сообщения
объединенного
института
ядерных
исследований
дубна

E4-83-574

R.A.Eramzhyan, M.Gmitro, T.D.Kaipov,
S.S.Kamalov, R.Mach

CROSS SECTIONS OF THE $^{12}C\ (\gamma,\pi^+),\ ^{12}B\ (1^+_{g.s.},\ 2^+_1)$ REACTION

1983

1. INTRODUCTION

The differential and integral cross sections of the nuclear pion photoproduction are known to be sensitive to the nuclear structure input. Recently /1/ we have shown that the calculation of the form factors of electric-type transitions (E2) in ^{12}C should use the continuity equation of the electromagnetic current ("Siegert method"). In that way one can account for several problems of the earlier investigations. In particular, as an example, we have calculated in paper /1/ the branching ratios $R(1^+)$ and $R(2^+)$ for the radiative capture of the resting pions $^{12}C(\pi^-,\gamma)^{12}B(J^+)$. For the first time the results agree with data both for the M1 and E2 transitions. In the present paper we shall consider the pion photoproduction process on ^{12}C leading to the lowest isovector levels in ^{12}B. The nuclear transition densities are those of ref. /1/. The method has been described recently /2/, it is parallel to the inelastic pion scattering formalism developed in ref. /3/. Here we summarise just a few working formulae.

Basic formalism of the photoproduction reaction and some aspects of pion rescattering in the pion photoproduction process are discussed in sect.2. The calculated photoproduction cross sections are compared with data in sect.3. The results obtained here together with those for the other pionic reactions on ^{12}C reported earlier and briefly summarised in sect.4.

2. CHARGED PION PHOTOPRODUCTION

Starting with the Lippman-Schwinger form of the pion-nucleus wave function one obtains /2/ for the partial photoproduction amplitude $F^{\gamma j}$ the expression

$$F^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma) = U^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma) - \sum_{m L'_\pi} \frac{1}{\pi} \int q'^2 dq' F^j_{nm}(qL_\pi, q'L'_\pi) \frac{G_m}{\mathfrak{M}_m} U^{\gamma j}_{m0}(q'L'_\pi, k\lambda L_\gamma), \tag{1}$$

where index γ is used for the quantities pertaining to the photoproduction process, k and λ are photon momentum and polarization, respectively, the Green function is defined as $G_m = P/(E_0 - E_m) - i\pi\delta(E_0 - E_m)$, and $\mathfrak{M}_m$ is the reduced mass.

The lack of any fundamental description for the off-shell production amplitude is indeed one of the main problems if one wants to calculate the integral in eq. (1). The Ansatz suggested in the pioneering work by Sauders[4] consists in assuming that $f_{\pi\gamma}$ is a smooth function of the momentum, then $f_{\pi\gamma}(q'...) \approx f_{\pi\gamma}(q...)$ and the integral is simplified. To compare with the earlier calculations we shall show the results obtained with this assumption. Frequently[5] the amplitude $f_{\pi\gamma}$ as fitted to the free data is taken also for the off-shell conditions. Such a procedure is maybe better suited[6] for BL than for CGLN amplitude (see discussion in ref.[6]) though the correctness of such off-shell continuation cannot be a priori ascertained in either case. A suggestion has also been made to shift the values of certain kinematical quantities[6] that would fulfil the conservation laws at the production vertex. At a deeper thought[6] it indeed simply shifts the problem of the off-shell dynamics to ad hoc kinematical assumptions. Apparently, specific models of the off-shell extrapolation should be constructed, similar to those discussed in ref.[3] for the scattering amplitude. We wish to come back to this problem in more detail in a further publication. In appendix A we display the formulae for the simple case of the on-shell pion propagation.

In terms of the amplitudes $F^{\gamma j}$ one finds for the photoproduction differential cross section the formula

$$\frac{d\sigma_{n0}}{d\Omega} = \frac{q}{2k}\,\frac{1}{2J_0+1}\sum_{M_0 M_n \lambda}\Big|\sum_{L_\pi L_\gamma j}\begin{bmatrix} L_\pi & J_n & j \\ M_\pi & M_n & j_z\end{bmatrix}\begin{bmatrix} L_\gamma & J_0 & j \\ \lambda & M_0 & j_z\end{bmatrix} \tag{2}$$

$$\times\, Y_{L_\pi M_\pi}(\Omega_{\vec q})\, F^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma)\Big|^2,$$

where $[\because]$ are the Clebsch-Gordan coefficients.

In the numerical work we have used the πN photoproduction amplitude by Chew, Goldberger, Low and Nambu[7] (CGLN) relativistically transformed into the π-nucleus centre-of-mass system. We note in passing that the effects of those transformations, though numerically very important in the (3,3)-resonance region[2], are much less pronounced in the near-threshold region ($T_\pi \sim 15\div40$ MeV) which we consider here. (Unfortunately, data are not yet available for the photoproduction on ^{12}C at higher energies).

2.1. Pion Rescattering Effects

At low pion energies, far from the (3,3) resonance region, one encounters serious difficulties when attempting to reproduce the elastic scattering data. Apparently, the problem is connected both with the omission of the absorption terms and with a poor knowledge of the πN form factors.

Fortunately, in the pion photoproduction calculation the disease is less accute since the pion distortion enters the (γ,π) calculation only once (in the final state) as compared with the double entree ϕ^*_π, ϕ_π in the scattering reaction. Naturally, the, the photoproduction calculation is less influenced by the errors in the description of the pion rescattering. In addition the relative importance of the pion rescattering term decreases very rapidly with falling energy. In ref.[2] we have seen that it is only about 15-20% below $T_\pi \approx 50$ MeV. To be more quantitative we shall compare our calculated quantities with the results of phenomenological analyses, which are available for the ^{12}C target in literature.

The reflection coefficients $\eta_{L_\pi} = |\exp(2i\delta_{L_\pi})|$ and phase-shifts δ_{L_π} in the lowest partial waves are shown in table 1.

Table 1

40 MeV π^+-nucleus scattering partial wave parameters deduces from the optical potential and from the partial wave analysis are compared with our calculation (case b) and the plane-wave (case a). η_ℓ and δ_ℓ represent the magnitude and phase of the strong scattering amplitude.

Case	η_0	η_1	η_2	δ_0	δ_1	δ_2
a	1.	1.	1.	0.	0.	0.
b	0.84	0.90	0.98	-3.6°	15.5°	4.0°
c	0.92	0.86	0.97	-12.9°	10.0°	2.9°
d	0.67	0.99	0.94	-15.9°	7.7°	3.4°

a) PWIA, b) present calculation; c) optical-model fit of ref.[8]; d) partial-wave analysis of ref.[8].

Apart from the trivial PWIA result, we quote our calculation and two results of the phenomenological analysis performed by Blecher et al.[8]. They correspond to the optical model fit and a partial-wave analysis. One observes that uncertainty of the two phenomenological sets is about the same as their difference with the calculated values. The respective pion photoproduction cross sections are shown in table 2. We have mentioned above that the difference between the PWIA and DWIA results is at most 20%. Since the uncertainly in δ_{L_π} seen in table 1 induces an error of this small correction only, we

Table 2

Photoproduction differential cross sections $d\sigma/d\Omega(J_n^+)$ in μb/sr at T_π = 42 MeV calculated in the laboratory frame. The cases a) through d) are the same as in table 1. Integrated cross section σ_T in μb is shown in the last column.

θ_{LAB}		30°	60°	90°	120°	150°	σ_T
$\frac{d\sigma}{d\Omega}(1^+)$	a)	1.417	0.494	0.072	0.	0.008	4.34
	b)	1.202	0.406	0.057	0.012	0.025	3.70
	c)	1.215	0.412	0.061	0.015	0.029	3.77
	d)	1.209	0.401	0.051	0.015	0.041	3.72
$\frac{d\sigma}{d\Omega}(2^+)$	a)	0.053	0.207	0.390	0.443	0.412	3.96
	b)	0.057	0.180	0.327	0.460	0.327	3.29
	c)	0.066	0.188	0.334	0.367	0.332	3.38
	d)	0.045	0.156	0.292	0.323	0.290	2.92

conclude that even at small pion energies our procedure ensures a realistic calculation of the pion photoproduction cross sections. This we demonstrate in figs.1-3.

3. $0^+ \to 1^+$ and $0^+ \to 2^+$ PHOTOPRODUCTION TRANSITIONS

First we consider the pion photoproduction in the $0^+ \to 1^+$ transition. The calculated cross sections for energies T_π = 32 MeV and T_π = 42 MeV are in fig.1 compared with experimental data. The on-shell calculations (full line) agree nicely with observations for $\theta \lesssim 90°$. The correct reproduction of data at small angles is a noticeable improvement over the calculation by Singham and Tabakin /5/. Their theoretical cross sections are by a factor of 1.5 ÷ 2 above data for $\theta \lesssim 30°$. In ref. /9/ it has been suggested that this disturbing feature may be connected with the use of the Blomqvist-Laget (BL) /11/ photoproduction amplitude in ref. /5/. We have checked this assumption by a direct calculation using the BL amplitude. For the energy T_π = = 42 MeV we found that the effect, though visible, does not exceed 18% at θ = 30°. At larger angles the ratio $(d\sigma/d\Omega)_{CGLN}/(d\sigma/d\Omega)_{BL}$ approaches quickly the value one.

From the comparison of the on-shell approximation with the Saunders Ansatz (dashed line in fig.1) one can see the main

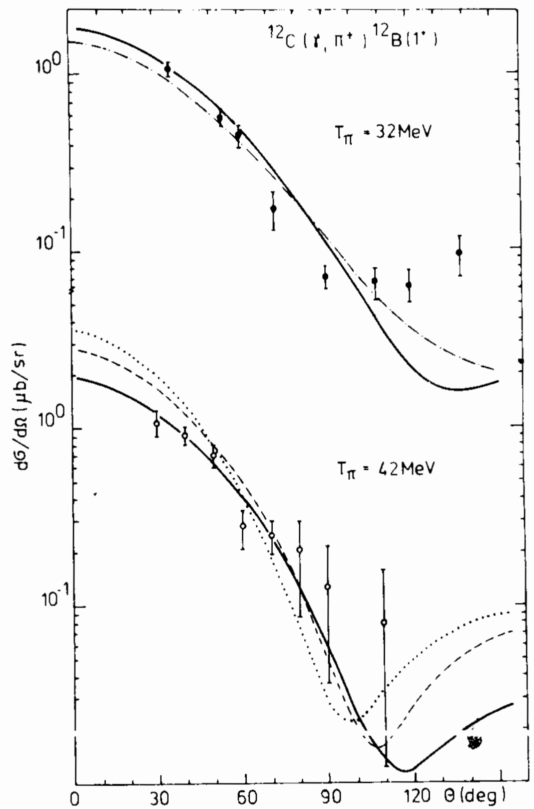


Fig.1. Pion angular distributions for the $^{12}C(\gamma,\pi^+)^{12}B(1^+, g.s.)$ reaction. The data at T_π = = 32 MeV (ref. /9/) and T_π = 42 MeV (ref. /10/) are compared with our on-shell (solid line: DH density, dash-dotted line: CK density of table 1 in ref. /1/) and Saunders-approximation (see the text) calculation (dashed line: DH density /1/). The dotted line corresponds to the result by Singham and Tabakin /5/ obtained with the DH density.

source of difference between our calculation and that of ref. /5/ Including the principal value integral in eq. (1) we have obtained the cross section which is rather close to the one calculated by Singham and Tabakin (dotted line). Similarly as for the pion scattering, we notice that the off-shell effects if taken into account under the standard assumptions may deteriorate the basically sound on-shell approximation even at small momentum transfer. At the same time at larger angles the on-shell result underestimates the data. The calculations at still lower pion energies (T_π = 17 MeV and T_π = 29 MeV) as shown in table 3 also signal definite problems. One concludes then, that the difficult problem of constructing an appropriate off-energy-shell extrapolation of the πN photoproduction amplitudes cannot be avoided if a quantitative theory is to be built up.

The integrated cross sections σ_T $(0^+ \to 1^+)$ are shown in fig.2. Two sets of data visualized by bars (1979, ref. /13/) and dots (1974, ref. /14/) are in mutual contradiction. Our calculations seem to support the data by Epaneshnikov /14/ and as we have

Table 3

Differential cross section of the $^{12}C(\gamma,\pi^+)$ $^{12}B(J^+_n)$ reaction at θ = 90° in μb/sr

J^{π}_n	T_π (MeV)	Present result CK a)	Present result DH b)	Haxton c)	Experimental result d)	Experimental result e)
1^+	17	0.22	0.24	0.22	0.14±0.03	-
	29	0.14	0.14	0.13	0.20±0.o3	0.072±0.012
2^+	17	0.16 d)	-	0.35	0.13±0.03	-
	29	0.24 d)	-	0.55	0.33±0.03	0.373±0.055

a) Cohen-Kurath wave functions, see ref. /1/ ; b) Dubach-Haxton wave function, see ref. /1/ ; c) as quoted in ref. /12/; d) ref. /12/; e) ref. /9/.

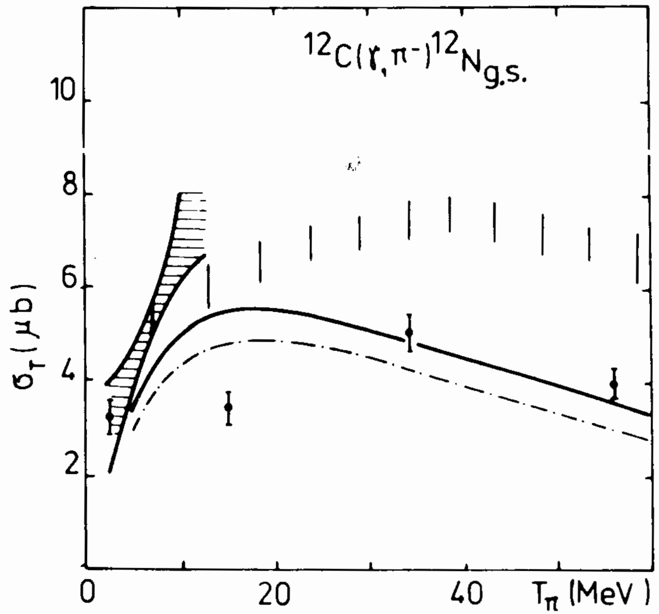
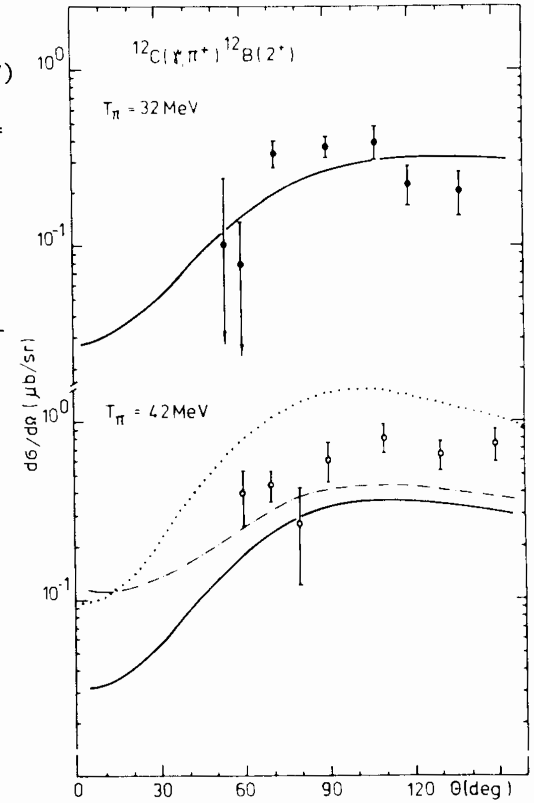


Fig.2. Energy dependence of the integrated cross section for the $^{12}C(\gamma,\pi^-)$ $^{12}N(1^+1)$ reaction. Calculations were performed with the CK density (dash-dotted line) and DH density (solid line). Experimental data are from Bosted et al. /13/ (bars), from Epaneshnikov et al. /14/ (dots), and from Bernstein et. al /15/ (hatched area).

Fig.3. Pion angular distributions for the $^{12}C(\gamma,\pi^+)$ ^{12}B (2^+, 0.95 MeV) reaction. The data at T_π = = 32 MeV (ref. /9/) and T_π = = 42 MeV (ref. /10/) are compared with our on-shell calculation (solid line) and with theoretical results by Singham and Tabakin /5/ (dotted line) and Nagl and Ueberall /16/ (dash-dotted line). CEC-modified Cohen-Kurath nuclear transition densities (see ref. /1/) have been used.



seen above they account correctly for $d\sigma/d\Omega$ at small angles, which give the dominant contribution to σ_T. The result by Singham and Tabakin /5/ is close to newer data /13/, that calculation overestimates, however, $d\sigma/d\Omega$ at $\theta \lesssim 30°$ as compared with the measurement by Shoda /10/, see fig.1. A new experiment for $\sigma_{\pi\gamma}(0^+\to 1^+)$ is indeed desirable to solve the problem.

Now we shall discuss the $0^+\to 2^+$ transition. The results shown in fig.3, pertain to the energies T_π = 32 MeV and T_π = = 42 MeV. The calculated cross sections are actually very close for the two energies, the difference being about 20% only. Since the data at T_π = 42 MeV are roughly 2-4 times as large as those at T_π = 32 MeV, one suspects possible problems with the normalization in at least one of the two data sets. It is then difficult to discuss the absolute values of calculated cross sections. The angular trend of data is, however, reproduced correctly. A comparison with two earlier theoretical works, which follows, seems to be instructive.

Singham and Tabakin [5] have observed that their shell model results for the $0^+ \to 2^+$ transition are as much as three times as large as those calculated by Nagl and Ueberall [16] using the Helm model, even though both densities are consistent with the measured E2 form factor. The contradiction is due to the neglect of CEC in ref. [5]. In deriving the Helm model parametrization Ueberall and collaborators [16] indeed use the Siegert theorem. Their procedure satisfies the continuity-equation constraint in the long wavelength approximation and is therefore closer to our way of extracting the E2 transition densities. Unfortunately, the Helm model, being a parametrization does not allow the calculation of the term proportional to $j_{J+1}(Qr)$. To check on this point we have repeated out fit for $|F_T(2^+)|^2$ and $|F_L(2^+)|^2$ omitting the corresponding term. The photoproduction cross section $d\sigma/d\Omega\ (0^+ \to 2^+)$ calculated with the transition density in this way is then very close to the result by Nagl and Ueberall [16].

4. CONCLUSIONS

The main point observed in our work concerns the sensitivity of pionic cross sections to the nuclear structure input. According to our experience it is actually meaningless to perform the pion scattering or photoproduction calculations without an extremely careful preparation of the nuclear transition densities. Any inconsistency in that respect can easily cause up to order-of-magnitude changes in the calculated results.

When extracting the nuclear transition densities we have observed [1] that the correct analysis of the electric form factors should include the continuity-equation constraint (CEC). We would like to call it Siegert method to distinguish from the well known Siegert theorem which in addition to CEC includes the assumption of the long wavelength regime.

The calculations of the inelastic pion scattering and pion photoproduction for the two isovector transitions $0^+ \to 1^+$ and $0^+ \to 2^+$ in the A = 12 nuclei have shown that the reaction mechanism models proposed recently [3,17], which are variants of the DWIA method, provide appropriate tools for the systematic analysis of data and planing of further experiments. The results of the on-shell approximation agree nicely with the observed cross-sections in the forward hemisphere. Off-energy shell terms, however, should be invoked if one aims at the quantitative understanding of data at large pion angles, approximately beyond $\theta = 100°$. We expect, however, that a realistic off-shell extrapolation of the πN scattering and photoproduction amplitudes should bring in only very small changes at small angles.

For the excitation of the first excited (2^+ T=1) level in ^{12}B a severe disagreement with data has been observed in earlier calculations of the radiative pion capture, (π,π'), and $(\gamma,\pi^\pm)$ reactions. Surprisingly enough all the queries were consistently accounted for by the CEC modification of the 2^+ wave function.

As an application of the present results we would like to suggest that the information on the structure of the 2^+ T = 1 level in ^{12}B can help to analyse the polarization measurements in the $^{12}C(\mu^-,\nu)^{12}B(J^\pi)$ reaction. The precision experiments performed by the Telegdi group [18] can aid in clarification of the basic structure of the weak-interaction Hamiltonian. The interpretation is marred by the absence of good experimental μ^--capture results for the partial transition into the 2^+ final channel. An accurate theoretical substitute is indeed wishful.

Disentangling the convection-current and spin-magnetization contributions to the transverse form factors remains to be a difficult problem. Little success can be expected in the calculations of pionic cross sections unless the good microscopic models of nuclear structure are employed, which predict correctly that ratio.

APPENDIX A

On-Shell Pion Photoproduction

Assuming the on-shell propagation of pions in eq. (1) allows up to put the photoproduction amplitude $F^{\gamma j}$ into the form

$$F^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma) = U^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma)\,[1 + iq_n F^{j}_{nn}(qL_\pi, qL_\pi)]. \qquad \text{(A1)}$$

If the phenomenological phase shifts δ_{L_π} are known, we can rewrite eq. (A1) as

$$F^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma) = e^{i\delta_{L_\pi}} \cos\delta_{L_\pi}\, U^{\gamma j}_{n0}(qL_\pi, k\lambda L_\gamma). \qquad \text{(A2)}$$

This last result resembles very much the Fermi-Watson theorem [19] well known in the elementary-particle physics. Unlike that case, the phase shifts δ_{L_π} are complex, however, in eq. (A2). The same result (A2) has been derived [20] in the dispersion-relation approach in the nuclear reaction theory.

APPENDIX B

The plane-wave partial amplitudes $U^{\gamma j}_{nm}$ are obtained after a relativistic transformation from the πN c.m. system to the π-nucleus c.m. system [3,21]. After some algebra they can be manipulated [2] into the form

$$U^{\gamma j}_{nm}(q'L'_{\pi}, k\lambda L_{\gamma}) = \sum_{sLJ} \int_{-1}^{1} dx.\ P^{\gamma sLJ}_{nm}(q'L'_{\pi}, k\lambda L_{\gamma}, x)\, M^{sLJ}_{nm}(Q), \tag{B1}$$

where $x = \cos\theta$, and Q is the momentum transfer, $Q^2 = q'^2 + k^2 - 2kq'x$. The coefficients P_{nm} are built up of the geometrical factors and the elementary πN photoproduction amplitudes.

The nuclear structure information is fully concentrated in the reduced matrix elements

$$M^{sLJ}_{nm}(Q) = \langle J_n \,||\, [\sigma^s \times Y_L]_J\, j_L(Qr)\, \tau^t \,||\, J_m \rangle, \tag{B2}$$

where Y_{LM} are spherical harmonics, J is the rang of the transition operator, $\sigma^0 = \tau^0 = 1$, $\sigma^1 = \sigma$, $\tau^1 = \tau$, σ and τ are Pauli matrices and $j_L(Qr)$ is the spherical Bessel function.

REFERENCES

1. Eramzhyan R.A. et al. JINR, E4-83-543, Dubna, 1983.
2. Eramzhyan R.A. et al. Journ.Phys.G: Nucl.Phys., 1983, vol.9, p.605.
3. Falomkin I.V. et al. Nuovo Cim., 1980, 57A, p.111; Gmitro M., Kvasil J., Mach R. Phys.Lett., 1982, 113B, p.205.
4. Saunders L.M. Nucl.Phys., 1968, B7, p.293.
5. Singham M.K., Tabakin F. Ann.Phys., 1981, 135, p.71.
6. Tiator L., Rej A.K., Drechsel D. Nucl.Phys., 1980, A333, p.343.
7. Chew G.F. et al. Phys.Rev., 1957, 106, p.1345.
8. Blecher M. et al. Phys.Rev., 1979, C20, p.1884.
9. Schmitt Ch. et al. Nucl.Phys., 1983, A395, p.435.
10. Shoda K., Ohashi H., Nakahara K. Phys.Rev.Lett., 1977, 39, p.1131.
11. Blomqvist I., Laget J.M. Nucl.Phys., 1977, A280, p.405.
12. Paras N. et al. Phys.Rev.Lett., 1979, 42, p.1455.
13. Bosted P.E., Blomqvist K.I., Bernstein A.M. Phys.Rev.Lett., 1979, 43, p.1473.
14. Epaneshnikov V.D., Kuznetsov V.M., Stukov O.I. Sov.J.Nucl. Phys., 1974, 19, p.242.
15. Bernstein A.M. et al. Phys.Rev.Lett., 1976, 37, p.819.
16. Nagl A., Ueberall H. Phys.Lett., 1980, 96B, p.254.

17. Gmitro M., Mach R., Zeit.f. Phys., 1979, A290, p.179.
18. Roesch L.Ph. et al. Phys.Rev.Lett., 1981, 46, p.1507.
19. Watson K.M. Phys.Rev., 1954, 95, p.228.
20. Shapiro I.S. In: Selected Topics in Nuclear Theory. (Ed. by F.Janouch). IAEA, Vienna, 1963.
21. Heller R., Bohannon G.E., Tabakin F. Phys.Rev., 1976, C13, p.742.

Received by Publishing Department
on August 8, 1983.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices - in US $, including the packing and registered postage

Эрамжян Р.А. и др.

E4-83-574

Эффективные сечения реакции $^{12}C(\gamma,\pi^+)^{12}B(1^+_{\text{осн.сост.}}, 2^+_1, 0{,}95$ МэВ)

Рассчитаны характеристики реакции фоторождения π -мезонов на ^{12}C, приводящей к образованию низших изовекторных 1^+ и 2^+ состояний ^{12}B. Применялись ядерные переходные плотности, полученные раньше; для анализа E2 перехода был использован метод Зигерта. Согласование ядерных переходных плотностей с данными по рассеянию электронов позволило улучшить результаты по сравнению с предыдущими работами. Рассчитанные характеристики реакции фоторождения хорошо согласуются с экспериментальными данными. При предположении о распространении пионов в ядре на энергетической поверхности удается устранить расхождение в дифференциальном сечении /в полтора-два раза/ в области малых углов для перехода на уровень 1^+. В случае перехода на уровень 2^+ объясняется большое различие /в три-четыре раза/ между расчетами, проведенными с использованием оболочечной модели и обобщенной модели Хельма.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Eramzhyan R.A. et al.

E4-83-574

Cross Sections of the $^{12}C(\gamma,\pi^+),^{12}B(1^+_{g.s.}, 2^+_1)$ Reaction

We calculate the pion photoproduction on ^{12}C into the lowest isovector 1^+ and 2^+ levels of ^{12}B. Nuclear transition densities used here have been extracted from (e,e') data; those for the E2 transition, using the Siegert method. The consistency with the electromagnetic data has allowed to reach an improvement in several instancies over the earlier results; the calculations agree nicely with experimental data. The assumption of an on-shell pion propagation in nuclei allows us to remove a discrepancy (1.5-2 times) at small angles for the transition to the 1^+ state. In the case of transition to the 2^+ state a large difference between the shell- and Helm-model calculations (3 to 4 times) is accounted for.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983