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INCLUSION OF NEUTRON-PROTON PHONONS INTO THE QUASIPARTICLE-PHONON NUCLEAR MODEL

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1. INTRODUCTION

In recent years the existence of the Gamow-Teller resonance has been confirmed experimentally, and the giant charge-exchange resonances of the electric type have been discovered in atomic nuclei [1,2]. Just a part of the strength of these states is observed experimentally as compared to the values following from the relevant sum rules. The strength fragmentation due to the coupling with more complex states and with the Δ-isobar-nucleon hole configuration [3-5] is thought to be one of the reasons for the missing strength of the charge-exchange resonances.

The strength fragmentation of the charge-exchange states can be studied within the quasiparticle-phonon nuclear model [6-8]. In ref. [9] the neutron-proton phonons are introduced to describe the $T_>$ giant dipole resonance. The characteristics of the charge-exchange states are correctly described in the RPA using the neutron-proton phonons. To describe the charge-exchange states in more detail, one should develop a mathematical apparatus for calculating the fragmentation of neutron-proton phonons.

In this paper we show how the neutron-proton (np) phonons are introduced into the quasiparticle-phonon nuclear model. A system of basic equations for even spherical nuclei including np phonons is derived. It is demonstrated that a wide class of diagrams is summed. The transition to an approximate system of equations used in the numerical calculations of the fragmentation of np phonons is realized.

2. THE MODEL HAMILTONIAN

The Hamiltonian of the quasiparticle-phonon nuclear model includes the average field as the Saxon-Woods potential, the pairing interactions, the multipole-multipole and spin-multipole- spin-multipole isoscalar and isovector, including charge-exchange interactions. The one-phonon states calculated in the RPA are used as basis states. The multipole forces are used to generate phonons with $J^\pi = 1^-, 2^+, 3^-, \ldots, 7^-$, and the spin-multipole to generate phonons with $J^\pi = 1^+, 2^+, 3^+, \ldots 7^+$. The neutron-proton multipole phonons are generated by the following part of the isovector interaction:

$$\kappa_1^{(\lambda)} (t_1^{(+)} t_2^{(-)} + t_1^{(-)} t_2^{(+)}) r_1^\lambda Y_{\lambda\mu}(\theta_1\phi_1) r_2^\lambda Y_{\lambda-\mu}(\theta_2\phi_2) \quad (1)$$

the radial dependence may be arbitrary, $t^{(\pm)} = t_x \pm i t_y$. Alongside with the phonon creation operator of multipolarity [8,10] λ

$$Q^+_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} \{\psi^{\lambda i}_{jj'} A^+(jj';\lambda\mu) - (-)^{\lambda-\mu} \phi^{\lambda i}_{jj'} A(jj';\lambda-\mu)\}, \quad (2)$$

where

$$A^+(jj';\lambda\mu) = \sum_{m,m'} \langle jm j'm' | \lambda\mu \rangle \alpha^+_{jm} \alpha^+_{j'm'} ,$$

and α^+_{jm} is the quasiparticle creation operator, we introduce the neutron-proton (np) phonon creation operator

$$\Omega^+_{\lambda\mu i} = \sum_{j_p j_n} \{\psi^{\lambda i}_{j_p j_n} A^+(j_p j_n;\lambda\mu) - (-)^{\lambda-\mu} \phi^{\lambda i}_{j_p j_n} A(j_p j_n;\lambda-\mu)\}, \quad (3)$$

where

$$A^+(j_p j_n;\lambda\mu) = \sum_{m_p m_n} \langle j_p m_p j_n m_n | \lambda\mu \rangle \alpha^+_{j_p m_p} \alpha^+_{j_n m_n} . \quad (4)$$

The single-particle states are specified by the quantum numbers jm, $j_p m_p$ for protons and $j_n m_n$ for neutrons. The np phonon operators satisfy the commutation relations

$$[\Omega_{\lambda'\mu' i'}, \Omega^+_{\lambda\mu i}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'} \sum_{j_p j_n} (\psi^{\lambda' i'}_{j_p j_n} \psi^{\lambda i}_{j_p j_n} - \phi^{\lambda' i'}_{j_p j_n} \phi^{\lambda i}_{j_p j_n}) -$$

$$- \sum_{\substack{j_n j'_n \\ m_n m'_n}} \ell^{j_n j'_n}_{m_n m'_n}(\lambda' i', \lambda i)\, \alpha^+_{j_n m_n} \alpha_{j'_n m'_n} - \quad (5)$$

$$- \sum_{\substack{j_p j'_p \\ m_p m'_p}} \ell^{j_p j'_p}_{m_p m'_p}(\lambda' i', \lambda i)\, \alpha^+_{j_p m_p} \alpha_{j'_p m'_p} ,$$

where

$$\ell^{j_n j'_n}_{m_n m'_n}(\lambda' i', \lambda i) =$$

$$= \sum_{j_p m_p} \{\psi^{\lambda' i'}_{j_p j'_n} \psi^{\lambda i}_{j_p j_n} \langle j_p m_p j'_n m'_n | \lambda'\mu' \rangle \langle j_p m_p j_n m_n | \lambda\mu \rangle -$$

$$- (-)^{\lambda+\lambda'-\mu-\mu'} \phi^{\lambda' i'}_{j_p j_n} \phi^{\lambda i}_{j_p j_n} \langle j_p m_p j'_n m'_n | \lambda' - \mu' \rangle \langle j_p m_p j_n m_n | \lambda - \mu \rangle \} .$$

The RPA equations for the energies $\Omega_{\lambda i}$ of the np phonons have the following form:

$$\mathcal{F}_{np}(\Omega_{\lambda i}) \equiv (1 - X^{\lambda i}_{11}(\Omega_{\lambda i}))(1 - X^{\lambda i}_{22}(\Omega_{\lambda i})) - (X^{\lambda i}_{12}(\Omega_{\lambda i}))^2 ,$$

$$X^{\lambda i}_{11}(\Omega_{\lambda i}) = \frac{2\kappa_1^{(\lambda)}}{2\lambda + 1} \sum_{j_p j_n} \frac{(f^{\lambda}(j_p j_n) u^{(+)}_{j_p j_n})^2 \epsilon_{j_p j_n}}{\epsilon^2_{j_p j_n} - \Omega^2_{\lambda i}} ,$$

$$X^{\lambda i}_{22}(\Omega_{\lambda i}) = \frac{2\kappa_1^{(\lambda)}}{2\lambda + 1} \sum_{j_p j_n} \frac{(f^{\lambda}(j_p j_n) u^{(-)}_{j_p j_n})^2 \epsilon_{j_p j_n}}{\epsilon^2_{j_p j_n} - \Omega^2_{\lambda i}} , \qquad (6)$$

$$X^{\lambda i}_{12}(\Omega_{\lambda i}) = - \frac{2\kappa_1^{(\lambda)}}{2\lambda + 1} \sum_{j_p j_n} \frac{(f^{\lambda}(j_p j_n))^2 u^{(+)}_{j_p j_n} u^{(-)}_{j_p j_n} \Omega_{\lambda i}}{\epsilon^2_{j_p j_n} - \Omega^2_{\lambda i}} ,$$

$$\psi^{\lambda i}_{j_p j_n} = \frac{\sqrt{\kappa_1^{(\lambda)}}}{\sqrt{\mathcal{Y}_{\lambda i}}} \frac{f^{\lambda}(j_p j_n)}{\epsilon_{j_p j_n} - \Omega_{\lambda i}} \frac{1}{\sqrt{2\lambda + 1}} (u^{(+)}_{j_p j_n} - y^{\lambda i} u^{(-)}_{j_p j_n}) , \qquad (7)$$

$$\phi^{\lambda i}_{j_p j_n} = \frac{\sqrt{\kappa_1^{(\lambda)}}}{\sqrt{\mathcal{Y}_{\lambda i}}} \frac{f^{\lambda}(j_p j_n)}{\epsilon_{j_p j_n} + \Omega_{\lambda i}} \frac{1}{\sqrt{2\lambda + 1}} (u^{(+)}_{j_p j_n} + y^{\lambda i} u^{(-)}_{j_p j_n}) , \qquad (8)$$

$$\mathcal{Y}^{-1/2}_{\lambda i} = \left\{ \frac{1 - X^{\lambda i}_{22}(\Omega_{\lambda i})}{- \frac{\partial}{\partial \Omega} \mathcal{F}_{np}(\Omega) |_{\Omega = \Omega_{\lambda i}}} \right\}^{1/2} ,$$

$$y^{\lambda i} = \frac{X^{\lambda i}_{12}(\Omega_{\lambda i})}{1 - X^{\lambda i}_{22}(\Omega_{\lambda i})} = \frac{1 - X^{\lambda i}_{11}(\Omega_{\lambda i})}{X^{\lambda i}_{12}(\Omega_{\lambda i})} . \qquad (9)$$

Here $\epsilon_{j_p j_n} = \epsilon_{j_p} + \epsilon_{j_n}$, ϵ_j is the quasiparticle energy, $u^{(\pm)}_{jj'} = u_j v_{j'} \pm u_{j'} v_j$, $v^{(\pm)}_{jj'} = u_j u_{j'} \pm v_j v_{j'}$. Formulae for the spin-multipole np phonons are the same. The only difference is that instead of the reduces matrix elements of the multipole operators

$$f^{\lambda}(j_p j_n) = \langle j_p || i^{\lambda} r^{\lambda} Y_{\lambda} t^{(-)} || j_n \rangle \qquad (10)$$

they contain the reduced matrix elements of the spin-multipole operators

$$f^{L-1,L}(j_p j_n) = \langle j_p || i^L r^{L-1} [Y_{L-1} \sigma]_L t^{(-)} || j_n \rangle . \qquad (10')$$

Taking into account the solutions of the secular equations for the one-phonon states, the QPM Hamiltonian is as follows:

$$H_M = \sum_{jm} \epsilon_j a^+_{jm} a_{jm} + H_{Mv} + H_{Mvq} + H_{sv} + H_{svq} + H_{cMv} + H_{cMvq} + H_{csv} + H_{csvq} , \qquad (11)$$

the explicit form of H_{Mv}, H_{Mvq}, H_{sv} and H_{svq} is given in refs. [8,10].

$$H_{cv} = - \sum_{\lambda \mu i i'} \frac{1 + y^{\lambda i} y^{\lambda i'}}{\sqrt{\mathcal{Y}_{\lambda i} \mathcal{Y}_{\lambda i'}}} \Omega^+_{\lambda \mu i} \Omega_{\lambda \mu i'} , \qquad (12)$$

$$H_{cvq} = - \sum_{\lambda \mu i} \sqrt{\frac{\kappa_1^{(\lambda)}}{\mathcal{Y}_{\lambda i}}} \sum_{j_p j_n} \frac{f^{\lambda}(j_p j_n)}{\sqrt{2\lambda + 1}} \{ [u_{j_p} u_{j_n} (-)^{j_n - j_p - \mu} B(j_n j_p ; \lambda - \mu) - \qquad (13)$$

$$- (-)^{\lambda - \mu} v_{j_p} v_{j_n} B(j_p j_n ; \lambda - \mu)] [(1 + y^{\lambda i}) \Omega^+_{\lambda \mu i} + (-)^{\lambda + \mu} (1 - y^{\lambda i}) \Omega_{\lambda - \mu i}] + h.c. \} ,$$

where

$$B(j_p j_n ; \lambda \mu) = \sum_{m_p m_n} (-)^{j_n + m_n} \langle j_p m_p j_n m_n | \lambda \mu \rangle a^+_{j_p m_p} a_{j_n - m_n} . \qquad (14)$$

The terms H_{csv} and H_{csvq} differ from (12) and (13) by the change of (10) by (10').

The model Hamiltonian is constructed so that all the operators $A(jj'; \lambda\mu)$, $A^+(jj'; \lambda\mu)$, $A(j_p j_n ; \lambda\mu)$, and $A^+(j_p j_n ; \lambda\mu)$ are expressed through the phonon operators, and the quasiparticle operators enter only in the form $B(jj'; \lambda\mu)$, $B(j_p j_n ; \lambda\mu)$, and $B(j_n j_p ; \lambda\mu)$. As well as in ref. [10], such a construction of the Hamiltonian overcomes the difficulties with the double counting. In the nuclear field theory [11] a special procedure has been developed for this purpose.

3. GENERAL FORM OF EQUATIONS DESCRIBING THE FRAGMENTATION OF NEUTRON-PROTON PHONONS

Now we get in the most general form the system of equations of the quasiparticle-phonon nuclear model for the description of the strength distribution of collective charge exchange states. The wave function of a doubly even spherical nucleus is taken in the form

$$\Psi_{\nu}(JM) = \{ \sum_i R_i(J\nu) \Omega^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \sum_{\mu_1 \mu_2} \langle \lambda_1 \mu_1 \lambda_2 \mu_2 | JM \rangle \Omega^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2} \} \Psi_0 , \qquad (15)$$

where Ψ_0 is the ground state wave function of a doubly even nucleus. The normalization condition is

$$(\Psi_\nu^*(JM)\,\Psi_\nu'(JM)) = 1 = \sum_i (R_i(J\nu))^2 + \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\mu))^2 + \tag{16}$$

$$+ \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_1' i_1' \lambda_2' i_2'}} P^{\lambda_1' i_1'}_{\lambda_2' i_2'}(J\nu)\, P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\, K^J_{\Omega Q}(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda_1 i_1, \lambda_2 i_2).$$

Here

$$\sum_{\mu_1\mu_2\mu_1'\mu_2'} \langle \lambda_1'\mu_1'\lambda_2'\mu_2' \mid JM\rangle \langle \lambda_1\mu_1\lambda_2\mu_2 \mid JM\rangle \times$$

$$\times \langle Q_{\lambda_2'\mu_2' i_2'}\, \Omega_{\lambda_1'\mu_1' i_1'}\, \Omega^+_{\lambda_1\mu_1 i_1}\, Q^+_{\lambda_2\mu_2 i_2}\rangle =$$

$$= \delta_{\lambda_1\lambda_1'}\delta_{i_1 i_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'} + K^J_{\Omega Q}(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda_1 i_1, \lambda_2 i_2), \tag{17}$$

$$K^J_{\Omega Q}(\lambda_2' i_2', \lambda' i' \mid \lambda i, \lambda_2 i_2) = -\sum_{\mu\mu_2\mu'\mu_2'} \langle \lambda'\mu'\lambda_2'\mu_2' \mid JM\rangle \langle \lambda\mu\lambda_2\mu_2 \mid JM\rangle \sum_{j_n m_n j_p m_p} \times$$

$$\times \Big\{ \sum_{j_p' m_p' j_p'' m_p''} \Big[\langle j_p' m_p' j_n m_n \mid \lambda'\mu'\rangle \langle j_p m_p j_n m_n \mid \lambda\mu\rangle\, \psi^{\lambda' i'}_{j_p' j_n}\, \psi^{\lambda i}_{j_p j_n} -$$

$$- (-)^{\lambda+\lambda'-\mu-\mu'} \langle j_p' m_p' j_n m_n \mid \lambda' -\mu'\rangle \langle j_p m_p j_n m_n \mid \lambda -\mu\rangle\, \phi^{\lambda' i'}_{j_p' j_n}\, \phi^{\lambda i}_{j_p j_n}\Big] \times$$

$$\times \Big[\langle j_p' m_p' j_p'' m_p'' \mid \lambda_2\mu_2\rangle \langle j_p m_p j_p'' m_p'' \mid \lambda_2' \mu_2'\rangle\, \psi^{\lambda_2 i_2}_{j_p' j_p''}\, \psi^{\lambda_2' i_2'}_{j_p j_p''} -$$

$$- (-)^{\lambda_2+\lambda_2'-\mu_2-\mu_2'} \langle j_p'' m_p'' j_p m_p \mid \lambda_2 -\mu_2\rangle \langle j_p' m_p' j_p'' m_p'' \mid \lambda_2' -\mu_2'\rangle\, \phi^{\lambda_2 i_2}_{j_p'' j_p}\, \phi^{\lambda_2' i_2}_{j_p' j_p''}\Big] +$$

$$+ \sum_{j_n' m_n' j_n'' m_n''} \Big[\langle j_p m_p j_n' m_n' \mid \lambda'\mu'\rangle \langle j_p m_p j_n m_n \mid \lambda\mu\rangle\, \psi^{\lambda' i'}_{j_p j_n'}\, \psi^{\lambda i}_{j_p j_n} -$$

$$- (-)^{\lambda+\lambda'-\mu-\mu'} \langle j_p m_p j_n' m_n' \mid \lambda' -\mu'\rangle \langle j_p m_p j_n m_n \mid \lambda -\mu\rangle\, \phi^{\lambda' i'}_{j_p j_n'}\, \phi^{\lambda i}_{j_p j_n}\Big] \times$$

$$\times \Big[\langle j_n' m_n' j_n'' m_n'' \mid \lambda_2\mu_2\rangle \langle j_n m_n j_n'' m_n'' \mid \lambda_2'\mu_2'\rangle\, \psi^{\lambda_2 i_2}_{j_n' j_n''}\, \psi^{\lambda_2' i_2'}_{j_n j_n''} -$$

$$- (-)^{\lambda_2+\lambda_2'-\mu_2+\mu_2'} \langle j_n'' m_n'' j_n m_n \mid \lambda_2\mu_2\rangle \times$$

$$\times \langle j_n' m_n' j_n'' m_n'' \mid \lambda_2'\mu_2'\rangle\, \phi^{\lambda_2 i_2}_{j_n'' j_n}\, \phi^{\lambda_2' i_2'}_{j_n' j_n}\Big]\Big\}. \tag{17'}$$

Now we calculate the average value of H_M over the state (15), and we get

$$(\Psi_\nu^*(JM)\, H_M \Psi_\nu(JM)) = \sum_i \Omega_{\lambda i}(R_i(J\nu))^2 +$$

$$+ \sum_{\substack{\lambda' i' \lambda i \\ \lambda_2' i_2' \lambda_2 i_2}} P^{\lambda' i'}_{\lambda_2' i_2'}(J\nu)\, P^{\lambda i}_{\lambda_2 i_2}(J\nu) \{ (\Omega_{\lambda i} + \omega_{\lambda_2 i_2}) [\delta_{\lambda\lambda'}\delta_{ii'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'} +$$

$$+ K^J_{\Omega Q}(\lambda_2' i_2', \lambda' i' \mid \lambda i, \lambda_2 i_2)] -$$

$$- \sum_{i''} \frac{1 + y^{\lambda' i'} y^{\lambda' i''}}{\sqrt{\mathcal{Y}_{\lambda' i'} \mathcal{Y}_{\lambda' i''}}} K^J_{\Omega Q}(\lambda_2' i_2', \lambda' i' \mid \lambda i \lambda_2 i_2) - \tag{18}$$

$$- \sum_{\lambda_3 i_3 i_3' \lambda_3'' i_2''} \frac{1 + y^{\lambda_3 i_3} y^{\lambda_3 i_3'}}{\sqrt{\mathcal{Y}_{\lambda_3 i_3} \mathcal{Y}_{\lambda_3 i_3'}}} K^J_{\Omega Q}(\lambda_2' i_2', \lambda' i' \mid \lambda_3 i_3, \lambda_2'' i_2'')\, K^J_{\Omega Q}(\lambda_2'' i_2'', \lambda_3 i_3' \mid \lambda_2 i_2) -$$

$$- \frac{1}{4} \sum_{r i_2''} \frac{X_r^{\lambda_2' i_2'} + X_r^{\lambda_2' i_2''}}{\sqrt{\mathcal{Y}^r_{\lambda_2' i_2'} \mathcal{Y}^r_{\lambda_2' i_2''}}} K^J_{\Omega Q}(\lambda' i', \lambda_2' i_2'' \mid \lambda_2 i_2, \lambda i) -$$

$$- \sum_{r \lambda_4 i_4 i_4' \lambda'' i''} \frac{X_r^{\lambda_4 i_4} + X_r^{\lambda_4 i_4'}}{\sqrt{\mathcal{Y}^r_{\lambda_4 i_4} \mathcal{Y}^r_{\lambda_4 i_4'}}} K^J_{\Omega Q}(\lambda' i', \lambda_2' i_2' \mid \lambda_4 i_4, \lambda'' i'')\, K^J_{\Omega Q}(\lambda'' i'', \lambda_4 i_4' \mid \lambda_2 i_2, \lambda i)\} +$$

$$+ 2 \sum_{i \lambda_1 i_1 \lambda_2 i_2} R_i(J\nu)\, P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\, W^{Ji}(\lambda_1 i_1, \lambda_2 i_2),$$

using the same notation as in ref.[10], and also

$$W^{Ji}(\lambda_1 i_1, \lambda_2 i_2) = W_1^{Ji}(\lambda_1 i_1, \lambda_2 i_2) + W_2^{Ji}(\lambda_1 i_1, \lambda_2 i_2) + W_3^{Ji}(\lambda_1 i_1, \lambda_2 i_2) + W_4^{Ji}(\lambda_1 i_1, \lambda_2 i_2), \tag{19}$$

$$W_1^{Ji}(\lambda_1 i_1, \lambda_2 i_2) = -\frac{1}{\sqrt{2}} \sum_{j_3 j_3' r} \frac{f^{\lambda_2}(j_3 j_3')\, v^{(-)}_{j_3 j_3'}}{\sqrt{\mathcal{Y}^r_{\lambda_2 i_2}}} \mathfrak{L}^{Ji}_{\lambda_1 i_1}(j_3 j_3'; \lambda_2 r), \tag{20}$$

$$W_2^{Ji}(\lambda_1 i_1, \lambda_2 i_2) =$$

$$= -\frac{1}{\sqrt{2}} \sum_{\substack{\lambda_3 i_3 j j' \\ \lambda_4 i_4 \tau}} \frac{f^{\lambda_3}(j_3 j_3') v^{(-)}_{j_3 j_3'}}{\sqrt{\mathcal{Y}^{\tau}_{\lambda_3 i_3}}} \mathfrak{L}^{Ji}_{\lambda_4 i_4}(j_3 j_3'; \lambda \tau) K^{J}_{\Omega Q}(\lambda_4 i_4, \lambda_3 i_3 | \lambda_2 i_2, \lambda_1 i_1), \qquad (20')$$

$$W_3^{Ji}(\lambda_1 i_1, \lambda_2 i_2) = -2 \sum_{j_p j_n} \sqrt{\frac{\kappa_1^{(\lambda)}}{\mathcal{Y}_{\lambda_1 i_1}}} \frac{f^{\lambda_1}(j_p j_n)}{\sqrt{2\lambda_1 + 1}} (1 - y^{\lambda_1 i_1}) \times$$

$$\times \{ u_{j_p} u_{j_n} L^{Ji}_{\lambda_2 i_2}(j_n j_p; \lambda_1) (-)^{j_n - j_p + \lambda_2 - J} -$$

$$- v_{j_p} v_{j_n} L^{Ji}_{\lambda i}(j_p j_n; \lambda_1) (-)^{\lambda_1 + \lambda_2 - J} \}, \qquad (21)$$

$$W_4^{Ji}(\lambda_1 i_1, \lambda_2 i_2) = -2 \sum_{\substack{\lambda_3 i_3 j_p j_n \\ \lambda_4 i_4}} \sqrt{\frac{\kappa_1^{(\lambda_3)}}{\mathcal{Y}_{\lambda_3 i_3}}} \frac{f^{\lambda_3}(j_p j_n)}{\sqrt{2\lambda_3 + 1}} (1 - y^{\lambda_3 i_3}) \times$$

$$\times \{ u_{j_p} u_{j_n} L^{Ji}_{\lambda_4 i_4}(j_p j_n; \lambda_3) (-)^{j_p - j_n + \lambda_4 - J} - v_{j_p} v_{j_n} L^{Ji}_{\lambda_4 i_4}(j_p j_n; \lambda_3) (-)^{\lambda_3 + \lambda_4 - J} \} \times$$

$$\times K^{J}_{\Omega Q}(\lambda_4 i_4; \lambda_3 i_3 | \lambda_1 i_1, \lambda_2 i_2). \qquad (21')$$

Here

$$\mathfrak{L}^{\lambda i}_{\lambda_4 i_4}(j_p j_p'; \lambda_3 \tau) = \sqrt{(2\lambda_3 + 1)(2\lambda_4 + 1)} \sum_{j_n} \{ \psi^{\lambda i}_{j_p j_n} \psi^{\lambda_4 i_4}_{j_p' j_n} (-)^{j_n + j_p + \lambda_3 + \lambda_4} \times$$

$$\times \begin{Bmatrix} j_p' & j_n & \lambda_4 \\ \lambda & \lambda_3 & j_p \end{Bmatrix} + \phi^{\lambda i}_{j_p' j_n} \phi^{\lambda_4 i_4}_{j_p j_n} (-)^{j_n + j_p + \lambda_4} \begin{Bmatrix} j_p' & j_n & \lambda \\ \lambda_4 & \lambda_3 & j_p \end{Bmatrix} \},$$

$$L^{\lambda i}_{\lambda_4 i_4}(j_n j_p; \lambda_3) = -\sqrt{(2\lambda_3 + 1)(2\lambda_4 + 1)} \{ \sum_{j_p'} \psi^{\lambda i}_{j_p' j_n} \psi^{\lambda_4 i_4}_{j_p j_p'} \begin{Bmatrix} j_p & j_p' & \lambda_4 \\ \lambda & \lambda_3 & j_n \end{Bmatrix} (-)^{\lambda_3 + \lambda_4 - \lambda} -$$

$$- \sum_{j_n'} \phi^{\lambda i}_{j_p j_n'} \phi^{\lambda_4 i_4}_{j_n' j_n} \begin{Bmatrix} j_p & j_n' & \lambda \\ \lambda_4 & \lambda_3 & j_n \end{Bmatrix} \}.$$

Hence, it is seen that an essential role is played by the interactions of quasiparticles with usual and np phonons determined by H_{Mvq} and H_{cMvq}.

Using the variational principle in the form

$$\delta \{ (\Psi^*_{\nu}(JM) H_M \Psi_{\nu}(JM)) - \eta_{\nu} [(\Psi^*_{\nu}(JM) \Psi_{\nu}(JM)) - 1] \} = 0 \qquad (22)$$

and after the transformations, we get the secular equation for finding the energies η_{ν} as the determinant in the space of two-phonon states

$$\det \| (\Omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_{\nu}) [\delta_{\lambda_1 \lambda_1'} \delta_{i_1 i_1'} \delta_{\lambda_2 \lambda_2'} \delta_{i_2 i_2'} +$$

$$+ K^{J}_{\Omega Q}(\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2)] -$$

$$- \sum_{i_1''} \frac{1 + y^{\lambda_1' i_1'} y^{\lambda_1' i_1''}}{\sqrt{\mathcal{Y}_{\lambda_1' i_1'} \mathcal{Y}_{\lambda_1' i_1''}}} K^{J}_{\Omega Q}(\lambda_2' i_2', \lambda_1' i_1'' | \lambda_1 i_1, \lambda_2 i_2) -$$

$$- \sum_{\substack{\lambda_3 i_3 i_3' \\ \lambda_2'' i_2''}} \frac{1 + y^{\lambda_3 i_3} y^{\lambda_3 i_3'}}{\sqrt{\mathcal{Y}_{\lambda_3 i_3} \mathcal{Y}_{\lambda_3 i_3'}}} K^{J}_{\Omega Q}(\lambda_2' i_2', \lambda_1' i_1' | \lambda_3 i_3, \lambda_2'' i_2'') K^{J}_{\Omega Q}(\lambda_2'' i_2'', \lambda_3 i_3' | \lambda_1 i_1, \lambda_2 i_2) -$$

$$- \frac{1}{4} \sum_{i_2'' \tau} \frac{X_{\tau}^{\lambda_2' i_2'} + X_{\tau}^{\lambda_2' i_2''}}{\sqrt{\mathcal{Y}^{\tau}_{\lambda_2' i_2'} \mathcal{Y}^{\tau}_{\lambda_2' i_2''}}} K^{J}_{\Omega Q}(\lambda_1' i_1', \lambda_2' i_2'' | \lambda_2 i_2, \lambda_1 i_1) -$$

$$- \frac{1}{4} \sum_{\substack{\lambda_4 i_4 i_4' \\ \lambda'' i'' \tau}} \frac{X_{\tau}^{\lambda_4 i_4} + X_{\tau}^{\lambda_4 i_4'}}{\sqrt{\mathcal{Y}^{\tau}_{\lambda_4 i_4} \mathcal{Y}^{\tau}_{\lambda_4 i_4'}}} K^{J}_{\Omega Q}(\lambda_1' i_1', \lambda_2' i_2' | \lambda_4 i_4, \lambda'' i'') K^{J}_{\Omega Q}(\lambda'' i'', \lambda_4 i_4' | \lambda_2 i_2, \lambda_1 i_1) -$$

$$- \sum_{i} \frac{W^{Ji}(\lambda_1 i_1, \lambda_2 i_2) W^{Ji}(\lambda_1' i_1', \lambda_2' i_2')}{\Omega_{Ji} - \eta_{\nu}} \| = 0. \qquad (23)$$

As in ref. /10/ let us illustrate eq. (23) by the diagrams in fig.1, 2. In the first term we take into account the diagrams a), b), c) and d) fig.1. In the next two terms depending on the characteristics $\mathcal{Y}_{\lambda i}$ of np phonons, the diagrams of type e) and f) are summed. In the other two terms depending on $\mathcal{Y}^{\tau}_{\lambda i}$, the diagrams of type g) and h) are summed. The terms $\sim W \cdot W$ allow for the diagrams a), b), c) and d), fig.2. Apart from the diagrams e), f), g), h), fig.1 and c), d), fig.2, we take also into account those diagrams in which part a) is changed by b), c) and d), fig.1.

4. SYSTEM OF APPROXIMATE EQUATIONS

This paper is aimed at deriving equations for the description of the np phonon fragmentation. The np phonon fragmentation

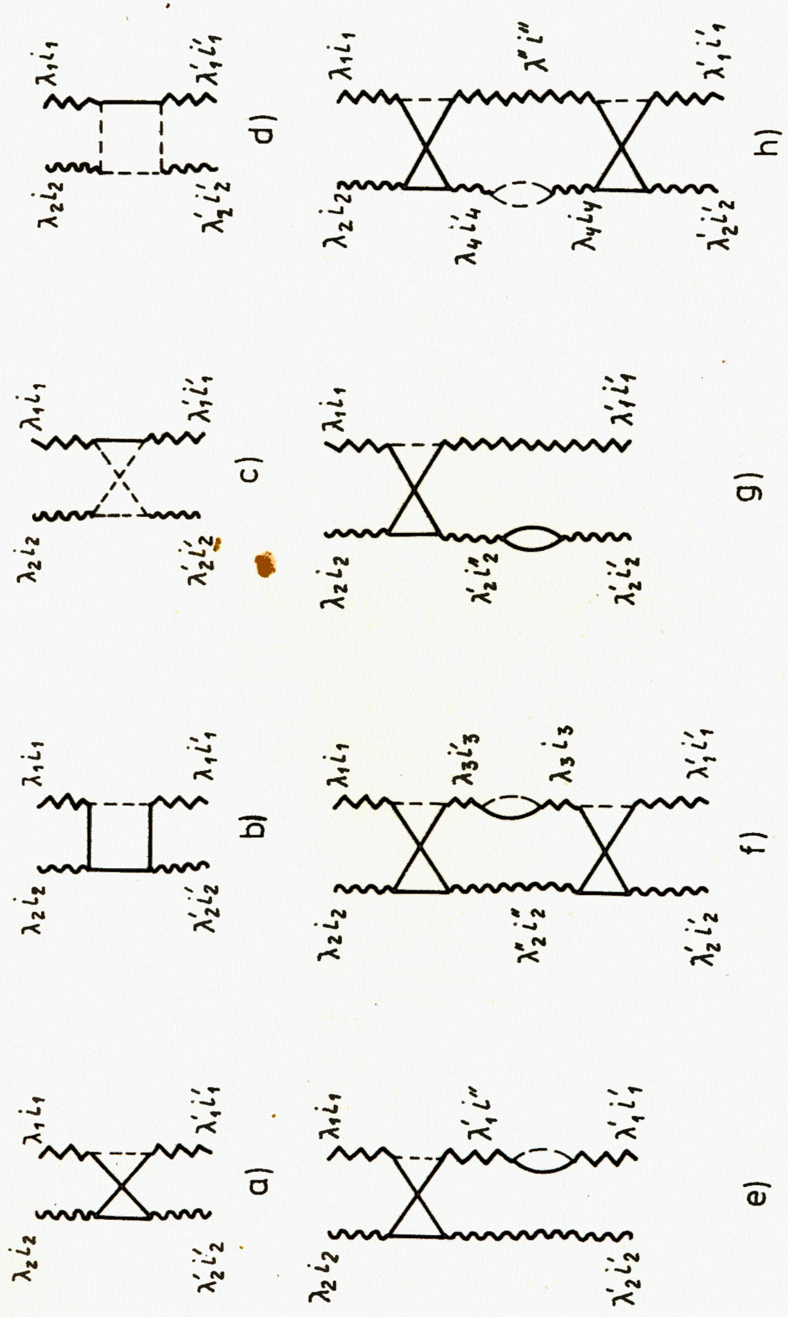

Fig.1. Diagrams in the space of two-phonon states. Notation: ~~~ is Q -phonon; ~~~ is Ω -np phonon; ——— - is neutron quasiparticle; ---- - is proton quasiparticle.

is determined by the coefficients $R_i(J\nu)$ which could be found by solving the secular equation (23). This equation is very complicated. In it many diagrams are summed which give a small contribution to the fragmentation of np phonons. Therefore, we pass to an approximate system of equations. For this purpose in (18) we, first, use only the terms proportional to $(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2$ and second, in the quadratic in $K^J_{\Omega Q}$ term we take one of $K^J_{\Omega Q}$ in the diagonal form. This approximation is possible due to the fact that the absolute values of the diagonal terms of $K^J_{\Omega Q}$ are considerably larger than those of the nondiagonal terms. In the normalization condition (16) we also use only the diagonal values of $K^J_{\Omega Q}$. As a result we get

$$\sum_i (R_i(J\nu))^2 + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 \{1 + K^J_{\Omega Q}(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)\} = 1 , \quad (24)$$

$$(\Psi^*_\nu(JM) H_M \Psi_\nu(JM)) = \sum_i \Omega_{\lambda i}(R_i(J\nu))^2 + \quad (25)$$

$$+ \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 (1 + K^J_{\Omega Q}(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)) \times$$

$$\times \{\Omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\Omega_{\lambda_1 i_1} + \Delta\omega_{\lambda_2 i_2}\} +$$

$$+ 2 \sum_{\substack{i \lambda_1 i_1 \\ \lambda_2 i_2}} R_i(J\nu) P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) W^{Ji}_0(\lambda_1 i_1, \lambda_2 i_2) \times$$

$$\times \{1 + K^J_{\Omega Q}(\lambda_2 i_2, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)\} ,$$

where

$$\Delta\Omega_{\lambda i} = -\sum_{i''} \frac{1 + y^{\lambda i} y^{\lambda i''}}{\sqrt{\mathcal{Y}_{\lambda i} \mathcal{Y}_{\lambda i''}}} K^J_{\Omega Q}(\lambda_2 i_2, \lambda i'' | \lambda i, \lambda_2 i_2) ,$$

$$\Delta\omega_{\lambda_2 i_2} = -\frac{1}{4} \sum_{i_2'' \tau} \frac{X^{\lambda_2 i_2}_\tau + X^{\lambda_2 i_2''}_\tau}{\sqrt{\mathcal{Y}^\tau_{\lambda_2 i_2} \mathcal{Y}^\tau_{\lambda_2 i_2''}}} K^J_{\Omega Q}(\lambda_1 i_1, \lambda_2 i_2'' | \lambda_2 i_2, \lambda_1 i_1) ,$$

$$W^{Ji}_0(\lambda_1 i_1, \lambda_2 i_2) = W^{Ji}_1(\lambda_1 i_1, \lambda_2 i_2) + W^{Ji}_3(\lambda_1 i_1, \lambda_2 i_2) .$$

Using the variational principle we get the following system of equations:

$$(\Omega_{\lambda i}-\eta_\nu)R_i(J\nu) + \sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\,W_0^{Ji}(\lambda_1 i_1,\lambda_2 i_2)\times \tag{26}$$

$$\times\{1+K^J_{\Omega Q}(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\}=0,$$

$$(\Omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\Omega_{\lambda_1 i_1}+\Delta\omega_{\lambda_2 i_2}-\eta_\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)+\sum_i R_i(J\nu)\,W_0^{Ji}(\lambda_1 i_1,\lambda_2 i_2)=0. \tag{27}$$

From eq. (26) we get R_i, substitute it into (27) and then get the secular equation in the space of two-phonon states in the following form:

$$\det\Big\|(\Omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\Omega_{\lambda_1 i_1}+\Delta\omega_{\lambda_2 i_2}-\eta_\nu)\delta_{\lambda_1\lambda_1'}\delta_{i_1 i_1'}\delta_{\lambda_2\lambda_2'}\delta_{i_2 i_2'}- \tag{28}$$

$$-\sum_i \frac{W_0^{Ji}(\lambda_1 i_1,\lambda_2 i_2)\,W_0^{Ji}(\lambda_1' i_1',\lambda_2' i_2')\{1+K^J_{\Omega Q}(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1\lambda_2 i_2)\}}{\Omega_{\lambda i}-\eta_\nu}\Big\|=0.$$

In this case the diagrams e), g), fig. 1 and a), b), c), fig.2 are taken into account. Moreover, a particular case of the diagrams c) with $\lambda_4=\lambda_2$, $i_4=i_2$, $\lambda_3=\lambda_1$ and $i_3=i_1$ is considered, i.e. when it contains no summation over the intermediate two-phonon states. In the case of the secular equation (28), less diagrams are summed in comparison with eq. (23). It

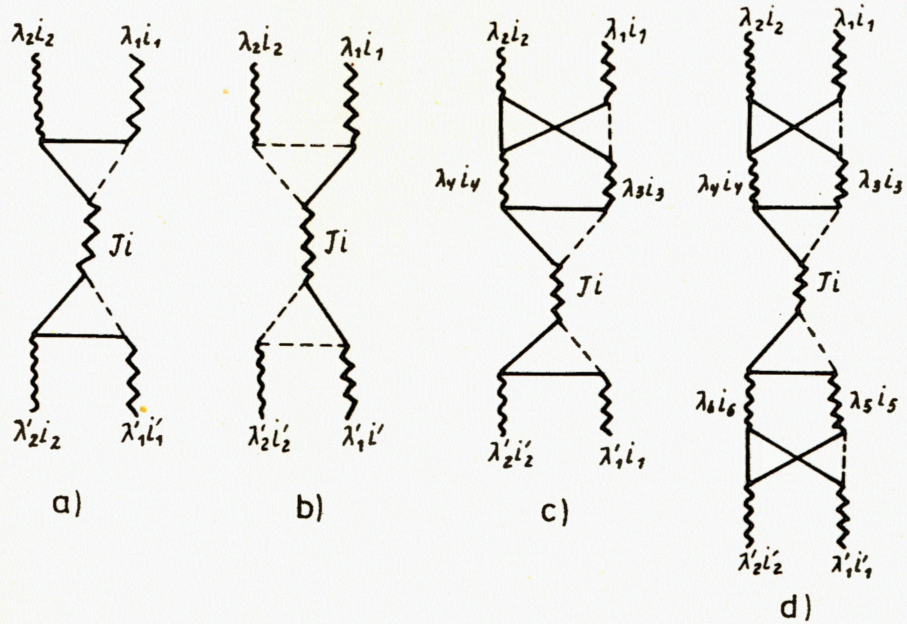

Fig.2. Diagrams allow for the quasiparticle-phonon interaction determined by H_{Mvq} and H_{cMvq}. Notation is as in fig.1.

should be emphasized that the fragmentation of np phonons is mainly caused by the diagrams a) and b), fig.2.

The secular equation in the space of one-phonon states has the following form:

$$\det\Big\|(\omega_{Ji}-\eta_\nu)\delta_{ii'}- \tag{29}$$

$$-\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}\frac{W_0^{Ji}(\lambda_1 i_1,\lambda_2 i_2)\,W_0^{Ji'}(\lambda_1 i_1,\lambda_2 i_2)\{1+K^J_{\Omega Q}(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\}}{\Omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\Omega_{\lambda_1 i_1}+\Delta\omega_{\lambda_2 i_2}-\eta_\nu}\Big\|=0.$$

In this case we take into account the diagrams given in fig.3 and the diagram c), fig.3, in which the parts of the diagram b) are instead of the parts of the diagram a), and moreover, the part a), fig.1 is changed by b), c) and d), fig.1.

Consider equation (29). The rank of the determinant is equal to the number of neutron-proton one-phonon states in the first term of the wave function (15). It is two orders as less as the rank of the determinants (23) and (28). The factor $(1+K^J_{\Omega Q}(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)$ is due to the inclusion of the Pauli principle in the two-phonon terms of the wave function (15). In the case of maximal violation of the Pauli principle $K^J_{\Omega Q}=-1$

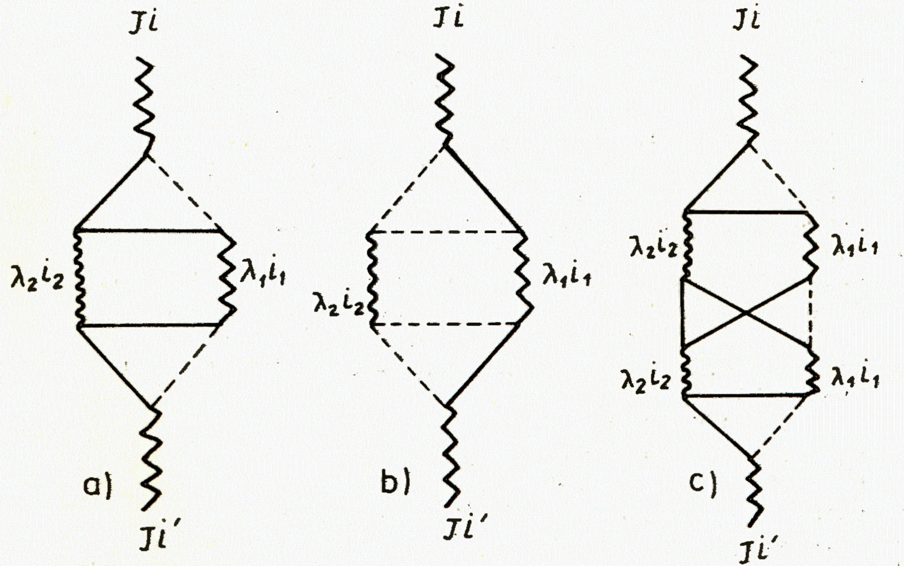

Fig.3. Diagrams in the space of one-phonon states. Notation is as in fig.1.

and the corresponding term is excluded from the sum over $\lambda_1 i_1 \lambda_2 i_2$. The two-phonon pole is shifted by $\Delta\Omega_{\lambda_1 i_1}+\Delta\Omega_{\lambda_2 i_2}$ due to the inclusion of the diagram c), fig.2, in which $\lambda_1 = \lambda_3$, $i_1 = i_3$, $\lambda_2 = \lambda_4$, and $i_2 = i_4$.

The widths of the giant charge-exchange resonances demonstrate the fragmentation of np phonons which are generated by the change-exchange part of the isovector multipole and spin-multipole forces. Their description does not need any new forces and new parameters. It is shown in this paper that the formalism of charge exchange states can be formulated so as to make it similar to that for describing the low-lying collective states and the collective states of the type of giant multipole and spin-multipole resonances in spherical nuclei. In the quasiparticle-phonon nuclear mode, when the wave function contains one- and two-phonon components, many diagrams are summed. The numerical calculations of the nuclear characteristics are made by the method of strength functions which are based at the first stage on equation (29) with $K^{J}_{\Omega Q} = 0$. The multipole and spin-multipole usual and np phonons are used as a basis in the calculations. The investigations of the charge-exchange resonances are still at the initial stage, and there are large possibilities for their description within the quasiparticle-phonon nuclear model.

REFERENCES

1. Doering R.R. et al. Phys.Rev.Lett., 1975, 35, p. 1691.
   Bainum D.E. et al. Phys.Rev.Lett., 1980, 44, p. 1751.
   Rapaport J. et al. Phys.Lett., 1982, B119, p. 61.
2. Horen D.J. et al. Phys.Lett., 1981, B99, p. 383.
   Gaarde C. et al. Nucl.Phys., 1981, A369, p. 258.
3. Bohr A., Mottelson B.R. Phys.Lett., 1981, B100, p. 10.
   Brown G.E., Rho M. Nucl.Phys., 1981, A372, p. 397.
4. Suzuki T., Gaarde C., Sagawa H. Phys.Lett., 1982, B116, p.91.
   Sagawa H., Nguen van Giai, Phys.Lett., 1982, B113, p. 119.
5. Bertsch C.F., Hamamoto I. Phys.Rev., 1982, C26, p. 1323.
   Fiebig H.R., Wambach J. Nucl.Phys., 1982, A386, p. 381.
6. Соловьев В.Г. Изв. АН СССР, сер.физ., 1971, 35, с. 666;
   ТМФ, 1973, 17, с. 90.
7. Соловьев В.Г. ЭЧАЯ, 1978, 9, с. 580.
   Soloviev V.G. Nukleonica, 1978, 23, p. 1149.
   Малов Л.А., Соловьев В.Г. ЭЧАЯ, 1980, 11, с. 301.
8. Вдовин А.И., Соловьев В.Г. ЭЧАЯ, 1982, 14, с. 237.
9. Кузьмин В.А., Соловьев В.Г. ЯФ, 1982, 35, с. 620.
10. Воронов В.В., Соловьев В.Г. ОИЯИ, Е4-83-52, Дубна, 1983.
11. Bes D.R. et al. Nucl.Phys., 1976, A260, p. 77.

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Включение нейтрон-протонных фононов в квазичастично-фононную модель ядра

Показано, как вводятся операторы нейтрон-протонных фононов в математический аппарат квазичастино-фононной модели ядра. С учетом решений секулярных уравнений для однофононных состояний преобразован гамильтониан модели. Получена в общем виде система основных уравнений и осуществлен переход к системе приближенных уравнений. Они предназначены для описания фрагментации нейтрон-протонных фононов и тем самым для вычисления характеристик зарядово-обменных гигантских резонансов в сферических ядрах.

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Soloviev V.G. E4-83-272

Inclusion of Neutron-Proton Phonons into the Quasiparticle-Phonon Nuclear Model

It is shown how the neutron-proton phonon operators are introduced into the quasiparticle-phonon nuclear model. The model Hamiltonian is transformed allowing for the solutions of secular equations for the one-phonon states. The system of basic equations is derived in the general form, and the transition to the system of approximate equations is realized. They are used to describe the fragmentation of neutron-proton phonons and thereby to calculate the charge-exchange giant resonance in spherical nuclei.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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