}(J\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \times \\
&\times \{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2})[\delta_{\lambda_1\lambda_1'}\delta_{i_1 i_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'} + \delta_{\lambda_1\lambda_2'}\delta_{i_1 i_2'}\delta_{\lambda_2\lambda_1'}\delta_{i_2 i_1'} + \\
&+ K^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2)] - \\
&- \frac{1}{4}\sum_{i_3\tau}\Big[\frac{X_M^{\lambda_1' i_1'}(\tau) + X_M^{\lambda_1' i_3}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_1' i_1'}\mathcal{Y}_\tau^{\lambda_1' i_3}}} K(\lambda_2' i_2', \lambda_1' i_3 | \lambda_1 i_1, \lambda_2 i_2) + \\
&+ \frac{X_M^{\lambda_2' i_2'}(\tau) + X_M^{\lambda_2' i_3}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_2' i_2'}\mathcal{Y}_\tau^{\lambda_2' i_3}}} K^J(\lambda_2' i_3, \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2)\Big] - \\
&- \frac{1}{4}\sum_{\substack{\lambda_3 i_3 i_3' \\ \lambda_4 i_4 \tau}} \frac{X_M^{\lambda_3 i_3}(\tau) + X_M^{\lambda_3 i_3'}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_3 i_3}\mathcal{Y}_\tau^{\lambda_3 i_3'}}} K^J(\lambda_4 i_4, \lambda_3 i_3' | \lambda_1 i_1, \lambda_2 i_2) \times \\
&\times K^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda_3 i_3, \lambda_4 i_4)\} + \\
&+ \sum_{\lambda_1 i_1 \lambda_2 i_2 i} R_i(J\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)[U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) + V^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)].
\end{aligned}
\qquad (13)
$$

The explicit form of U is given in ref. [11]. It is represented by the diagram of fig.2a. The function V becomes zero at $K^J = 0$; it is represented by the diagrams 2b and 2c.

Using the variational principle in the form

$$
\delta\{(\Psi_\nu^*(JM)H_M\Psi_\nu(JM)) - \eta_\nu[(\Psi^*(JM)\Psi_\nu(JM)) - 1]\} = 0 \qquad (14)
$$

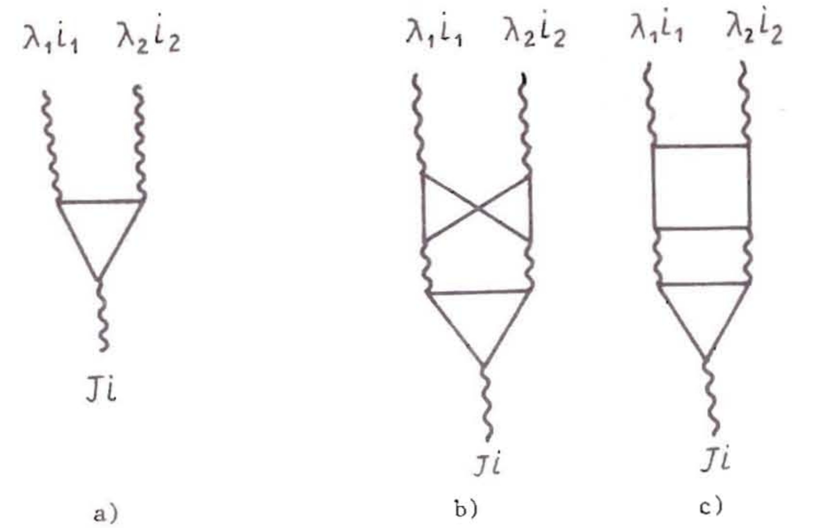


Fig.2. Diagrams taking into account the coupling of one- and two-phonon states.

we get the system of equations. From the first equation

$$
(\omega_{Ji} - \eta_\nu) R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)[U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) + V^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)] = 0 \qquad (15)
$$

we find $R_i(J\nu)$, substitute it into the second one and get the secular equation for defining the energies η_ν as a determinant in the space of two-phonon states

$$
\begin{aligned}
\det\|(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_\nu)[\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_1 i_1'}\delta_{i_2 i_2'} &+ \\
+ \delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'}\delta_{i_1 i_2'}\delta_{i_2 i_1'} + K^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2)] &- \\
- \frac{1}{4}\sum_{i_3\tau}\Big[\frac{X_M^{\lambda_1' i_1'}(\tau) + X_M^{\lambda_1' i_3}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_1' i_1'}\mathcal{Y}_\tau^{\lambda_1' i_3}}} K^J(\lambda_2' i_2', \lambda_1' i_3 | \lambda_1 i_1, \lambda_2 i_2) &+
\end{aligned}
$$

$$+\frac{X_M^{\lambda_2' i_2'}(r) + X_M^{\lambda_2' i_3}(r)}{\sqrt{\mathcal{Y}_r^{\lambda_2' i_2'}\mathcal{Y}_r^{\lambda_2' i_3}}} K^J(\lambda_2' i_3, \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2)] -$$

$$-\frac{1}{4}\sum_{\substack{\lambda_3 i_3 i_3' \\ \lambda_4 i_4 r}} \frac{X_M^{\lambda_3 i_3}(r) + X_M^{\lambda_3 i_3'}(r)}{\sqrt{\mathcal{Y}_r^{\lambda_3 i_3}\mathcal{Y}_r^{\lambda_3 i_3'}}} K^J(\lambda_4 i_4, \lambda_3 i_3' | \lambda_1 i_1, \lambda_2 i_2) \times \qquad (16)$$

$$\times K^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda_3 i_3, \lambda_4 i_4) -$$

$$-\sum_i \frac{(U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) + V_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji))(U_{\lambda_2' i_2'}^{\lambda_1' i_1'}(Ji) + V_{\lambda_2' i_2'}^{\lambda_1' i_1'}(Ji))}{\omega_{Ji} - \eta_\nu} \| = 0 .$$

Let us illustrate eq. (16) by the diagrams. In the first terms we take into account the diagrams a) and b), fig.3, and then the diagrams c) and b) are taken into account. The terms with UU are represented by the diagram e); with UV, by the diagram f); and with VV, by the diagram g). Moreover, there are diagrams which are obtained from the diagrams c), d), f) and g) by changing part a) by b). Such an illustration is conditional, especially as the vertex parts are different in each case.

4. SYSTEM OF APPROXIMATE EQUATIONS

The secular equation (16) is very complicated, for medium and heavy nuclei the rank of the determinant is $10^3 - 10^5$. It can be solved under an essential reduction of the one-phonon basis. Generally, many graphs are taken into account, which give a small contribution to the fragmentation of one-phonon states. To calculate the fragmentation of two-phonon states one should take into account the three-phonon terms of the wave function. Therefore, we pass to an approximate system of equations. For this purpose in expression (13) we use only the terms proportional to $(P_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji))^2$, and in the quadratic in K^J term we take one of K^J in the diagonal form. This limitation in (13) is possible due to the fact that the absolute values of the diagonal terms K^J are considerably larger than those of the nondiagonal terms.

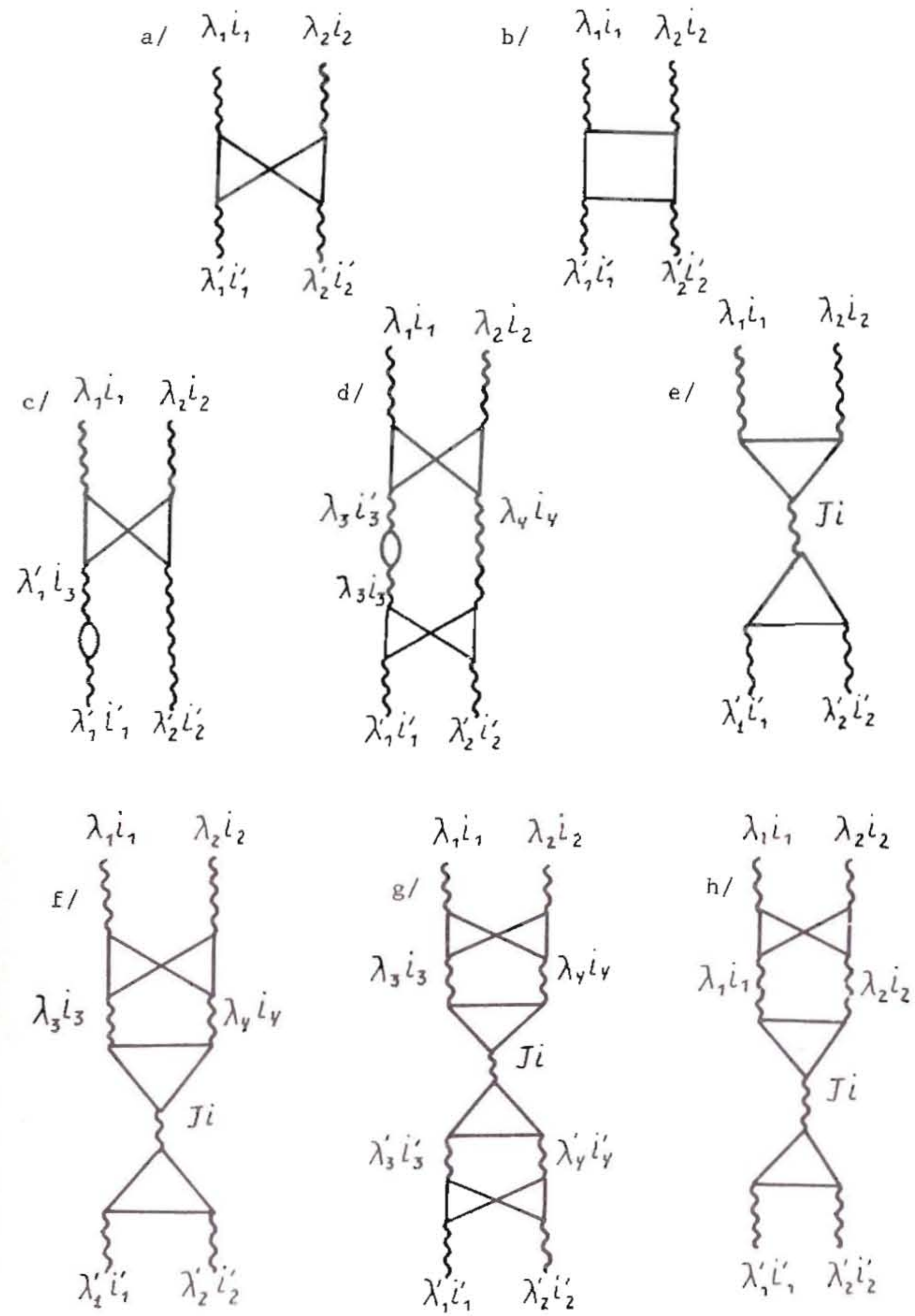


Fig.3. Diagrams in the space of two-phonon states.

In the normalization condition (12) we also use only the diagonal values of K^J. As a result we get

$$\sum_i (R_i(J\nu))^2 + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 [1 + \frac{1}{2} K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)] = 1, \tag{17}$$

$$(\Psi^*_\nu(JM) H_M \Psi_\nu(JM)) = \sum_i \omega_{Ji} R^2_i(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 \times$$

$$\times (1 + \frac{1}{2} K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2)) + \tag{18}$$

$$+ \sum_{\lambda_1 i_1 \lambda_2 i_2 i} R_i(J\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)(1 + \frac{1}{2} K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)),$$

where

$$\Delta\omega(\lambda_1 i_1, \lambda_2 i_2) = -\frac{1}{8} \sum_{i_3 \tau} [\frac{X^{\lambda_1 i_1}_M(\tau) + X^{\lambda_1 i_3}_M(\tau)}{\sqrt{\mathcal{Y}^{\lambda_1 i_1}_\tau \mathcal{Y}^{\lambda_1 i_3}_\tau}} K^J(\lambda_2 i_2, \lambda_1 i_3 | \lambda_1 i_1, \lambda_2 i_2) +$$

$$+ \frac{X^{\lambda_2 i_2}_M(\tau) + X^{\lambda_2 i_3}_M(\tau)}{\sqrt{\mathcal{Y}^{\lambda_2 i_2}_\tau \mathcal{Y}^{\lambda_2 i_3}_\tau}} K^J(\lambda_2 i_3, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_3)]. \tag{19}$$

Using the variational principle we get the following system of equations:

$$(\omega_{Ji} - \eta_\nu) R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) \times$$

$$\times [1 + \frac{1}{2} K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)] = 0, \tag{20}$$

$$2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2) - \eta_\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) +$$

$$+ \sum_i R_i(J\nu) U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = 0. \tag{21}$$

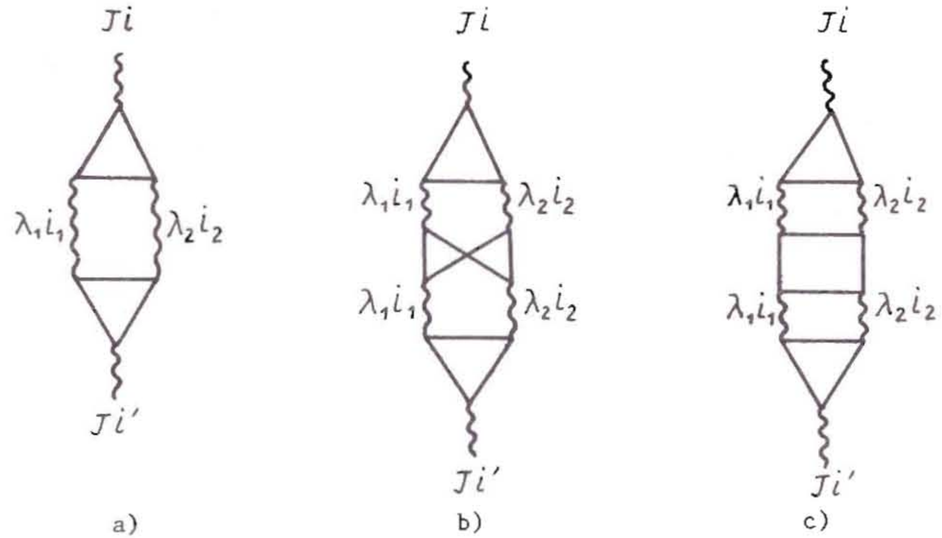


Fig.4. Diagrams in the space of one-phonon states.

Now the secular equation can be derived both in the space of two-phonon states and in the space of one-phonon states. The secular equation in the space of two-phonon states is

$$\det \| (\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2) - \eta_\nu) \times$$

$$\times (\delta_{\lambda_1 \lambda_1'} \delta_{i_1 i_1'} \delta_{\lambda_2 \lambda_2'} \delta_{i_2 i_2'} + \delta_{\lambda_1 \lambda_2'} \delta_{i_1 i_2'} \delta_{\lambda_2 \lambda_1'} \delta_{i_2 i_1'}) - \tag{22}$$

$$- \frac{1}{2} \sum_i \frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) U^{\lambda_1' i_1'}_{\lambda_2' i_2'}(Ji)}{\omega_{Ji} - \eta_\nu} (1 + K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1 \lambda_2 i_2)) \| = 0.$$

In this case the diagrams c), e), and h) in fig.3 are taken into account. The diagram h) differs from f) as it contains no summation over the intermediate two-phonon states, since only the diagonal value of K^J is used. This means that less diagrams are summed in comparison with eq. (16).

The secular equation in the space of one-phonon states has the following form:

$$\det \| (\omega_{Ji} - \eta_\nu)\delta_{ii'} - \frac{1}{2}\sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)\, U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')\left(1 + \frac{1}{2}K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)\right)}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2) - \eta_\nu} \| = 0. \qquad (23)$$

In the space of one-phonon states the diagrams given in fig.4 are taken into account. As it has been shown in ref./15/, for the numerical calculations in the nuclear field theory only particular cases of the diagram a) in fig.4 are taken into account, i.e., when one of the intermediate phonons is changed by the two-quasiparticle states and the other by strongly collectivized phonons.

Let us consider eq. (23). The rank of the determinant is equal to the number of one-phonon states in the first term of the wave function (11). It changes from 20 to 200 and is twice as less as the rank of the determinants (16) and (22). The factor $(1 + \frac{1}{2}K^J(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2))$ is caused by the Pauli principle in the two-phonon terms of the wave function (11). In the case of maximal violation of the Pauli principle $K^J = -2$ and the corresponding term is excluded from the sum over $\lambda_1 i_1$, $\lambda_2 i_2$. The two-phonon pole $\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2}$ is shifted by taking into account diagrams of the type of fig.3c. As has been shown /20/ this shift is large for the first two-phonon collective states of deformed nuclei. The shift $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$ is small for the collective phonons forming giant resonances of different types. The shift $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$ is equal to zero in the case $K^J = 0$.

In the case when the terms proportional to $a^+ a$ are neglected in the commutator (2) $K^J = 0$ and the secular equation (23) is

$$\det \| (\omega_{Ji} - \eta_\nu)\delta_{ii'} - \frac{1}{2}\sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)\, U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_\nu} \| = 0. \qquad (24)$$

In refs./11,13/ the fragmentation of one-phonon and two-quasiparticle states has been studied by solving eq. (24) and calculating the corresponding wave function with excluding the terms in which the Pauli principle is violated. When calculating the widths of giant resonances and neutron strength functions, such an approximation turned out to be rather good.

5. CONCLUSION

We have presented the systems of equations for the case of multipole phonons, spin-multipole and in some cases of charge-exchange phonons. If the spin-multipole - spin-multipole interactions are taken into account, the Hamiltonian (5) is added by the terms H_{Sv} and H_{Svq}, which have the form similar to (6) and (7). New constants $\kappa_0^{(\lambda L)}$ and $\kappa_1^{(\lambda L)}$ are introduced, in (6) $X_M^{\lambda i}(r)$ is changed by

$$X_S^{Li}(r) = \frac{1}{2L+1}\sum_{jj'}^{\tau} \frac{(f^{(L-1,L)}(jj')\, u^{(-)}_{jj'})^2\, \epsilon_{jj'}}{\epsilon^2_{jj'} - \omega^2_{Li}},$$

$$f^{(L-1,L)}(jj') = \langle j \| R_{L-1}(r)\, i^{L-1} \sum_{\mu,\rho=0,\pm 1} \langle 1\rho\lambda\mu | LM\rangle \sigma_\rho Y_{\lambda\mu} \| j'\rangle .$$

and in (7) $v^{(-)}_{jj'}$ is changed by $v^{(+)}_{jj'}$. The charge-exchange phonons are introduced by using the matrix elements

$$f^{(\lambda)}(j_p j_n) = \langle j_p \| R_\lambda(r)\, i^\lambda Y_{\lambda\mu}\, \tau^{(-)} \| j_n\rangle ,$$

$$f^{(\lambda L)}(j_p j_n) = \langle j_p \| R_\lambda(r)\, i^\lambda \sum_{\mu,\rho=0,\pm 1} \langle 1\rho\lambda\mu | LM\rangle \sigma_\rho Y_{\lambda\mu}\, \tau^{(-)} \| j_n\rangle .$$

The fragmentation of one-phonon and two-quasiparticle states and the properties of spherical nuclei determined by this fragmentation are calculated using the basis constructed of multipole and spin-multipole phonons. The success in calculating nuclear properties at intermediate and high excitation energies is attributed to the use of the strength function method /8-11/. It should be mentioned that the quasiparticle-phonon nuclear model has large potentialities for a detailed description of the properties of excited states in spherical and deformed nuclei.

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Соловьев В.Г., Воронов В.В. E4-83-58
Основные уравнения квазичастично-фононной модели ядра

Для четно-четных сферических ядер в общем виде получена система основных уравнений квазичастично-фононной модели ядра для случая, когда волновые функции включают одно- и двухфононные компоненты. Осуществлен переход к приближенной системе уравнений, используемой в численных расчетах. Показано, что в квазичастично-фононной модели имеется возможность просуммировать широкий класс диаграмм, включая диаграммы, обычно учитываемые в теории ядерных полей.

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Soloviev V.G., Voronov V.V. E4-83-58
Basic Equations of the Quasiparticle-Phonon Nuclear Model

A system of basic equations of the quasiparticle-phonon nuclear model for even-even spherical nuclei is derived in the general form. The transition to an approximate system of equations used in the numerical calculations is realized. It is demonstrated that a wide class of diagrams can be summed within the quasiparticle-phonon nuclear model. The model is shown to have large possibilities for a more detailed description of the nuclear characteristics at intermediate and high excitation energies.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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