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SCATTERING AND BREAK UP OF DEUTERONS ON ALPHA PARTICLES IN A REALISTIC THREE-BODY MODEL

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1. INTRODUCTION

In the study of direct nuclear reactions, one of the most employed projectiles is the deuteron. It is widely used, both for inelastic scattering and transfer processes. Apart from technical advantages, the main reason for this is the comparatively simple structure of the deuteron, i.e., that it is a two-particle system with only one bound state.

The information obtained in deuteron induced reactions is overwhelming. Also, in recent times, some details in the theoretical description of these reactions (polarization, D-state component), which are of importance for the interpretation of the data, seem to be better understood.

However, there remains one main problem, which is common for all transfer reactions and scattering of complex particles. The description of such reactions is in general given in terms of the Born approximation (DWBA) which can essentially be shown to be valid only for two body problems. It is therefore highly important to study such reactions using the correct mathematical procedures of many-body scattering, as developed particularly by Faddeev and his co-workers [1]. In this way one may both establish the limits of validity of older approaches, and get the tools for extending the calculations beyond these limits.

Here again, the deuteron scattering and reaction processes present the simplest possible problems of this type. If the target can be treated as inert, these processes are of three body type and they can be handled by means of the Faddeev equations, which are at present the only ones which seem to be solvable in realistic cases.

The simplest scattering problem of this type is the deuteron-α scattering. The tight binding of the α-particle makes it possible to neglect its internal degrees of freedom to a larger extent than with any other nucleus, i.e., up to energies 22 MeV in the CM system. At the same time, the small size of the alpha-particle means that in this energy range, only a few angular momentum channels will contribute essentially to the scattering.

This problem is therefore the first which should be attacked, if a firmer bases for understanding the scattering and reations with the more complex targets and projectiles shall be constructed.

The questions which arise when a N-body problem - in the present case N = 6 - is approximated by a 3-body problem have been discussed by a number of authors, see particularly the recent work of Schmid[6] and Bencze et al.[7], and references therein. Basically two different points of view may here be taken. Either one may, as in[7], start with the N-body formalism. This actually involves coupled equations for components which result from splitting the system in 2,3,4,etc. "clusters" of interacting (not necessarily bound) particles. Hence, by elimination and truncation, it leads to an approximation scheme, where the system may subsequently be described in terms of 2 clusters, 3 clusters,etc., with effective interactions between them. Effects of other fragmentations may be taken into account as higher order corrections.

A precise account of the different fragmentation channel is particularly important when they open up, whereas below threshold their influence is small. In our case, the most important neglected channel is presumably $^6Li \rightarrow {}^3H + {}^3He$. The main justification for the neglect of this is its rather high threshold, 15.79 MeV.

The inclusion of antisymmetrization in this approach is a little complicated. In this respect, the other approach[6] may be simpler, since the wavefunction is given as a sum of terms containing antisymmetrized products of cluster functions with functions of their relative coordinates (like the Jacobian coordinates below). The clusters are in general distorted, so that only asymptotically they correspond to the free ones.

In calculations it is convenient to truncate the complete system of cluster wave functions, and, e.g., use a generalized Kohn variation principle to calculate amplitudes of concrete processes like transfer, etc.

The main principle is that from the fundamental interactions, acting between the particles, as well as from the Pauli principle, effective interactions between the clusters are derived. These interactions must depend on the internal structure of the clusters, including the distortion. They will in general be both energy dependent and of multiparticle character, i.e., there will be not only two-cluster, but also three-cluster forces, etc. The present method can be viewed as a special case of that of ref.[6], with the distortion of the alpha particle neglected. This crucial approximation, which can also be called a neglect of the virtual break-up of this particle, is again mainly justified by the high threshold for real break up. We therefore write the wave function as an antisymmetrized product of an α-particle function χ and a function ψ of the Jacobian coordinates, describing the distances between the center of mass of the α-particle and the two other ones. The variation is performed on ψ only, and the antisymmetry leads to elimination of ψ -components which are not orthogonal to the internal neutron (or proton) χ -functions, which are approximated as 1s states with respect to the center of mass of the α-particle. In this limit, which also means, that center of mass problems are only treated approximately, there are only two-body interactions

between the clusters. Exchange terms are now similar to those in nucleon-α scattering and may be approximated in a similar way.

Some attempts to solve the deuteron-α scattering problem by means of the Faddeev equations have been made recently[2,3]. These attempts are based on the usual formalism of the Faddeev equations, by means of scattering amplitudes and Green's functions given in momentum space. Interactions are then generally used which have simple forms in this space, particularly sums of separable terms. Of these approaches, that of Charnomordic[3] is presumably the most extensive. There, the n-p interaction is simulated by ($\bar{L}$ = angular momentum of relative motion)

$$V_{L,L'}(p,p') = \sum_{i(1 \le i \le I)} \lambda_i \, g^i_L(p) \, g^i_{L'}(p')$$

$$g^i_L(p) = p^{L+S_i} / (p^2 + \beta^2_L(i))^{\frac{L+m_i}{n_i}}$$

or

$$g^i_L(p) = \frac{p^L (\alpha_i + \gamma_i p^2)}{\prod_{j=0}^{n_i+1} (1 + \beta^i_{L,j} p^2)} \qquad (1.1)$$

and the alpha-nucleon interaction by an expression similar to the latter one.

Results are obtained which are in fair agreement with experimental scattering data.

However, in this approach, as well as in others with sums of separable terms, one meets with a number of difficulties.

The first is of mathematical nature. The Coulomb interaction cannot be well approximated by a finite sum of separable terms. Alt and Sandhas[4] have suggested a scheme for solving the Faddeev equations with a potential which is the sum of a separable and a Coulomb terms. This method seems, however to meet with some mathematical difficulties, and the only rigorous solution of the Faddeev problem with Coulomb interactions as well as

short range ones seems at the moment to be that of Merkuriev[5] who uses an extension of the method also employed in the present work.

The second problem is connected with the exclusion principle. A complete antisymmetric wavefunction cannot be constructed in any realistic calculation with more than four particles. However, the antisymmetrization can, as discussed above, be approximated by saying, that certain occupied states (here roughly corresponding to the bound states in a shell model description of the alpha-particle) are forbidden in the nucleon-alpha scattering.

This forbiddenness can be effectuated by a repulsive potential which is (infinitely) strong and separable, since it corresponds to a projection operator for the corresponding single particle state. In a calculation like that of ref.[3], the most near lying will therefore be to use a repulsive potential for l s. In the present calculation, the alpha-nucleon potential is attractive and local. It is chosen so as to reproduce the alpha-nucleon phase shifts. The same is the case with the repulsive potential of ref.[3], but this is only possible in a limited sense, as is seen from Levinson's theorem applied to the occupied state.

The difficulties mentioned above are thus overcome in the present calculation, by integrating the Faddeev equations in configuration space, and using local interactions together with a device corresponding to the exclusion principle. This method has the further advantage, since the matrices representing both the kinetic energies and the potentials are sparse, that a diagonalization is facilitated. The calculations are discussed in details in the next section.

In the present context, however, the main progress which is obtained with the use of local potentials, is that, in that way, the road is prepared for a connection to the great bulk of nu-

clear physics calculations. In bound state problems, the shell model potentials, and in scattering problems, the optical model potentials are well established nucleus-nucleon potentials, and parameters have been found, which give overall fits to a vast number of data and which with small modifications can be made to fit bound state and scattering data for any concrete nucleous. Also nucleon-nucleon interactions have been found, which reproduce all properties of the deuteron, as well as the appropriate phase shifts in a very large energy range.

Therefore, if the task is to understand the limitations and successes of previous nuclear reaction calculations, and to find their natural extensions, it is extremely useful to be able to use those interactions in the correct Faddeev treatment of the problem.

It is of particular interest for the present calculation, that a set of local interactions has been found which reproduce not only the deuteron wave function and the nucleon-nucleon as well as nucleon-alpha scattering data, but also the binding energy and charge radius of ^{6}Li[8]. We shall in the following make use of these potentials, just as the ^{6}Li wave functions have to a certain extent been used to check the programs used in the present calculations.

When we write the α-nucleon interaction as a local potential which reproduces the α-nucleon phase shift, as a consequence we must also approximate the forbidden 1s state by a bound state in the same potential, as mentioned in ref.[8].

2. GENERAL FORMALISM

Since the method of solving the Faddeev equations in coordinate space has been explained in details in other works[7], we shall here give a rather brief description of this formalism.

The system of equations to be solved can be written

$$(E - H_0 - V_\alpha)\psi^\alpha = V_\alpha(\psi^p + \psi^n) , \qquad (2.1)$$

$$(E - H_0 - \tilde{V}_p)\psi^p = \tilde{V}_p(\psi^n + \psi^\alpha) , \qquad (2.2)$$

$$(E - H_0 - \tilde{V}_n)\psi^n = \tilde{V}_n(\psi^\alpha + \psi^p) . \qquad (2.3)$$

Here the interactions are, as usual, indicated by the particle which does not take part in them. So V_α is the neutron-proton interaction. The interaction between the neutron and the alpha-particle (which is here treated as an elementary particle) is denoted V_p, $\tilde{V}_p$ being a modified interaction which takes into account the exclusion principle. For the neutron $\tilde{V}_n$ is defined analogously.

The three equations are solved with boundary conditions corresponding to the interactions which they contain. So, e.g., in the description of deuteron-alpha scattering, with possible inclusion of deuteron break-up, part of ψ^α will asymptotically correspond to plane waves, and part to outgoing waves of the relative deuteron-alpha motion (multiplied with a deuteron function of the internal coordinate). The α,n,p case is particular by the non-existence of bound nucleon-alpha systems, so the two other components have asymptotically only outgoing three-particle waves.

The total wave function is given by

$$\psi = \psi^\alpha + \psi^p + \psi^n . \qquad (2.4)$$

The three components are often referred to as the wave function in the three channels, α, p and n; it must be remem-

bered, however, that Ψ^α also contains such parts where the deuteron has been subject to break-up.

The concrete solution of the coupled equations (2.1) (2.2) (2.3) is carried out in Jacobian coordinates, corresponding to the different channels with

$m_p = m_n = m$ and $m_\alpha = \omega m$ $(= 4m)$

we have

$$\begin{aligned}
\underline{x}_\alpha &= \left(\frac{1+\omega}{2\omega}\right)^{\frac{1}{2}} (\underline{r}_p - \underline{r}_n), \\
\underline{y}_\alpha &= \left(\frac{2(\omega+1)}{\omega+2}\right)^{\frac{1}{2}} ((\underline{r}_p+\underline{r}_n)/2 - \underline{r}_\alpha), \\
\underline{x}_p &= \underline{r}_n - \underline{r}_\alpha, \\
\underline{y}_p &= (\omega(\omega+2))^{\frac{1}{2}} (\omega\underline{r}_\alpha + \underline{r}_n - (\omega+1)\underline{r}_p), \\
\underline{x}_n &= \underline{r}_\alpha - \underline{r}_p, \\
\underline{y}_n &= (\omega(\omega+2))^{\frac{1}{2}} (\omega\underline{r}_\alpha + \underline{r}_p - (\omega+1)\underline{r}_n).
\end{aligned} \tag{2.5}$$

In these coordinates, angular momentum components are introduced:

$$\Psi^\nu(\underline{x}_\nu, \underline{y}_\nu) = \sum_i \frac{\psi_i^\nu(x_\nu, y_\nu)}{x_\nu y_\nu} Z_i(\hat{y}_\nu, \hat{x}_\nu),$$

$$Z_i(\hat{y}, \hat{x}) = \left\{ \mathcal{Y}^{\lambda_i}(\hat{y}) \otimes \left[\chi^{s_i} \otimes \mathcal{Y}^{\ell_i}(\hat{x}) \right]^{\Sigma_i} \right\}^{J_i M_i} \tag{2.6}$$

$$\nu = \alpha, p, n$$

When this is inserted in (2.1), (2.2), (2.3), a system of coupled differential two-dimensional equations for the ψ_i is obtained. For limited total energies, the centrifugal barriers and the limited extension of the nuclear forces make it possible to neglect the couplings due to V_ν ($\underline{x}_\nu$, $\underline{\sigma}_\nu$) for higher λ, ℓ-values; this limits the number of coupled equations (for $E \lesssim$ 10 MeV $\lambda, \ell > 2$ contributes very little for all ν). A similar argument is also valid for the term in $\tilde{V}_n$ and $\tilde{V}_p$, which take care of the exclusion principle. For the Coulomb forces, however, the argument is not applicable. A rigorous treatment of these forces is postponed to later work; in this

paper wee shall content ourselves with a perturbative approach, which seem justified at most energies.

In that case, as a first approximation, the forces acting on the neutron and the proton are identical and we may reduce the number of coupled channels (ν, i) by introducing the operator of exchanging the neutron and proton P_{pn}.

$$\begin{aligned}
&P_{pn}\Psi^\alpha = \Psi^\alpha, \; P_{pn}\Psi^p = \Psi^n, \; P_{pn}\Psi^n = \Psi^p \\
&P_{pn}\Psi = \Psi = \Psi^\alpha + (1 + P_{pn})\Psi^p
\end{aligned} \tag{2.7}$$

The nuclear forces used were taken so as to reproduce the nucleon-alpha and nucleon-nucleon scattering data for lower energies, as well as the properties of the deuteron. The principle of approximating the nucleon-alpha interaction in the antisymmetrized system by a two-body potential together with an exclusion of the 1s state in their relative motion, and the use of this approximation in the six particle system was discussed in [8]. This however is now treated in a more efficient manner, as discussed below.

Since the alpha-nucleon and nucleon-nucleon force of [8] were very carefully chosen to fit the two-body data, and since they furthermore lead to very good fits to most of the properties of the ^{6}Li ground state, we shall here just use the same forces subject to a slightly different treatment of the Coulomb force problem. Keeping the forces constant, we shall concentrate our efforts on the scattering problem, with particular attention to the way in which the Pauli forbiddenness is treated.

3. THE EXCLUSION PRINCIPLE

In ref. [8] the exclusion principle was handled by projecting the Faddeev components into the allowed space. Let the

projection operator for the forbidden neutron state be

$$\Gamma_p = |u(\underline{x}_p)\rangle\langle u(\underline{x}_p)|, \qquad (3.1)$$

where $u(x_p)$ is the wave function of the 1s state, and the corresponding operator for the proton Γ_n, then in ref.[8] the components which are found by iteration $\psi^\nu_m \to \psi^\nu_{m+1}$ are in each step replaced by

$$P\psi^\nu_m \approx (1-\Gamma_p)(1-\Gamma_n)\psi^\nu_m \qquad (3.2)$$

$$(P \equiv P_{\text{allowed space}},\ P^2 = P)$$

A generalization of this method to the scattering problem seems highly complicated, so it was proposed in ref.[8] to use instead an algorithm similar to the one suggested by Bang and Gareev[10]. The same idea was successfully applied to similar problems by Kukulin et al.[11]

Such an algorithm is the main ingradient which has to be added when few body methods are to be applied to scattering and reactions of complex nuclei. The case of elastic scattering was discussed in ref.[11], we shall here look at the modifications of the method which are necessary for reaction problems.

Without the Pauli principle, the $\tilde{V}_p$, $\tilde{V}_n$ of (2.2), (2.3) reduce to V_p, V_n, and the coupled equations imply that ψ of (2.4) satisfy the Schrödinger equation

$$(E - H)\psi = 0 \qquad (3.3)$$

with

$$H = H_0 + V_\alpha + V_p + V_n . \qquad (3.4)$$

In a similar way, we want that (2.1), (2.2), (2.3) imply

$$P(E - H)P\psi = 0 . \qquad (3.5)$$

We may, however, instead of (3.5) solve the inhomogeneous equation

$$(E-H)\psi = u_n(\underline{x}_n)F_n(\underline{y}_n) + u_p(\underline{x}_p)F_p(\underline{y}_p), \qquad (3.6)$$

where one chooses the F's so that

$$\int u_n(\underline{x}_n)\psi\, d\underline{x}_n = \int u_p(\underline{x}_p)\psi\, d\underline{x}_p = 0 . \qquad (3.7)$$

We see that when (3.7) is satisfied, we may write

$$\psi = P\psi . \qquad (3.8)$$

When we insert this in (3.6), and operate on both sides of it with P, we get back (3.5), since

$$Pu_n) = Pu_p) = 0. \qquad (3.9)$$

Once more, instead of (3.6), we solve the system of coupled equations

$$(E - H_0 - V_p)\psi^p = V_p(\psi^n + \psi^\alpha) + u_p F_p , \qquad (3.10)$$

$$(E - H_0 - V_n)\psi^n = V_n(\psi^p + \psi^\alpha) + u_n F_n , \qquad (3.11)$$

$$(E - H_0 - V_\alpha)\psi^\alpha = V_\alpha(\psi^p + \psi^n) . \qquad (3.12)$$

Again, neglecting the differences between u_p and u_n as well as between V_p and V_n, this system reduces to two coupled equations, plus a symmetry between ψ^p and ψ^n.

For the numerical calculation, it is practical to expand the function F(y) on some fixed system of independent functions $f_i(y)$

$$F(y) = \sum_i c_i f_i(y) ; \qquad (3.13)$$

A convenient choice of f_i will be the so-called spline functions; in the present calculations cubic splines were used.

Since the system of equations (3.10) - (3.12) is linear, it may be solved by replacing the $F_\nu(y_\nu)$ (ν=n,p) by $f_i(y_\nu)$ solving the inhomogeneous equations for each i-value, and then determining the c_i so that the total wave functions is orthogonal to u_n and u_p.

Let the solution, obtained by replacing F(y) by $f_i(y)$ be $\psi^{(i)}$, and $\psi^{(0)}$ be a solution with no inhomogeneous term. Then a solution of (3.10) - (3.12) is

$$\psi = \psi^{(0)} + \sum_i c_i \psi^{(i)} . \qquad (3.14)$$

So, from (3.7) we get

$$\{\langle u|\psi^{(0)}\rangle + \sum_i c_i \langle u|\psi^{(i)}\rangle\} = 0 . \qquad (3.15)$$

If the f_i are orthonormal, we may by multiplication with $f_j(y)$ and integration over y obtain (3.15) in the form

$$a_j + \sum_i c_i a_{ij} = 0 . \qquad (3.16)$$

We see that this determines the c's up to a common normalization contained in $\psi^{(0)}$.

The possibility of expanding F_n, F_p on square integrable functions is seen as follows.

We have, e.g.,

$$\begin{aligned} F_p(y_p) &= \int dx_p\, u(x_p)\{(E-H_0-V_p)\psi^p \\ &\qquad - V_p(\psi^n+\psi^\alpha)\} \\ &= (E-\varepsilon_0-T_{y_p})\int dx_p\, u(x_p)\psi^p \\ &\quad - \int dx_p\, u(x_p)\, V_p(\psi^n+\psi^\alpha) , \end{aligned} \qquad (3.17)$$

or, using (3.7)

$$F_p(y_p) = \int dx_p\, u(x_p)(H_0-E)(\psi^n+\psi^\alpha) . \qquad (3.18)$$

Now, $u(x_p)\to 0$ for $x_p\to\infty$; and as is seen from (2.5), large values of y_p and finite values of x_p correspond to large values of x_n and x_α. But we see from (3.11) and (3.12), that

$$(E-H_0)\psi^\nu \underset{x_\nu\to\infty}{\to} 0 \quad (\nu = n,\alpha) . \qquad (3.19)$$

So $F_p(y_p)\to 0$ for $y_p\to\infty$.

Taking the potentials to be of finite range, it will actually be the exponential decay of $u(x_\nu)$ as function of x_ν which determines the asymptotic behaviour of $F_p(y)$ (as a similar exponential function).

We see, that the modified interaction of (2.2) can be written as

$$\begin{aligned} \tilde{V}_p(\psi^n+\psi^\alpha) &= V_p(\psi^n+\psi^\alpha) \\ &+ \int dx_p\, u(x_p)(H_0-E)(\psi^n+\psi^\alpha) , \end{aligned} \qquad (3.20)$$

and similarly for $\tilde{V}_n$. Since $u_p(x_p)$ is of finite extension, we see that the boundary conditions of (2.1) - (2.3) or (3.10) - (3.12) are similar to those of the usual Fadeev equations with no Pauli principle, provided the forbidden states are excluded in the boundary conditions (e.g., for x_p finite, $y_p\to\infty$).

This is the justification for saying, that the solution of (3.10) - (3.12) with the proper boundary conditions is what is

needed for the scattering problem with the exclusion principle, in the same sense as the usual Faddeev equation provided the solution of the three-body scattering problem.

The method presented here can be obtained from the method of adding a term, $\lambda(\Gamma_p + \Gamma_\alpha)$ to the Hamiltonian and letting go to infinity[10,11]. Actually, the spline expansion may be applied for finite values λ, obtaining a linear equation for determination of the c's, which in the limit of infinite becomes identical to (3.16).

It is interesting to note that, since Γ_ν is a typical example of a separable potential, the procedure just mentioned can be viewed as a tool for solving the Faddeev equations with a potential, which is the sum of a local and a separable part. So also for this task, the method of solving the equations in coordinate space turns out to be efficient.

4. TREATMENT OF THE COULOMB INTERACTIONS

Although the product of the charges in d+ α scattering is small, the Coulomb force has an important role in determining for example the positions and widths of resonances. The Coulomb energy in the Faddeev ground state is 0.77 MeV[8], and the shifts to features in the scattering phases are expected to be a large fraction of this. Because the Faddeev equations are otherwise so accurate, it is essential to include some good approximation to these Coulomb effects on the nuclear phase shifts.

We have found it satisfactory to use the screened-Coulomb method described in Goldberger & Watson[12] and used in 3-nucleon scattering by Alt[4]. In this procedure the scattering equations are first solved for nuclear forces plus Coulomb forces which have been screened by multiplication with a cut-off

shape $(1+\exp((r-R)/a))^{-1}$. The scattering S matrix elements can then be found for this screened Coulomb force alone, as the cut-off radius R has been chosen inside the radial mesh. Although as R increases both scattering phases diverge logarithmically, their difference becomes constant and equal to the nuclear phase shift that we require. Since in the present work we mainly need the effects of Coulomb distortion in the interior and surface regions of the target, and since we are looking only at partial waves up to $\ell = 2$ at incident momenta up to k = 1 fm^{-1}), it is sufficient to take a cut-off radius such as 10 fm, which is 5 times the size of the target, and which is hence also much greater than L/k = 2 fm. A cut-off diffuseness of 2 fm was used. Because the neutron and proton were assumed earlier to have a symmetric wavefunction, we have distributed the deuteron's charge symmetrically over its components. This neglects the higher multipoles of Coulomb distorting force and will perhaps change the break-up distributions slightly, but should only weakly affect the elastic scattering for the lower partial waves.

When there are several incoming partial waves coupled during the interaction, the above procedure is generalized, so that the nuclear matrix S_N is found from the nuclear + Coulomb S_{Nc} and the screened-Coulomb-only S_c by the unitary transformation $S_N = S_c^{-\frac{1}{2}}$, S_{Nc} $S_c^{-\frac{1}{2}}$. The S_N is then analysed into phase shifts, in the standard manner which in the2 channel case involves finding δ_1, δ_2, and ϵ such that

$$S_N = u \begin{pmatrix} e^{2i\delta_1} & 0 \\ 0 & e^{2i\delta_2} \end{pmatrix} u^+ , \qquad (4.1)$$

$$\text{where} \quad u = \begin{pmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{pmatrix} , \qquad (4.2)$$

must be the second unitary transformation.

5. NUMERICAL RESULTS FOR DEUTERON-ALPHA-SCATTERING

As mentioned above, we have in this work used the same nucleon-nucleon and nucleon-alpha interactions as in the work[8] on the ground state of ^{6}Li.

These interactions are, for the . p and n channels given in Table 1. The bound 1s states were calculated with the potential of Table 1. The angular momentum components of the present calculation used in the different channels are given in Table 2.

For J = 1, λ_α = 0, 2, the phase shifts given by equations (4.1) and (4.2) are shown in Fig. 1.

For λ_α = 2, J = 2,3, the coupling is negligible and the phase shifts are given in Fig. 2.

The differential break-up cross sections are in principle obtainable from our wave function, although they present some numerical problems; but the total reaction cross reactions are obtained from the imaginary term in the phase shifts (in earch channel)

$$\sigma_{in} = \frac{\pi}{k^2_{a,\alpha}} (1 - \eta^2) . \qquad (5.1)$$

Since the only inelastic process which is possible in the present model, and in reality, for energies smaller than 22 MeV, is deuteron break-up, and since $\lambda = 0, \lambda = 2$ is very dominating in the scattering at these energies, (5.1) with the relevant η gives the total break-up cross section.

The numerical work was done with a network of points in a system of hyperspherical coordinates (R, ϑ), where R takes 60 steps of length .3 fm, whereas ϑ goes in steps of 0.7143°.

The fits to the experimental phase shifts of Figs. 1 and 2 are seen to be somewhat superior to those of previous calculations[3]. The region where $\delta_{\lambda=0}$ and $\delta_{\lambda=2}$ are near to

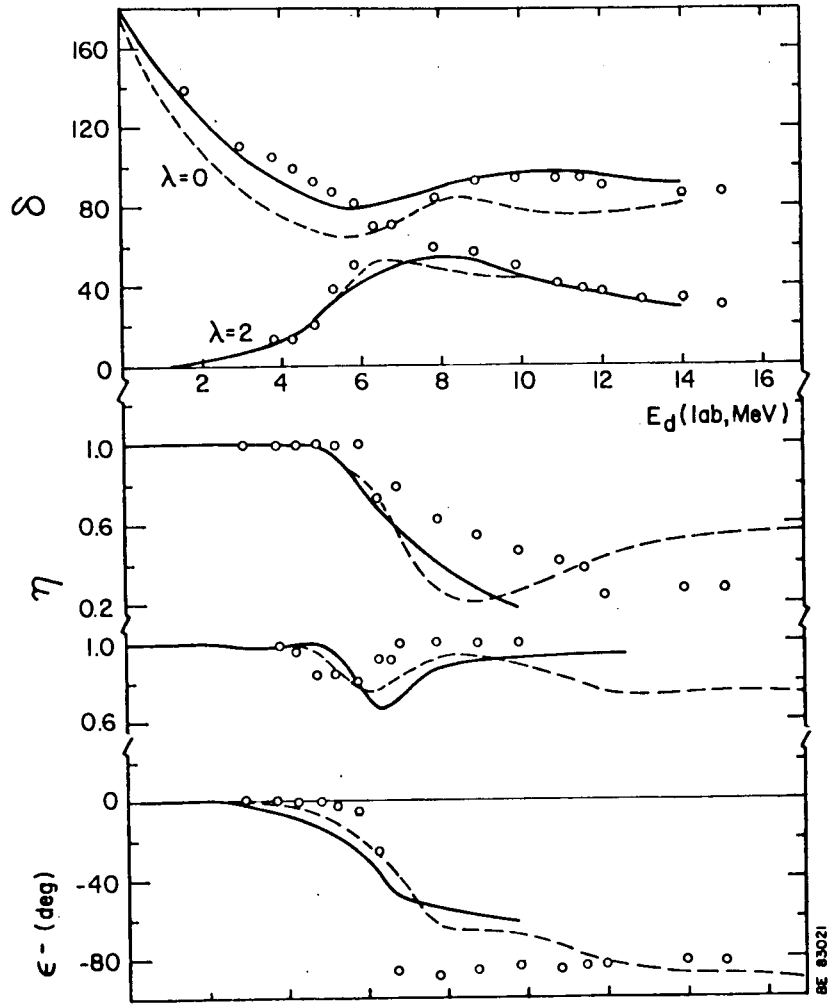

Fig. 1. The phase shifts $\delta(\lambda_\alpha = 0)$ and $\delta(\lambda_\alpha = 2)$ for J = 1. The real part of the phase shifts are denoted δ, whereas η = exp (Im (δ)) and ϵ is given by equation (4.1). The experimental values, given by circles are those of ref.[12]. The dotted line corresponds to neglect of the spin orbit force, the full line is with the full interaction. Upper η-curve: λ = 0.

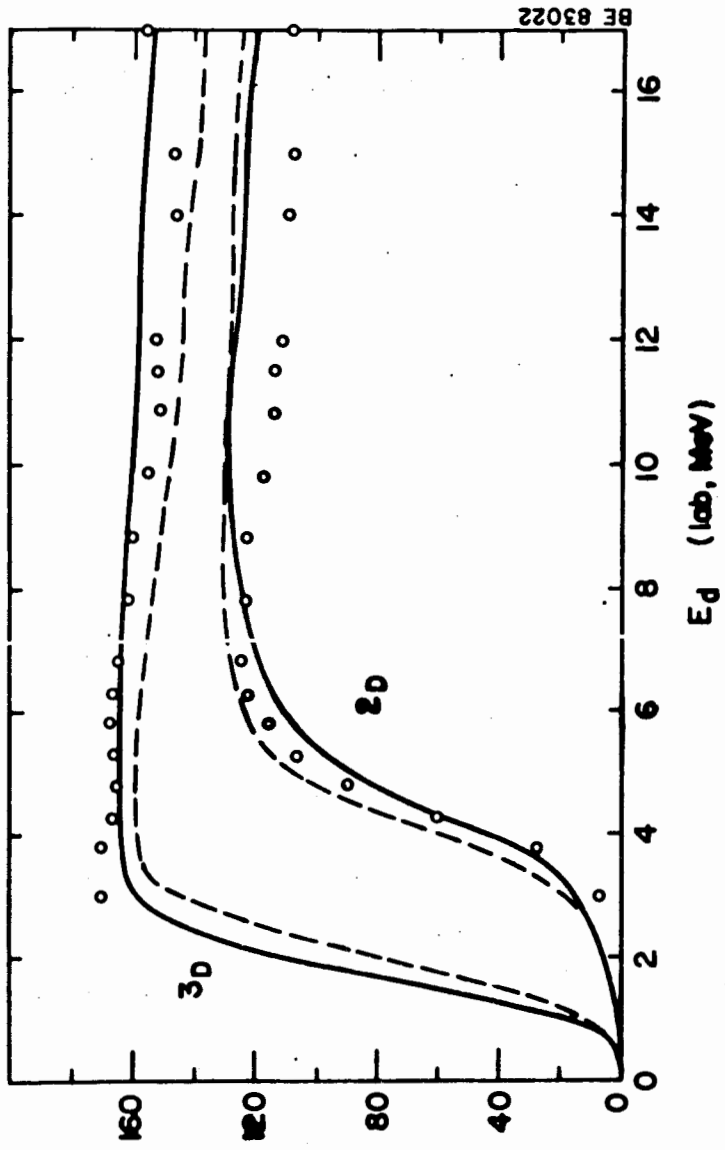

Fig. 2. The phase δ for $\lambda = 2$, $J = 2$ and 3, explanation as for fig. 1.

Table 1. The nuclear potentials.

$$V_n = V_p = V_{\alpha n} = \frac{-43.0}{1+\exp((r-R_0)/a_0)} + \frac{\vec{l}\cdot\vec{s}}{r}\frac{d}{dr}\frac{40.0}{1+\exp((r-R_1)/a_1)} \text{ MeV}$$

$R_0 = 2.0$ fm, $a_0 = 0.70$ fm,

$R_1 = 1.5$ fm, $a_1 = 0.35$ fm.

each other seems to present a problem both for the separable potential method and for ours, but this is a region where the analysis of experiments presents some problems too, due to a strong sensitivity to the small S_{02} (T_{02}). Actually the experimental phase shifts of Keller and Haeberli[13] are in this region much nearer to both of the two theoretical sets, than those of Grüebler et al., given in fig. 1.

It should be remembered, however, that our main task is not to give a particularly good fit to the deuteron-alpha scattering phase shifts (therefore also no variation of the potentials was attempted), but rather to use this as a test case for a method with a much wider range of applications.

6. CONCLUSIONS

As we have seen above, the method of solving the Faddeev equations in coordinate space, which has had so many successes in genuine three particle problems, is by the introduction of the pseudo-potentials, easily extended to the scattering of complex particles. Excellent fits are in this way obtained for low energy deuteron scattering and break-up.

The same method, with slight modifications, were also used for calculating the ground state of ^{6}Li. In this case,

Table 2 The angular momentum components on the left side of the Faddeev equations. The crosses indicate the couplings due to the potentials.

[λ	(S	(L	σ)	j)	Σ]	J	Kind	Potentials								
0	1/2	0	1/2	1/2	1	1+	BETA	X	0	0	0	0	0	0	0	0
1	1/2	1	1/2	1/2	0	1+	BETA	0	X	0	0	0	0	0	0	0
1	1/2	1	1/2	1/2	1	1+	BETA	0	0	X	0	0	0	0	0	0
1	1/2	1	1/2	3/2	1	1+	BETA	0	0	0	X	0	0	0	0	0
1	1/2	1	1/2	3/2	2	1+	BETA	0	0	0	0	X	0	0	0	0
0	0	0	1	1	1	1+	ALPHA	0	0	0	0	0	X	X	0	0
0	0	2	1	1	1	1+	ALPHA	0	0	0	0	0	X	X	0	0
2	0	0	1	1	1	1+	ALPHA	0	0	0	0	0	0	0	X	X
2	0	2	1	1	1	1+	ALPHA	0	0	0	0	0	0	0	X	X

The two incoming channels are Nos. 6, 8.
Pauli-blocked channels are No. 1.

a complete agreement with the earlier calculations of Bang and Gignoux[6] was obtained. Since the method of taking into account the exclusion principle, presented here, is much simpler than that of ref[8], it should be preferred in future calculations.

The perspectives for such future calculations are quite wide. Among bound state systems, the states of any nucleus, consisting of a closed shell plus two nucleons, may be calculated by the present methods. Since such nuclei play a large role as test cases for nuclear transfer reactions, and as model cases for nucleon clusterization phenomena, a number of attempts have been made to obtain precise wave functions for such cases[10] like ^{18}O, ^{42}Ca, etc.

However, these calculations, known under the name of extended basis shell model calculations, meet with particularly great difficulties when the two particles outside the closed shell are a neutron and a proton, which in the periphery of the nucleus will form a deuteron cluster. Nuclei of such a type are ^{18}F, ^{42}Ti, etc. In such cases, the present method should be able to lead to correct answers.

The most interesting example is, though the scattering of deuterons on closed shell nuclei, including break-up and stripping processes. As an example, one may think of $^{16}O + d \rightarrow {}^{16}O + d$, $^{17}O + p$, $^{17}F + n$ and $^{16}O + n + p$.

It is obvious that the description of such scattering processes is more complicated than that of $^4He + d$, given here. However, the difficulty is not of principle, but rather like also for the bound state cases, a question of including sufficiently many componenets of different angular momenta in the calculations. With the present generation of computers, this seems feasible, and such calculations are actually under preparation.

In many cases, the usual DWBA calculations may of course be sufficient, but the main problem seems just to be to establish the limits of their applicability.

Also, in the calculation of break up cross sections, the distorted wave methods meet with particular difficulties, which one may hope to overcome with the present method.

In most reactions with heavier nuclei, the neglect of core degrees of freedom, inherent in the few body treatment may be dubious. For the cases mentioned above, this is hopefully not the case. Anyhow, many approximate ways of taking the core degrees of freedom into account exist, whereas the three body aspects of nuclear reactions as well as the local clusterization phenomena are sparsely treated in literature.

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Рассеяние и развал дейтрона на α-частице
в реалистической модели трех тел

Исследуется рассеяние дейтрона на α-частице в модели трех тел с учетом дейтронного канала развала. Численно решается система связанных уравнений Фаддеева в конфигурационном пространстве. Используемые силы являются реалистическими: взаимодействие нуклона с α-частицей взято в виде потенциала Саксона-Вудса и его производной для спин-орбитальных сил. Нуклон-нуклонные силы - типа мягкого кора. В обоих случаях взаимодействия воспроизводят экспериментальные данные при низких энергиях рассеяния /и связанные состояния/. Принцип Паули учтен приближенно, за исключением состояний, в которых один или два нуклона могут находиться в $1s$-состоянии в нуклон-α-частичном потенциале.

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Bang J. et al.

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Scattering and Break up of Deuterons on Alpha Particles
in a Realistic Three-Body Model

Deuteron-alpha particle scattering is calculated in a 3-body model, taking into account the deuteron break-up. The method of calculation is to solve the coupled Faddeev equations in configuration space. The forces are realistic. The nucleon-alpha interaction consists of a Woods-Saxon potential and a Woods-Saxon derivative spin-orbit. The nucleon-nucleon force is of the supersoft core type. In both cases the interactions reproduce low energy scattering /and Bound state/ data. The Pauli principle is taken into account in an approximate way, excluding the states where one or two nucleons are bound in $1s$-states by the nucleon-alpha interaction.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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