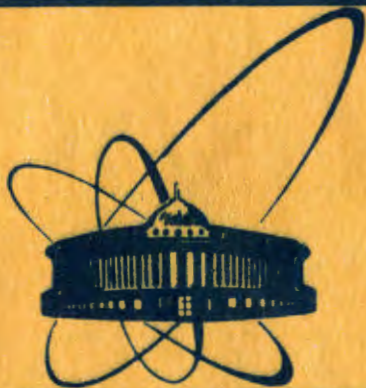


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СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

5120/83

E4-83-446

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**RADIATIVE MUON CAPTURE:
IMPULSE APPROXIMATION
AND THE CONTINUITY EQUATION**

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1983

1. INTRODUCTION

Though studied intensively for nearly thirty years, the radiative muon capture (RMC) still belongs to the poorly understood nuclear processes. There remain both a serious discrepancy between measured and calculated yields of hard γ -rays^{/1/} and the disagreement between different theoretical approaches^{/2,3/}. If taken literally, the experimental results would dictate a strong upward (RMC on ^{16}O) or downward (RMC on ^{40}Ca) renormalization^{/1/} of the pseudoscalar coupling constant g_p of weak interactions. It is clear that a certain numerically important piece of the theoretical analysis is missing in the recent RMC calculations.

In the present paper we argue that the difficulties might be connected with the currently used form of the impulse approximation (IA): Indeed one cannot exclude a possible importance of the meson exchange currents (MEC) in RMC. In particular, the analysis of RMC on ^3He (ref.^{/4/}) and ^{12}C (ref.^{/5/}) has shown that the so-called "elementary particle treatment" (EPT) provides considerably higher photon yields than the detailed IA calculations. We definitely subscribe to all conceptual criticism (see, e.g., ref.^{/4/}) directed against the application of EPT. Nevertheless, the difference mentioned is very systematic and it easily lends itself as an indication of strong MEC in RMC.

The explicit calculation of the MEC corrections in RMC is indeed very difficult. The early estimate^{/6/} has been criticised^{/7/} on general grounds. The calculation by Akhmedov^{/8/} suggests a strong renormalization of g_p ; it is, however, based on the infinite nuclear matter properties and techniques. An alternative procedure would be to employ the coupling provided by the continuity equation of the electromagnetic current. Using it, an important portion of the MEC corrections might be effectively included in terms of the one-body operators, i.e., in IA. We shall follow here that path.

Actually, the well-known Siegert theorem^{/9/} tells us that the probabilities of the electric transitions $B(\text{EL})$, even if originally subject to large MEC corrections, can reliably be calculated via a modified IA: Using continuity equation

$$\vec{\nabla} \cdot \vec{j}^{(ef)} + \frac{\partial j_4^{(ef)}}{\partial x_4} = 0 \quad (1)$$

for the electric current ($\vec{j}^{(e\ell)}$) and charge ($j_4^{(e\ell)}$) densities and the long wavelength regime ($q \rightarrow 0$) one can express the needed nuclear current operator in terms of the nuclear charge operator; the IA is expected to work well for the latter one.

One cannot apply the Siegert theorem for the nuclear radiative processes at larger values of the momentum transfer q . Nevertheless, the continuity equation (1) still allows us to modify IA. Such a procedure has been used in the analysis of the electric-type (EL) transitions induced by electron scattering^{/10/}. In some cases, e.g., for the E2 transition to the 16.11 MeV $T=1$ level in ^{12}C the effects are numerically dramatic, though they might be moderate in other cases. Similarly, the analysis of the deuteron photodisintegration reaction $\gamma + d \rightarrow n + p$ has shown^{/11/} that the large MEC effects which arise in that case can be effectively included in IA for a broad energy interval ($E_\gamma \leq 150$ MeV) via the continuity-equation constraint (CEC).

The radiative muon capture differs from both (e, e') and $\gamma + d \rightarrow n + p$ reactions since RMC is controlled by two currents: instead of the single electromagnetic current $j_\nu^{(e\ell)}(\mathbf{x})$ we deal with the time-ordered product of $j_\nu^{(e\ell)}(\mathbf{x})$ and weak current $J_\lambda(\mathbf{y})$ (two-step reaction). The purpose of the present work is to develop a formalism which allows one to include the continuity-equation constraint in such a case. It is done in the next section.

2. RADIATIVE MUON CAPTURE

For the RMC reaction off the nuclear state $|N\rangle$ with the creation of the state $|N'\rangle$

$$\mu^-(\mathbf{p}^{(\mu)}) + N(\mathbf{p}) \rightarrow \nu(\mathbf{p}^{(\nu)}) + N'(\mathbf{p}') + \gamma(\mathbf{k}, \epsilon) \quad (2)$$

we divide the amplitude into a lepton radiating part T_ℓ and the hadronic part T_h : $T = T_\ell + T_h$. Since T_ℓ is well established (see, e.g., ref.^{/12/}) we shall be interested in T_h only. With the isospin indices and operators omitted it can be represented in the form

$$T_h(\mu) = \frac{ieG\cos\theta_C}{\sqrt{2}} \sqrt{\frac{1}{2E_\gamma}} \iint d^4x d^4y e^{i\ell \cdot y} e^{-ik \cdot x} \quad (3)$$

$$\times \epsilon_\nu^*(\mu) \langle N'(\mathbf{p}') | T[j_\nu^{(e\ell)}(\mathbf{x}) J_\lambda(\mathbf{y})] | N(\mathbf{p}) \rangle \mathcal{L}_\lambda(\mathbf{p}^{(\mu)}, \mathbf{p}^{(\nu)}),$$

$$\mu = \pm 1,$$

where $\ell = \mathbf{p}^{(\mu)} - \mathbf{p}^{(\nu)}$, $\mathcal{L}(\mathbf{p}^{(\mu)}, \mathbf{p}^{(\nu)})$ is the lepton weak current, $j_\nu^{(e\ell)}(\mathbf{x})$ and $J_\lambda(\mathbf{y})$ are the electromagnetic and weak nuclear current and

$$T[j_\nu^{(e\ell)}(\mathbf{x}) J_\lambda(\mathbf{y})] = j_\nu^{(e\ell)}(\mathbf{x}) J_\lambda(\mathbf{y}) + \theta(\mathbf{y}_4 - \mathbf{x}_4) [J_\lambda(\mathbf{y}), j_\nu^{(e\ell)}(\mathbf{x})] \quad (4)$$

is the time-ordered product. The other notation should be obvious. We use the Pauli metric, $A_\mu = (\vec{A}, iA_0)$.

Conventionally the theory of RMC is discussed in terms of the effective Hamiltonians^{/12/}, then

$$T_h(\mu) = \frac{ieG\cos\theta_C}{\sqrt{2}} \sqrt{\frac{1}{2E_\gamma}} 2\pi \delta(E_f - E_i) \quad (5)$$

$$\times \int d^3x e^{-ik \cdot \vec{x}} \langle N'(\mathbf{p}') | \vec{\epsilon}^*(\mu) \cdot \vec{H}_h(\vec{x}) + \vec{\epsilon}_4^*(\nu) H_{h4}(\vec{x}) | N(\mathbf{p}) \rangle.$$

The Hamiltonians $\vec{H}_h(\vec{x})$ and $H_{h4}(\vec{x})$ correspond to the time-ordered products of the weak current with the electromagnetic current and charge operators, respectively (see eq. (2)). In the impulse approximation they are represented as sums over all nucleons of the one-body nucleon operators, that is

$$\vec{H}_h(\vec{x}) = \sum_{i=1}^A \vec{h}(\vec{x}_i) \delta(\vec{x} - \vec{x}_i) \quad (6)$$

and

$$H_{h4}(\vec{x}) = \sum_{i=1}^A h_4(\vec{x}_i) \delta(\vec{x} - \vec{x}_i). \quad (7)$$

As was mentioned above, the relations (6) and (7) are valid only if the MEC corrections are negligible. The validity of such an assumption for $H_{h4}(\vec{x})$ seems to be beyond doubt. One observes, however, that for the real photon $\epsilon_4^*(\mu) = 0$, and we should deal in eq. (5) with the Hamiltonian $\vec{H}_h(\vec{x})$. To improve the approximation stated by eq. (6), we shall use eq. (1) to express $\vec{H}_h(\vec{x})$ in terms of $H_{h4}(\vec{x})$ wherever possible as we have outlined in the Introduction. This should indeed be done by manipulating eq. (3), where the electromagnetic current $j_\nu^{(e\ell)}(\mathbf{x})$ is still singled out.

To perform this programme, we shall expand the photon wave in eq. (3) as a sum of two terms

$$\vec{\epsilon}^*(\mu) e^{-ik \cdot \vec{x}} = \vec{A}^*(\mu, \vec{k} \cdot \vec{x}) + \vec{V} S^*(\mu, \vec{k} \cdot \vec{x}) \quad (8)$$

(see Appendix A) and simplify the expressions which contain the gradient operator ($\vec{V} S^*$). To this end we shall use (i) the

electromagnetic current conservation, eq. (1), and (ii) the SU(2)×SU(2) current algebra commutators

$$\delta(y_4 - x_4)[J_\lambda(y), j_4^{(el)}(\mathbf{x})] = \delta^4(y - \mathbf{x})J_\lambda(y). \quad (9)$$

The amplitude T_h is divided in correspondence with eq. (8) as

$$T_h(\mu) = \frac{ieG\cos\theta_C}{\sqrt{2}} \sqrt{\frac{1}{2E_\gamma}} [M_\lambda^{(A)}(\mu) + M_\lambda^{(\nabla S)}(\mu)] \mathcal{L}_\lambda, \quad \mu = \pm 1. \quad (10)$$

Substituting (8) into (3) and integrating by parts we obtain

$$\begin{aligned} M_\lambda^{(\nabla S)}(\mu) &= \iint d^4x d^4y e^{i\ell \cdot y} e^{E_\gamma y^4} \vec{\nabla} S^*(\mu, \vec{k} \cdot \vec{x}) \langle N' | T[j_4^{(el)}(\mathbf{x})J_\lambda(y)] | N \rangle \\ &= \iint d^4x d^4y e^{i\ell \cdot y} e^{E_\gamma y^4} S^*(\mu, \vec{k} \cdot \vec{x}) \langle N' | T\left[\frac{\partial j_4^{(el)}(\mathbf{x})}{\partial x_4} J_\lambda(y)\right] | N \rangle, \end{aligned} \quad (11)$$

where eq. (1) has already been used. Applying eq. (9) and integrating again by parts we have

$$\begin{aligned} M_\lambda^{(\nabla S)}(\mu) &= \iint d^4x d^4y e^{i\ell \cdot y} e^{E_\gamma y^4} S^*(\mu, \vec{k} \cdot \vec{x}) \\ &\times \left\{ \frac{\partial}{\partial x_4} \langle N' | T[j_4^{(el)}(\mathbf{x})J_\lambda(y)] | N \rangle + \langle N' | J_\lambda(y) | N \rangle \delta^4(\mathbf{x} - y) \right\} \\ &= \iint d^4x d^4y e^{i\ell \cdot y} e^{E_\gamma y^4} S^*(\mu, \vec{k} \cdot \vec{x}) \\ &\times \left\{ -E_\gamma \langle N' | T[j_4^{(el)}(\mathbf{x})J_\lambda(y)] | N \rangle + \langle N' | J_\lambda(y) | N \rangle \delta^4(\mathbf{x} - y) \right\}. \end{aligned} \quad (12)$$

The final expression for $T_h(\mu)$ then reads

$$\begin{aligned} T_h(\mu) &= \frac{ieG\cos\theta_C}{\sqrt{2}} \sqrt{\frac{1}{2E_\gamma}} \mathcal{L}_\lambda(p^{(\mu)}, p^{(\nu)}) \iint d^4x d^4y e^{i\ell \cdot y} e^{E_\gamma y^4} \\ &\times \left\{ \vec{A}^*(\mu, \vec{k} \cdot \vec{x}) \langle N'(p') | T[j_4^{(el)}(\mathbf{x})J_\lambda(y)] | N(p) \rangle \right. \\ &- S^*(\mu, \vec{k} \cdot \vec{x}) E_\gamma \langle N'(p') | T[j_4^{(el)}(\mathbf{x})J_\lambda(y)] | N(p) \rangle \\ &\left. + S^*(\mu, \vec{k} \cdot \vec{x}) \langle N'(p') | J_\lambda(y) | N(p) \rangle \delta^4(\mathbf{x} - y) \right\}. \end{aligned} \quad (13)$$

In terms of the effective Hamiltonian introduced in eq. (5) the full RMC amplitude can be written in the form

$$T(\mu) = \frac{ieG\cos\theta_C}{\sqrt{2}} \sqrt{\frac{1}{2E_\gamma}} 2\pi \delta(E_f - E_i) \quad (14)$$

$$\begin{aligned} &\left\{ \int d^3x e^{-i\vec{k} \cdot \vec{x}} \langle N'(p') | \vec{H}_\ell(\vec{x}) \cdot \vec{\epsilon}^*(\mu) | N(p) \rangle \right. \\ &+ \int d^3x \langle N'(p') | \vec{A}^*(\mu, \vec{k} \cdot \vec{x}) \cdot \vec{H}_h(\vec{x}) + \\ &\left. S^*(\mu, \vec{k} \cdot \vec{x}) [-E_\gamma H_{h4}(\vec{x}) + H_{capt}(\vec{x})] | N(p) \rangle \right\}. \end{aligned}$$

Here $\vec{H}_\ell(\vec{x})$ corresponds to the muon radiating RMC diagram and $H_{capt}(\vec{x})$ is the ordinary-muon-capture Hamiltonian. Explicit expressions of the Hamiltonians \vec{H}_ℓ , \vec{H}_h , H_{h4} , and H_{capt} are given in Appendix B.

The analysis of the RMC with the new form of amplitude as given by eq. (14) is indeed computationally much more complicated than the standard one. In eq. (14) there have appeared additional terms which resemble the off-shell ordinary muon capture; instead of one term in eq. (5) we have now three groups of operators. This will lead to a number of interference terms. Still, we think that the calculation is worth of further efforts if it takes into account, at least partly, the MEC effects. The numerical work for the $^{16}\text{O}(\mu^-, \nu\gamma)$ and $^{40}\text{Ca}(\mu^-, \nu\gamma)$ reactions is in progress, and we shall report the results elsewhere.

The scheme developed in the present paper may indeed be applied to other radiative two-current reactions. An application to the nuclear pion photoproduction and the radiative pion capture is being prepared.

APPENDIX A

There are several representations which allow us to extract a gradient part from the photonic wave $\vec{\epsilon}(\mu)\exp(i\vec{k} \cdot \vec{x})$. Here we have used the decomposition by Rose^{13/}, the coordinate system is such that the third axis is oriented along the photon momentum \vec{k} . Then

$$\vec{\epsilon}(\mu) e^{i\vec{k} \cdot \vec{x}} = \vec{A}(\mu, \vec{k} \cdot \vec{x}) + \vec{\nabla} S(\mu, \vec{k} \cdot \vec{x}), \quad (A.1)$$

$$\begin{aligned} \vec{A}(\mu, \vec{k} \cdot \vec{x}) &= \\ &= -\sqrt{2\pi} \sum_{\ell=1}^{\infty} i^\ell (2\ell+1)^{1/2} [\mu \vec{T}_{\ell, \mu}(\Omega_{\vec{x}}) j_\ell(kx) + \frac{i\vec{k}\vec{x}}{\sqrt{\ell(\ell+1)}} Y_{\ell, \mu}(\Omega_{\vec{x}}) j_\ell(kx)] \end{aligned} \quad (A.2)$$

$$= -\sqrt{2\pi} \sum_{\lambda=0, \pm 1} \vec{e}_\lambda \sum_{\ell=1}^{\infty} i^\ell (2\ell+1)^{1/2} j_\ell(kx) \left\{ \begin{array}{ccc} \ell & 1 & \ell \\ \mu-\lambda & \lambda & \mu \end{array} \right\} Y_{\ell, \mu-\lambda}(\Omega_{\vec{x}})$$

$$-\frac{i\mathbf{k}\mathbf{x}}{\sqrt{2\ell+1}}\left[\sqrt{\frac{1}{\ell}}\begin{bmatrix}\ell+1 & 1 & \ell \\ \mu-\lambda & \lambda & \mu\end{bmatrix}Y_{\ell+1\ \mu-\lambda}(\Omega_{\vec{x}})-\sqrt{\frac{1}{\ell+1}}\begin{bmatrix}\ell-1 & 1 & \ell \\ \mu-\lambda & \lambda & \mu\end{bmatrix}Y_{\ell-1\ \mu-\lambda}(\Omega_{\vec{x}})\right] \quad (\text{A.3})$$

$$\mathbf{S}(\mu, \vec{k}, \vec{x}) = -\sqrt{2\pi} \sum_{\ell=1}^{\infty} i^{\ell+1} \left[\frac{2\ell+1}{\ell(\ell+1)} \right]^{1/2} \frac{1}{k} \left(1 + \mathbf{x} \frac{d}{d\mathbf{x}} \right) j_{\ell}(k\mathbf{x}) Y_{\ell\mu}(\Omega_{\vec{x}}),$$

$$\vec{e}_{\lambda} = \vec{e}(\lambda), \lambda = \pm 1; \quad \vec{e}_0 = \hat{\mathbf{k}},$$

where $j_{\ell}(k\mathbf{x})$ are the spherical Bessel functions, \vec{T} are vector harmonics

$$\vec{T}_{J\ell M}(\Omega_{\vec{x}}) = \sum_{\mu=0, \pm 1} \begin{bmatrix} \ell & 1 & J \\ M-\mu & \mu & M \end{bmatrix} Y_{\ell M-\mu}(\Omega_{\vec{x}}) \vec{e}_{\mu} \quad (\text{A.4})$$

and $Y_{\ell m}$ are usual spherical harmonics. The symbol $\begin{bmatrix} \dots \\ \dots \end{bmatrix}$ stands for the Clebsch-Gordan coefficient.

Other decompositions have been suggested, e.g., by Foldy^{/14/} and by Eisenberg and Greiner^{/15/}. Joenpera^{/15/} compared the three decompositions and found, according certain criteria, the expansion (A.1)-(A.3) to be most suitable among them. Since the choice of criteria is certainly not unique, an interesting question arises of whether some optimal (for our purpose) expansion of $\vec{e}_{\mu} \exp(i\vec{k} \cdot \vec{x})$ can be constructed. The first step towards such an expansion has been done by Friar and Fallieros^{/17/}.

APPENDIX B

Here we display the nucleon operators h (cf. eqs. (6) and (7)) obtained via the non-relativistic reduction from the standard RMC and muon-capture diagrams^{/12/}. For the muon-radiating diagram of RMC one finds

$$\begin{aligned} \vec{h}_{\ell}(\vec{x}_j) \cdot \vec{e}^*(\mu) &= \bar{\chi}^{(\nu)}(\vec{x}_j) \frac{1}{m_{\mu}} [1_{\ell} - \vec{\sigma}_{\ell} \cdot \hat{\mathbf{p}}_{\ell}^{(\nu)}] \\ &\times \{ \delta_{\mu 1} g_V [-(1 + \frac{\vec{\mathbf{s}} \cdot \vec{\mathbf{k}}}{2m}) \vec{\sigma}_{\ell} \cdot \vec{e}^*(\mu) + \frac{E_{\nu}}{2m} (\vec{\sigma}_{\ell} \cdot \hat{\mathbf{k}} \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\mu) + \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\mu))] 1_j \\ &+ [-\delta_{\mu 1} g_A (\vec{e}^*(\mu) - i \vec{\sigma}_{\ell} \times \vec{e}^*(\mu)) + \delta_{\mu 1} (g_A - g_P^{\ell}) \vec{\sigma}_{\ell} \cdot \vec{e}^*(\mu) \frac{\vec{\mathbf{s}}}{2m} \end{aligned} \quad (\text{B.1})$$

$$+ (g_V + g_M) \frac{\delta_{\mu 1}}{2m} (i \vec{\mathbf{s}} \times \vec{e}^*(\mu) - \vec{\sigma}_{\ell} \cdot \vec{\mathbf{s}} \vec{e}^*(\mu) + E_{\nu} \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\mu) \vec{\sigma}_{\ell}) \vec{\sigma}_j \} r_{-} \chi^{(\mu)}(\vec{x}_j).$$

The vector \vec{A}^* has been decomposed in (A.2) into three components defined by the unit vectors $\vec{e}^*(-1)$, $\vec{e}^*(1)$ and $\hat{\mathbf{k}}$. Correspondingly,

$$\begin{aligned} \vec{h}(\vec{x}_j) \cdot \vec{e}^*(\lambda) &= \bar{\chi}^{(\nu)}(\vec{x}_j) \frac{1}{2m} [1_{\ell} - \vec{\sigma}_{\ell} \cdot \hat{\mathbf{p}}^{(\nu)}] \{ (g_A r_{\lambda} - g_V - g_M r_{\lambda} \frac{E_{\nu}}{2m}) \vec{\sigma}_{\ell} \cdot \vec{e}^*(\lambda) \\ &+ g_M r_{\lambda} \frac{E_{\nu}}{2m} \vec{\sigma}_{\ell} \cdot \hat{\mathbf{k}} \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\lambda) + g_P r_{\lambda} \frac{E_{\nu}}{2m} \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\lambda) \} 1_j \quad (\text{B.2}) \end{aligned}$$

$$\begin{aligned} &+ [(\lambda g_A - g_V - g_M) i \vec{\sigma}_{\ell} \times \vec{e}^*(\lambda) + g_P \frac{i\lambda}{2m} \vec{\mathbf{s}} \times \vec{e}^*(\lambda) + g_P^{\ell} \vec{\sigma}_{\ell} \cdot \vec{e}^*(\lambda) \frac{\vec{\mathbf{s}}}{m_{\mu}} \\ &+ (g_V r_{\lambda} - g_A - g_P^N) \vec{e}^*(\lambda) + g_P^N \frac{2E_{\nu}}{(p'-p)^2 + m_{\pi}^2} \hat{\mathbf{p}}^{(\nu)} \cdot \vec{e}^*(\lambda) \vec{\mathbf{s}} \} \vec{\sigma}_j \} r_{-} \chi^{(\mu)}(\vec{x}_j), \end{aligned}$$

$$r = \mu_p - \mu_n, \quad \lambda = \pm 1,$$

and

$$\begin{aligned} \vec{h}(\vec{x}_j) \cdot \hat{\mathbf{k}} &= \bar{\chi}^{(\nu)}(\vec{x}_j) \frac{1}{2m} [1_{\ell} - \vec{\sigma}_{\ell} \cdot \hat{\mathbf{p}}^{(\nu)}] \{ -g_V (1 + \vec{\sigma}_{\ell} \cdot \hat{\mathbf{k}}) 1_j \\ &+ [(-g_A \vec{\sigma}_{\ell} - (g_A + g_P^N) \hat{\mathbf{k}} - (g_V + g_M) i \vec{\sigma}_{\ell} \times \hat{\mathbf{k}} \\ &- g_P^N (\frac{1}{2m} - \frac{2yE_{\nu}}{(p'-p)^2 + m_{\pi}^2}) \vec{\mathbf{s}} + g_P^{\ell} \vec{\sigma}_{\ell} \cdot \hat{\mathbf{k}} \frac{\vec{\mathbf{s}}}{m_{\mu}}] \vec{\sigma}_j \} r_{-} \chi^{(\mu)}(\vec{x}_j). \quad (\text{B.3}) \end{aligned}$$

The Hamiltonian h_4 is obtained by assuming the radiation of a time-like photon:

$$h_4(\vec{x}_j) = \bar{\chi}^{(\nu)}(\vec{x}_j) i [1_{\ell} - \vec{\sigma}_{\ell} \cdot \hat{\mathbf{p}}^{(\nu)}] [g_V (\frac{1}{2mE_{\nu}} (2m - E_{\nu} + E_{\nu}) - \frac{1}{m} \vec{\sigma}_{\ell} \cdot \hat{\mathbf{k}}) 1_j$$

$$\begin{aligned}
& + \left(\frac{g_A}{E_\gamma} \vec{\sigma}_\ell + \frac{g_M + g_V}{2m} \frac{E_\nu}{E_\gamma} i \vec{\sigma}_\ell \times \hat{\vec{p}}^{(\nu)} + \frac{g_P - g_A}{2m} \frac{E_\nu}{E_\gamma} \hat{\vec{p}}^{(\nu)} + \frac{g_P}{m_\mu} \frac{\vec{s}}{2m} \right) \\
& + g_P^N \frac{2E_\nu \vec{s}}{(\vec{p}' - \vec{p})^2 + m_\pi^2} - \frac{g_A}{m} \vec{k} - \frac{g_A}{2m} \vec{\sigma}_\ell \cdot \vec{\sigma}_j] r_{-X}^{(\mu)}(\vec{x}_j).
\end{aligned} \quad (B.4)$$

The muon-capture Hamiltonian

$$\begin{aligned}
h_{\text{capt}}(\vec{x}_j) &= \bar{\chi}^{(\nu)}(\vec{x}_j) i [1_\ell - \vec{\sigma}_\ell \cdot \hat{\vec{p}}^{(\nu)}] \{ (g_V - g_V \frac{\vec{s} \cdot \vec{\sigma}_\ell}{2m}) 1_j + [g_A \vec{\sigma}_\ell \\
& + \frac{g_V + g_M}{2m} i \vec{\sigma}_\ell \times \vec{s} - g_A \frac{\vec{s}}{2m} - g_P^l \frac{1}{m} (\vec{\sigma}_\ell \cdot \vec{k} E_\gamma + E_\gamma - m_\mu) \frac{\vec{s}}{2m}] \vec{\sigma}_j \} r_{-X}^{(\mu)}(\vec{x}_j)
\end{aligned} \quad (B.5)$$

enters into eq. (14) with the off-shell momentum $\vec{q} = \vec{p}' - \vec{p} = \ell - \vec{k}$ (the on-shell value is $q^{\text{nonrad}} = \ell$). Combining eqs. (B.4) and (B.5) one finds

$$\begin{aligned}
-E_\gamma h_4 + h_{\text{capt}} &= \bar{\chi}^{(\nu)}(\vec{x}_j) \frac{i}{2m} [1_\ell - \vec{\sigma}_\ell \cdot \hat{\vec{p}}^{(\nu)}] \{ g_V (1 + \vec{\sigma}_\ell \cdot \vec{k}) E_\gamma 1_j \\
& + [(g_V + g_M) i E_\gamma \vec{\sigma}_\ell \times \vec{k} - g_P^N E_\nu \hat{\vec{p}}^{(\nu)} + g_A E_\gamma \vec{k} + g_A E_\gamma \vec{\sigma}_\ell \\
& + \vec{s} (g_P^l - g_P^l \frac{E_\gamma}{m_\mu} \vec{\sigma}_\ell \cdot \vec{k} - g_P^N \frac{2E_\nu E_\gamma}{(\vec{p}' - \vec{p})^2 + m_\pi^2})] \vec{\sigma}_j \} r_{-X}^{(\mu)}(\vec{x}_j),
\end{aligned} \quad (B.6)$$

where 1_ℓ and 1_j ($\vec{\sigma}_\ell$ and $\vec{\sigma}_j$) are unit (spin) operators acting in the spaces of lepton and j -th nucleon, respectively. The weak and magnetic form factors are denoted by g_i ($i = V, M, A, P$) and μ_p, μ_n , respectively; $g_P^N \equiv g_P(\ell^2)$, $g_P^l \equiv g_P((\ell - \vec{k})^2)$; $\vec{a} \equiv \vec{a}/|\vec{a}|$, $\vec{s} = \vec{p}^{(\nu)} + \vec{k}$, m is the nucleon mass.

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Received by Publishing Department
on June 28, 1983.

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E4-83-446

Радиационный захват мюонов:
импульсное приближение и уравнение непрерывности

Предложен новый способ построения эффективного гамильтониана радиационного захвата мюонов ядрами в импульсном приближении. При этом использовано уравнение непрерывности электромагнитного тока, что позволило уменьшить поправки, связанные с учетом мезонных обменных токов. Этот способ можно непосредственно использовать при рассмотрении других радиационных процессов, в которых участвуют два тока, в частности, при рассмотрении процесса фоторождения пиона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

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E4-83-446

Radiative Muon Capture:
Impulse Approximation and the Continuity Equation

A modified form of the impulse-approximation effective Hamiltonian of the nuclear radiative muon capture is developed. In the derivation we have used the electromagnetic-current continuity equation $\vec{\nabla} \cdot \vec{j}^{(el)} + \partial j_4^{(el)} / \partial x_4 = 0$ to minimise the meson-exchange-current corrections. The method is directly applicable to other radiative reactions controlled by two currents, e.g., to the pion photoproduction process.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983