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**ON SURFACE CLUSTERING  
AND PAULI PRINCIPLE EFFECTS  
IN ALPHA DECAY**

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## 1. INTRODUCTION

The main part of alpha decay width calculations is performed in the frame of two variants of theory which lead, respectively, to the R matrix<sup>/1,2/</sup> and to the integral formulas<sup>/3,4,5/</sup>. These two variants of the theory are analysed in sections 2,3 and it is shown that the essential problem with them is the fact that the model wave functions used in calculations do not describe correctly the nuclear surface. Then, having in mind that at the nuclear surface the Pauli principle effects and clustering must be important, a way to account for them is proposed. An integro-differential equation is obtained for the alpha particle-final nucleus relative motion wave function in section 4. A method for calculating the nonlocal potentials appearing in this equation and the first numerical results for these potentials are given in section 5.

## 2. THE R MATRIX FORMULA FOR ALPHA DECAY WIDTHS

The asymptotical behaviour of the wave function of a spherical alpha emitter with only one alpha channel open must have the form

$$\Psi_{I_i M_i} \rightarrow \sqrt{\frac{\Gamma \cdot k}{2Q}} \hat{G} \{ \chi_\alpha [\Psi_f^{I_f M_f} Y_{LM}]_{I_i M_i} \frac{G_L + iF_L}{R_\alpha} \}. \quad (1)$$

Here the functions  $\chi_\alpha$ ,  $\Psi_f^{I_f M_f}$  describe the internal motion of alpha particle and final nucleus,  $Y_{LM}$  is the spherical harmonic describing their angular relative motion, while  $G_L$  and  $F_L$  are the irregular and regular Coulomb wave functions depending on the distance  $R_\alpha$  between the centres of mass of the fragments. The constant  $\sqrt{\Gamma k/2Q}$  is obtained from the continuity equation, considering the state  $\Psi_{I_i M_i}^{I_i M_i}$  quasistationary, and contains the alpha width  $\Gamma$ , the c. of m. energy of the alpha particle  $Q$  and the corresponding wave number  $k$ . The operator  $\hat{G}$  stands for the antisymmetrization between the  $N-2$  neutrons and  $Z-2$  protons of the final nucleus and the 2 protons and 2 neutrons in the alpha particle.

For spontaneous alpha decay  $\Gamma \ll Q$  and, consequently, under the barrier, the function  $F_L$  is by  $10 \div 20$  orders of magnitude

smaller than the function  $G_L$ . Thus, the imaginary part of function  $\Psi^{I_i M_i}$  gives very small contributions to the matrix elements and can be neglected. At the same time, potentials acting between alpha particle and final nucleus can be considered real.

In the frame of the R-matrix theory for alpha decay<sup>1,2/</sup> it is stated that the Hamiltonian of the A-particle system can be described for overlapping fragments ( $R_a \leq R_c$ ) by the shell model and for nonoverlapping fragments ( $R_a \geq R_c$ ) by the optical model

$$H = \begin{cases} H_{sm} & \text{for } R_a \leq R_c \\ H_{opt} = H_a + H_f + T_{R_a} + V_{opt}(R_a) & \text{for } R_a \geq R_c. \end{cases} \quad (2)$$

Here the parameter  $R_c$  is the channel radius. Obviously, the results must not depend on this parameter, i.e., there must exist a region where the logarithmic derivatives of the shell model wave function

$$(H_{sm} - E)\Psi_{sm} = 0 \quad (3)$$

and of the optical model wave function

$$(H_{opt} - E)\{ \chi_a [\Psi_f^{I_f M_f} Y_{LM}]_{I_i M_i} \frac{g_L}{R_a} \} \\ g_L \rightarrow G_L \quad (4)$$

are equal. If this matching condition would be satisfied on a sphere of radius  $R_c$ , the alpha decay widths could be obtained equating the amplitudes of the model functions on this sphere:

$$\Psi_{sm}^{I_i M_i} |_{R_a=R_c} = \sqrt{\frac{\Gamma k}{2Q}} \{ \chi_a [\Psi_f^{I_f M_f} Y_{LM}]_{I_i M_i} \frac{g_L}{R_a} \} |_{R_a=R_c} \quad (5)$$

But, concrete calculations show that such a sphere does not exist. Eq. (5), projected on the channel wave function  $\chi_a [\Psi_f^{I_f M_f} Y_{LM}]_{I_i M_i}$ , reads

$$J_{if}^L(R_c) = \sqrt{\frac{\Gamma k}{2Q}} \frac{g_L(R_c)}{R_c} \left[ \binom{N-2}{2} \binom{Z-2}{2} \right]^{-1/2}, \quad (6)$$

where the function  $J_{if}^L$  is the overlap integral

$$J_{if}^L(R_c) = \langle \chi_a [\Psi_f^{I_f M_f} Y_{LM}]_{I_i M_i} | \Psi_{sm}^{I_i M_i} \rangle |_{R_a=R_c} \quad (7)$$

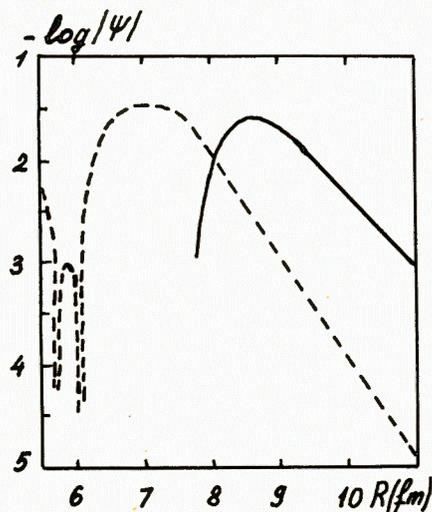


Fig.1. The overlap integral  $R * J_{if}^L$  and the function  $g_L$  (continuous line) for the nucleus  $^{206}\text{Rn}$ . The overlap integral is calculated with account for pairing correlations<sup>5/</sup>.

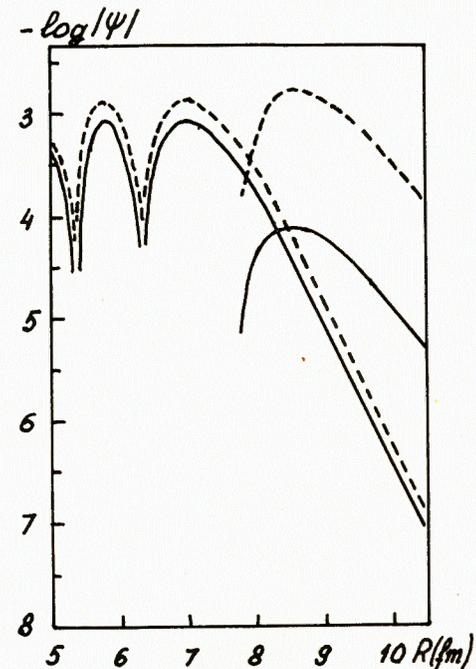


Fig.2. The overlap integral  $R * J_{if}^L$  and the function  $g_L$  for the transitions  $1^- \rightarrow 2^- (L=2)$  and  $9^- \rightarrow 2^- (L=8)$  (continuous line) for the nucleus  $^{208}\text{Bi}$ . The overlap integral is calculated with configuration mixing<sup>4/</sup>.

and the factor  $\left[ \binom{N-2}{2} \binom{Z-2}{2} \right]^{-1/2}$  appears as a consequence of

the antisymmetrization. This is the only effect of the antisymmetrization operator  $\hat{Q}$  for channel radii  $R_c$  where the fragments are well separated. Some typical cases for functions  $R_a * J_{if}^L$

and  $\sqrt{\frac{\Gamma_{exp} k}{2Q}} * g_L$  are represented in fig.1 and 2. It is obvious

that there are no points  $R_a = R_c$  at which they would match. This means that if the shell model describes well the system in the interior and the optical model at the exterior, there exist an intermediate region at the nuclear surface where none of them does work. (The dismatching demonstrated here leads to the strong channel radius dependence of alpha decay widths calculated by the R matrix formula). The shell model Hamiltonian in eq. (2) is unable to account for surface alpha clustering,

while the optical model Hamiltonian in eq. (2) does not account for Pauli principle effects.

An attempt was made to account for Pauli principle effects in the R matrix formalism<sup>/6/</sup>. It was stated that the channel radius  $R_c$  must lie in the region where the fragments are overlapping. For such distances  $R_c$  the effect of the antisymmetrization operator in eq. (5) is some more complicated. Projection of eq. (5) on the channel wave function yields

$$J_{if}^L(R) = \sqrt{\frac{\Gamma k}{2Q}} \frac{1}{R} \int N_f^L(R, R') g_L(R') dR' = \sqrt{\frac{\Gamma k}{2Q}} \hat{N} g_L, \quad (8)$$

where

$$N_f(R, R') = \langle \delta(R - R_a) \chi_\alpha^{I_f M_f} Y_{LM} ]_{I_i M_i} | \hat{Q} \{ \delta(R' - R_a) \chi_\alpha^{I_f M_f} Y_{LM} ]_{I_i M_i} \} \rangle \quad (9)$$

(For distances where the fragments are well separated function  $N_f^L(R, R')$  takes on the form  $\delta(R - R')$ . Then, it was proposed<sup>/6/</sup> to rewrite formula (8) as

$$\hat{N}^{-1/2} J_{if}^L = \hat{N}^{1/2} g_L \quad (10)$$

and from qualitative considerations to assume that  $\hat{N}^{1/2} g_L$  is the solution of the usual radial optical model Hamiltonian with local potentials. Here the difficult problem of calculating function  $N_f^L(R, R')$  arises. On the other hand, this procedure may contain some principal and mathematical indeterminations<sup>/7/</sup>. At the time, the problem of matching internal and external wave functions is not solved. Function  $\hat{N}^{-1/2} J_{if}^L$  falls down too rapidly at the nuclear surface, just like  $J_{if}^L$  does. (See figs. 1 and 2 of ref.<sup>/8/</sup>).

### 3. THE INTEGRAL FORMULA FOR ALPHA DECAY WIDTH

Another way to calculate alpha decay widths is to use the integral formula<sup>/3/</sup>

$$\Gamma = 2\pi \binom{N-2}{2} \binom{Z-2}{2} | \langle \chi_\alpha^{I_f M_f} Y_{LM} ]_{I_i M_i} \mathcal{F}_L | V_{\alpha f}^{\text{nucl}} | \Psi^{I_i M_i} \rangle |^2 \quad (11)$$

Here  $\mathcal{F}_L$  is the regular Coulomb wave function normalized to  $\delta$ -function in energy, while the potential  $V_{\alpha f}^{\text{nucl}}$  stands for the nuclear part of the alpha particle-final nucleus interaction.

This formula was obtained<sup>/3/</sup> comparing the wave function of the decaying nucleus with the resonant scattering wave functions. A simplified way to obtain this formula is the following. Let us rewrite the Hamiltonian of the system in the form

$$H = H_\alpha + H_f + T_{R_\alpha} + V_{\alpha f}^{\text{nucl}} + V_{\alpha f}^{\text{coul}}(R_\alpha). \quad (12)$$

The only approximation here is that the Coulombian part of the interaction is considered central. Projection of the corresponding Schrödinger equation on the channel, gives

$$\left( \frac{\hbar^2}{2m} \left( -\frac{d^2}{dR_\alpha^2} + \frac{L(L+1)}{R_\alpha^2} \right) + V_{\alpha f}^{\text{coul}}(R_\alpha) - Q \right) R_\alpha J_{if}^L = R_\alpha \langle \chi_\alpha^{I_f M_f} Y_{LM} ]_{I_i M_i} | V_{\alpha f}^{\text{nucl}} | \Psi^{I_i M_i} \rangle. \quad (13)$$

Writing along the equation for the regular Coulomb wave function  $\mathcal{F}_L$ , multiplying the first equation (13) by  $R_\alpha \mathcal{F}_L$ ; the second, by  $R_\alpha J_{if}^L$ , subtracting them and integrating over  $R_\alpha$  formula (11) is obtained, under the condition that the asymptotical condition (1) is satisfied (the imaginary part is neglected), i.e., that the overlap integral has the correct asymptotical behaviour

$$J_{if}^L \longrightarrow \sqrt{\frac{\Gamma k}{2Q}} \frac{G_L}{R} \binom{N-2}{2}^{-1/2} \binom{Z-2}{2}^{-1/2}. \quad (14)$$

As this formula does not depend on any free parameters, it permitted to perform consistent calculations for a large number of spherical alpha emitters<sup>/4,5/</sup>. But, in these calculations, the functions  $\Psi^{I_i M_i}$  for the alpha emitters were approximated by shell model wave functions. It was shown<sup>/5/</sup> that, working with Woods-Saxon one-nucleon functions and taking into account nucleon-nucleon correlations, the relative alpha decay width values are well described for a large number of nuclei and for all types of alpha transitions. But, the absolute values of the theoretical alpha decay widths are by a factor of 100 smaller than the experimental ones. In order to understand this phenomenological result, let us rewrite formula (11) (in which function  $\Psi^{I_i M_i}$  is approximated by  $\Psi_{\text{sm}}^{I_i M_i}$ ) as follows

$$\Gamma = 2\pi \left| \int J_{if}^L(R_\alpha) V_{\alpha f}^{\text{nucl}}(R_\alpha) \mathcal{F}_L(R_\alpha) R_\alpha dR_\alpha \right|^2 \binom{N-2}{2} \binom{Z-2}{2}. \quad (15)$$

Here the approximation  $V_{\alpha f}^{\text{nucl}} = V_{\alpha f}^{\text{nucl}}(R_\alpha)$  is used. It was shown<sup>/4,9/</sup> that such an approximation does not change essentially the results. From formula (15) it is clear that the relative values of alpha decay widths (besides their energetical dependence

which is accounted for via the function  $\mathcal{F}_L$ ) are determined mainly by the position and amplitude of the last maximum of the integral overlap  $J_{if}^L$ . Thus, the fact that relative values are in accordance with experiment means that this function is correct up to this last maximum. In terms of the many body functions, this means that the shell model with nucleon-nucleon correlations approximates well the decaying nucleus function up to the surface. The discrepancy in absolute values can be explained by the same fact as the strong channel radius dependence of the R matrix results: the function  $J_{if}^L$  falls down too rapidly at the nuclear surface, i.e., the shell model gives no good description for this region. Thus, we are lead to the same conclusion as in section 2: the shell model wave functions cannot account for surface alpha clustering, but, in addition, we can conclude that they give a good norm for the amplitude of alpha particle preformation probability up to the nuclear surface.

On the other hand, integrating formula (15) from the exterior up to the last point, where the interaction  $V_{af}^{nucl} = V_{opt}(R_a)$ , and consequently using instead of function  $J_{if}^L(R_a)$  the function

with correct asymptotical form  $\sqrt{\frac{\Gamma_{exp} k}{2Q}} \frac{g_L(R_a)}{R_a}$ , it was shown<sup>/10/</sup> that the outer and surface region are very important for the absolute values of alpha decay widths.

#### 4. DESCRIPTION OF THE SURFACE OF HEAVY ALPHA EMITTERS

The analyses given in the preceding sections showed that nor the shell model, neither the optical model can describe the nuclear surface. For a good description of this transitional region, surface alpha clustering and Pauli principle effects must be accounted for. This idea can be argued also by experimental evidence for surface clustering, following from nuclear reaction data<sup>/11/</sup> and by the nuclear matter calculations<sup>/12/</sup> which showed that alpha clustered nuclear matter is energetically favoured for small nucleon densities.

Having in mind all these arguments, let us write the Hamiltonian of the decaying system by adding to the shell model Hamiltonian a term accounting for surface alpha clustering

$$H = H_{sm} + (1 - \frac{\rho}{\rho_0}) V_a, \quad (16)$$

where  $\rho$  is the nucleon density and  $\rho_0$  its value in the centre of the nucleus, while  $V_a$  is the interaction potential forming the alpha particle

$$(\sum_{i=1}^3 T_{\xi_i} + V_a - \epsilon_a) \chi_a = 0. \quad (17)$$

The Hamiltonian (16) reduces to shell model Hamiltonian for the interior region, while in the exterior it can be rewritten as

$$H = H_a + H_f + T_{R_a} + V_{af} - \frac{\rho}{\rho_0} V_a. \quad (18)$$

For great distances  $R_a$  the last term is zero, while  $V_{af}$  averaged on the channel wave function must coincide with the real part of the local alpha-nucleus optical potential.

In order to obtain the solution of the Schrödinger equation with Hamiltonian (16) in the whole space, by diagonalizing it in a shell model basis, states with four particles in the continuum must be included. But, the discussion in section 3 hints to the fact that even the shell model wave functions obtained in usual truncated bases are good approximations up to some point  $R_s$  in the vicinity of the nuclear surface. In the external region the solution of Hamiltonian (16)-(17) coincides with the optical model wave function. In the transitional surface region, since here alpha clustering must take place, it is natural to look for the solution of the Schrödinger equation by means of the resonant group method<sup>/14/</sup> (RGM), i.e.,

$$(H-E) \mathcal{Q} \{ \chi_a [ \Psi_f^{I_f M_f} Y_{LM} ]_{I_i M_i} \frac{u_L(R_a)}{R_a} \} = 0; \quad u_L \rightarrow G_L. \quad (19)$$

If the truncation of the shell model basis for the interior region and the RGM function for the surface are good approximations, we must have in the vicinity of the sphere with radius  $R_s$  the internal and external solutions identical up to a constant, i.e.,

$$\Psi_{sm} |_{R_a=R_s} = \sqrt{\frac{\Gamma k}{2Q}} \mathcal{Q} \{ \chi_a [ \Psi_f^{I_f M_f} Y_{LM} ]_{I_i M_i} \frac{u_L(R_a)}{R_a} \} |_{R_a=R_s}. \quad (20)$$

Then, the alpha decay width  $\Gamma$  should be obtained from the ratio of the overlap integral and the RGM relative motion function at this point, because the channel projection of eq. (20) yields

$$J_{if}^L(R_s) = \sqrt{\frac{\Gamma k}{2Q}} \frac{1}{R_s} \int N_f^L(R, R') u_L(R') dR', \quad (21)$$

where the function  $N_f^L(R, R')$  is defined by eq. (9).

Let us consider for the beginning the case of an alpha emitter in the vicinity of the double magical  $^{208}\text{Pb}$  for which it can be assumed that the Hamiltonian  $H_{sm}$ , as well as  $H_f$ , is well

described by the independent particle shell model. Then,

developing the function  $\chi_\alpha Y_L \frac{u_L(R_\alpha)}{R_\alpha}$  in series of four particle shell model wave functions

$$\Psi_n^{LM} = [ [\Psi_{n_1 \ell_1 j_1} \Psi_{n_2 \ell_2 j_2} ]_{j_{12}}^{(a)} [\psi_{n_3 \ell_3 j_3} \psi_{n_4 \ell_4 j_4} ]_{j_{34}}^{(a)} ]_{LM} \quad (22)$$

(symbols  $n_1, \ell_1, j_1; n_2, \ell_2, j_2$  stand for proton state quantum numbers; while  $n_3, \ell_3, j_3; n_4, \ell_4, j_4$ , for neutron quantum numbers; (a), for two particle antisymmetrization; and the parenthesis, for angular momentum coupling), accounting for the antisymmetrization operator  $\mathcal{Q}$  and taking in mind that  $\Psi_n$  form a complete orthonormalized basis, the RGM equation for the relative motion wave function takes on the form

$$\left[ \frac{\hbar^2}{2m} \left( -\frac{d^2}{dR^2} + \frac{L(L+1)}{R^2} \right) + V(R) + V_C(R) \right] u_L + \int [V_P(R, R') + V_{PC}(R, R')] u_L(R') dR' = 0, \quad (23)$$

where

$$V(R) = \langle \chi_\alpha Y_{LM} | \sum_{i=1}^4 V_i | \chi_\alpha Y_{LM} \rangle \quad (24)$$

is a local potential obtained by averaging the selfconsistent field potentials  $V_i$  and

$$V_C(R) = -\langle \chi_\alpha | V_\alpha | \chi_\alpha \rangle \frac{\rho}{\rho_0} \quad (25)$$

is a local potential following from the account for clustering. The nonlocal potential  $V_P(R, R')$  appears due to the account for the Pauli principle and has the form

$$V_P(R, R') = \sum_{n < F} g_n^L(R) g_n^L(R') \left( \sum_{i=1}^4 \epsilon_i^{n_0} - \sum_{i=1}^4 \epsilon_i^n \right), \quad (26)$$

where the functions  $g_n^L(R)$  are overlap integrals of shell model and alpha particle functions:

$$g_n^L(R) = \langle \chi_\alpha Y_L | \Psi_n \rangle. \quad (27)$$

The sums  $\sum_{i=1}^4 \epsilon_i^n$  and  $\sum_{i=1}^4 \epsilon_i^{n_0}$  stand for the energy of the current four-particle state  $n$  and, respectively, for the four-particle state  $n_0$  just about the Fermi levels of the final nucleus. The second nonlocal potential is a mixed effect of the Pauli principle and clustering

$$V_{PC}(R, R') = RR' \left( 1 - \frac{\rho(R)}{\rho_0} \right) \sum_{n < F} V_n^L(R) g_n^L(R'), \quad (28)$$

where

$$V_n^L(R) = \langle \chi_\alpha Y_{LM} | V_\alpha | \Psi_n^{LM} \rangle. \quad (29)$$

The symbol  $n > F$  in eq. (26) and (28) stands for the complementary sum to that in which all four particles in the state  $n$  are taken over the Fermi surface of the final nucleus.

The solution of the integrodifferential equation (23) with degenerate integral kernels (26) and (28) can be represented in the following form<sup>/13/</sup>

$$u_L = u_L^0 + \sum_{n < F} \langle R g_n | u_L \rangle u_L^n, \quad (30)$$

where  $u_L^0$  is the solution of the homogeneous differential equation with local potentials, while  $u_L^n$  are solutions of the inhomogeneous differential equations

$$\left( \frac{\hbar^2}{2m} \left( -\frac{d^2}{dR^2} + \frac{L(L+1)}{R^2} \right) + V(R) + V_C(R) - Q \right) u_L^n = \quad (31)$$

$$= R \left( \sum_{i=1}^4 \epsilon_i^{n_0} - \sum_{i=1}^4 \epsilon_i^n \right) g_n^L(R) + R \left( 1 - \frac{\rho(R)}{\rho_0} \right) V_n^L(R).$$

The solutions  $u_L^n$  have in the internal region a form resembling to the inhomogeneity, while at the surface they turn into functions with the correct boundary condition ( $u_L \rightarrow G_L$ ) for the quasistationary state. Thus, one can expect that the sum in eq. (30) can yield a continuous transition from the overlap integral  $J_{if}^L$  to the radial function of relative motion with correct asymptotic form (see figs. 1 and 2).

## 5. FORM OF THE NONLOCAL POTENTIALS

To solve the integrodifferential equation (23)-(29) by the method based on eqs. (30)-(31) is quite cumbersome since the sum over four particle states  $n < F$  contains a very large number of slowly converging terms.

The same problem arises in the calculation of the nonlocal potentials  $V_P(R, R')$  and  $V_{PC}(R, R')$ . Nonlocal potentials of this type were calculated only for light nuclei<sup>/14/</sup> and then, instead of solving the integrodifferential equation, methods

of projecting out the states forbidden by the Pauli principle were used, i.e., the orthogonality condition model<sup>/15/</sup> or the method of orthogonalizing potentials<sup>/16/</sup>.

In the present case we can make use of the fact that for an internal alpha particle wave function of the gaussian type, as deduced from electron scattering on the <sup>4</sup>He nucleus and usually worked within alpha decay calculations<sup>/1,2,4,5/</sup>, the overlap integrals  $g_n^L(R)$  may be represented as a finite sum of harmonic oscillator functions  $\mathcal{R}_{NL}$

$$g_n^L(R) = \sum_N C_{NL}^n \mathcal{R}_{NL}(2R). \quad (32)$$

For the coefficients  $C_{NL}^n$  analytical formula can be obtained. They are a particular case of the Talmi-Moshinsky coefficients generalized for four particles.

Thus, the formula for the nonlocal potential  $V_P(R, R')$  takes on the form

$$V_P(R, R') = RR' \sum_{NN'} M_{NN'}^E \mathcal{R}_{NL}(2R) \mathcal{R}_{N'L}(2R'), \quad (33)$$

$$M_{NN'}^E = \sum_{n < F} C_{NL}^n C_{N'L}^n \left( \sum_{i=1}^4 \epsilon_i^{n_0} - \sum_{i=1}^4 \epsilon_i^n \right). \quad (34)$$

Neglecting the spin-orbit interaction for nucleus we have  $\sum_{i=1}^4 \epsilon_i^{n_0} - \sum_{i=1}^4 \epsilon_i^n = (N_{max} - N_{max}^n) \hbar \omega$ , where  $N_{max}$  is defined by the relation  $\sum_{i=1}^4 (2n_i + l_i) = 2N_{max} + L$ .

In order to evaluate the potentials following from the account for clustering, for the interaction  $V_a$  a sum of two-nucleon potentials was used and the nucleon-nucleon potentials were approximated by simple potential wells<sup>/17/</sup>. Thus, we get

$$V_C(R) = V_0 \frac{\rho(R)}{\rho_0}, \quad (35)$$

$$V_{PC}(R) = V_0 \left(1 - \frac{\rho(R)}{\rho_0}\right) RR' \sum_{NN'} M_{NN'} \mathcal{R}_{NL}(2R) \mathcal{R}_{N'L}(2R'), \quad (36)$$

$$M_{NN'} = \sum_{n < F} C_{NL}^n C_{N'L}^n, \quad (37)$$

were  $V_0$  is about - 84 MeV.

Calculations performed for <sup>208</sup>Pb as a final nucleus gave the results represented in fig.3. For the nonlocal potentials the diagonal, i.e.,  $R=R'$  part is represented. Summation in eq. (34) and (37) was done up to  $N=N'=16$ . This gave a good convergence for the  $V_{PC}$  potential and for the internal part, up to 6 fm, for the potential  $V_P$ . Addition of further terms will make the surface well in  $V_P$  some more deep.

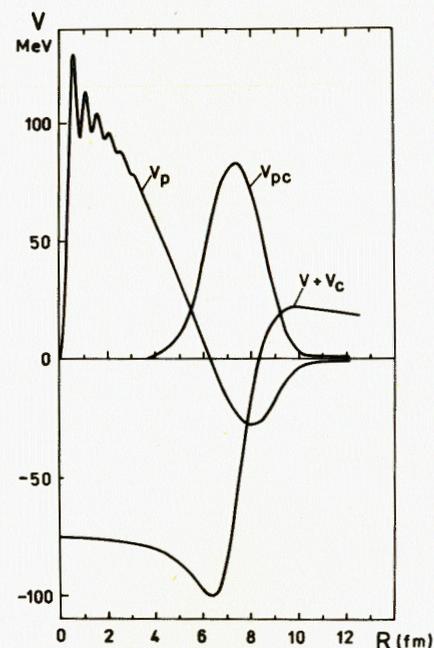


Fig.3. The local and nonlocal potentials (for  $R=R'$ ) of alpha-nucleus interactions calculated for a nucleus in vicinity of <sup>208</sup>Pb.

The potential  $V_P$  following from the Pauli principle forms a wall inside the nucleus and a small negative well at the surface also. Thus, the account for the Pauli principle will transform the alpha-nucleus potential into a surface well. The local term following from clustering  $V_C$  renormalize the well depth in the interior region making it more shallow. (Potential  $V$  has a depth of about 207 MeV). The nonlocal term  $V_{PC}$  following from clustering works as a wall in the

surface region, dissipating the alpha particle. It compensates also the surface well formed by the Pauli principle in such a way that the position and amplitude of the barrier, which is experimentally determined from elastic scattering, will not be changed by the introduction of the nonlocal terms. But, it must be underlined that the nonlocality of potentials  $V_P$  and  $V_{PC}$  is strong and, as it can be seen from formulas (30) and (31), it can change chaotically the form of the relative motion function  $u_L$  at the surface.

## 6. CONCLUSIONS

As far, a model is proposed to account for surface clustering in heavy nuclei and a way of calculating exactly nonlocal potentials arising from Pauli principle and clustering is worked out for nuclei in the vicinity of <sup>208</sup>Pb.

The next step in this work will be to solve the integro-differential equation and to verify if the internal and external functions are matching, i.e., if the truncation of internal and external bases for solving the Schrödinger equation is adequate. Then, the form of the clustering potential introduced in the Hamiltonian (16) must be verified in connection with other phenomena for which surface clustering can be important, e.g., in elastic scattering of alpha particles on heavy nuclei and alpha transfer reactions.

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#### REFERENCES

1. Mang H.J. Ann.Rev.Sci., 1964, 14, p. 1.
2. Zeh H.D. Z.Phys., 1963, 175, p. 490.
3. Kadmsky S.G., Furman V.I. JINR, P4-8729, Dubna, 1975.
4. Furman V.I. et al. JINR, E4-11286, E4-11287, Dubna, 1978.
5. Holan S. Studii si cercetari de fizica, 1976, 8, p. 767.
6. Fliessbach T. Z.Phys., 1975, A272, p. 39.
7. Bulgac A. et al. JINR, E4-81522, Dubna, 1981.
8. Fliessbach T., Mang H.J. Nucl.Phys., 1976, A263, p. 75.
9. Dumitrescu T. et al. Izv. Ak.Nauk SSSR, seria fiz., 1978, 42, p. 1982.
10. Kadmsky S.G. et al. JINR, P4-8730, Dubna, 1975.
11. Hodgson P.E. "Growth Points in Nuclear Physics", Pergamon Press, 1980. vol. 1. 3.
12. Brink D.M., Castro J.J. Nucl.Phys., 1973, A216, p. 109.
13. Collatz L. "The Numerical Treatment of Differential Equations", Springer-Verlag, Berlin, 1960.
14. Wildermuth K., Tang Y.C. "A Unified Theory of the Nucleus", Vieweg-Braunschweig, 1977.
15. Saito S. Progr. Theoret Phys., 1969, 41, p. 705.
16. Kukulín V.J., Neudatchin V.G., Smirnov Yu.F. Particles and Nuclei. M., 1979, vol. 10, part 6.
17. Bohr A., Mottelson B. "Nuclear Structure" W.A. Benjamin, Inc. 1969. vol. 1. p. 242.

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Поверхностная кластеризация и эффекты принципа Паули в  $\alpha$ -распаде

Отмечена важность правильного описания поверхностной области ядра при рассмотрении  $\alpha$ -распада. Предложена модель, явно учитывающая  $\alpha$ -кластеризацию и эффекты принципа Паули, существенные в поверхностной области. Разработан способ решения основного интегродифференциального уравнения модели на основе использования осцилляторного оболочечного базиса и метода Колатца. Первые численные результаты получены для нелокального потенциала взаимодействия альфа-частицы с дочерним ядром.

Работа выполнена в Лаборатории нейтронной физики ОИЯИ.

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On Surface Clustering and Pauli Principle Effects in Alpha Decay

The importance of the correct description of nuclear surface region in alpha decay calculations is pointed out. A model is proposed for taking into account surface clustering and Pauli principle effects which are essential in this region. A method for calculating the nonlocal potentials appearing in this model is worked out and the first numerical results for them are reported.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research, Dubna 1983