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**HIGH-LYING SINGLE-PROTON STATES  
IN SPHERICAL NUCLEI**

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At present the one-nucleon transfer reactions are a powerful tool for studying a high-lying part of the spectrum of odd-A spherical nuclei. The resonance-like structures caused by the excitation of deep-lying hole states have been observed and studied in detail in the neutron pick-up reactions <sup>1/</sup>. The study of proton hole states has been undertaken in the proton pick-up reaction <sup>2/</sup>. A new important step has been recently made towards the use of the one-nucleon transfer reactions for the study of high-lying nuclear excitations. A new type of the resonance-like structure has been observed while studying the proton stripping reactions ( $\alpha, t$ ) and ( $^3\text{He}, d$ ) in  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$  at the excitation energies  $E_x \approx 5 \div 12 \text{ MeV}^{3,4/}$ . The analysis of triton and deuteron angular distributions and other reasonings led the experimenters to the conclusion that these structures are due to the excitation of high-lying single-proton states with large momenta. The spins and parities of excited levels were presumably determined, and some quantitative data on the distribution of single-particle strength have been obtained.

The data on the strength distribution of deep-lying hole states have been successfully described within the quasiparticle-phonon nuclear model of Soloviev (QPM)<sup>5/</sup>. The results of calculation within this model are in good agreement with the experimental data of various groups. In the present note using the QPM we investigate the strength distribution of high-lying single-proton states in  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$  and compare it with the experimental data.

The QPM formalism has been systematically expounded in papers <sup>5/</sup>. We shall list the basic assumptions of the QPM as applied to spherical odd-A nuclei <sup>6,7/</sup>:

1) The model Hamiltonian includes a phenomenological average field of protons and neutrons as the Saxon-Woods potential, the monopole pairing forces with constant matrix elements in the particle-particle channel, the separable multipole and spin-multipole forces with the isoscalar and isovector components in the particle-hole channel. In the present paper the radial dependence of forces is chosen in the form of  $\partial U / \partial r_1 \times \partial U / \partial r_2$ , where  $U$  is the central part of the Saxon-Woods potential.

2) At the first step the phonon excitations of a doubly even core are calculated within the RPA. At the same time the Hamiltonian parameters are determined from the data on low-lying excitations and giant resonances.

3) The interaction of an odd quasiparticle with the phonon excitations of the core in the second quantization representation has the form

$$H_{qph} = \sum_{\lambda i} \sum_{j_1 j_2} \Gamma(j_1 j_2 \lambda i) \{ (Q_{\lambda \mu i}^+ + (-)^{\lambda-\mu} Q_{\lambda -\mu i}) B(j_1 j_2 \lambda -\mu) + h.c. \}, \quad (1)$$

$$B(j_1 j_2 \lambda -\mu) = \sum_{m_1 m_2} (-)^{j_2+m_2} \langle j_1 m_1 j_2 m_2 | \lambda -\mu \rangle a_{j_1 m_1}^+ a_{j_2 -m_2},$$

where  $Q_{\lambda \mu i}^+$  and  $Q_{\lambda \mu i}$  are the phonon creation and annihilation operators with momentum  $\lambda$ , its projection  $\mu$  and number  $i$ ;  $a_{jm}^+$  and  $a_{jm}$  are the quasiparticle creation and annihilation operators with the shell quantum numbers  $n|j$  (denoted by one index  $j$ ) and the projection of the total momentum  $m$ . The quantity  $\Gamma(j_1 j_2 \lambda i)$ , the expression for which can be found in refs.<sup>6,7/</sup>, determines the strength of the quasiparticle-phonon interaction. It depends on the structure of phonons and matrix elements of the effective forces and contains no special parameters. In this paper we have taken into account the interaction of an odd quasiparticle with one- and two-phonon excitations of the core. The wave function of the  $\nu^{\text{th}}$  excited state of the odd-A nucleus with spin and parity  $J^\pi$  has the form

$$\Psi_\nu(JM) = \{ C_{J\nu} a_{JM}^+ + \sum_j D_j^{\lambda i}(J\nu) [a_{jm}^+ Q_{\lambda \mu i}^+]_{JM} \} + \quad (2)$$

$$+ \sum_j \sum_{i_1 i_2} F_j^{\lambda_1 i_1 \lambda_2 i_2} [a_{j_1 m_1}^+ [Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+]_{i_1 m_1}]_{JM} \Psi_0$$

( $\Psi_0$  is the ground state wave function of a doubly even nucleus). As it is seen from (2), in the approximation when the commutator  $[a_{jm}, Q_{\lambda \mu i}^+] = 0$ , only the following matrix elements of the interaction operator:  $\langle \Psi_0 | a_{JM} | H_{qph} | [a_{jm}^+ Q_{\lambda \mu i}^+]_{JM} \Psi_0 \rangle$  and  $\langle \Psi_0 | [a_{j_1 m_1} Q_{\lambda_1 \mu_1 i_1}]_{JM} | H_{qph} | [a_{j_2 m_2}^+ [Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+]_{i_1 m_1}]_{JM} \Psi_0 \rangle$  differ from zero. Just these matrix elements have been taken into consideration in the calculations. The analysis of some corrections to this approximation can be found in papers<sup>8/</sup>. Their influence on the distribution of the one-quasiparticle component  $C_{J\nu}$  at intermediate and high excitation energies is insignificant.

The number of states (2) at the excitation energies  $5 \div 12$  MeV is very large. So, taking into consideration all single-particle bound and quasibound (with width  $\Gamma \ll E_{n|j}$ ) states of an odd

particle and all one- and two-phonon core excitations with an energy lower than 15 MeV produced by the phonons with spins and parities  $\lambda^\pi = 1^\pm \div 7^\pm$ , up to an energy of  $E_x = 15$  MeV one can observe 200-500 "quasiparticle plus phonon" and  $(25 \div 40) \cdot 10^3$  "quasiparticle plus two phonons" states (depending on the spin  $J$ ). The coefficients  $D_j^{\lambda i}(J\nu)$  of the wave function (2) are the solutions of a homogeneous system of equations of the corresponding rank (200-500). The coefficients in this system of equations are nonlinear functions of the state (2) energy  $\eta_{J\nu}$ . Hence, it is seen the calculation of the energy and structure of states (2) is a very cumbersome task from the computational point of view.

A more simple and profound is the calculation of the strength function  $C^2(E_x)$  which describes a change of an averaged value of the one-quasiparticle component in the wave function (2) as a function of the excitation energy  $E_x$ <sup>9/</sup>

$$C^2(E_x) = \frac{\Delta}{2\pi} \sum_\nu \frac{C_{J\nu}^2}{(E_x - \eta_{J\nu})^2 + \Delta^2/4}. \quad (3)$$

The advantages of such an approach, the methods for calculating the strength function and the criteria of choosing the parameter  $\Delta$  are discussed in detail in papers<sup>7,10,11/</sup>. In this paper we used the value of  $\Delta = 0.5$  MeV.

Prior to the discussion of the results of calculation, we present the experimental quantitative data on the distribution of the single-proton strength<sup>3/</sup>. The resonance structures observed in the reaction cross sections were approximated by the Gauss curves. Two peaks were distinguished in <sup>145</sup>Eu. In the first, more narrow peak with the parameters  $E_{\text{max}1} = 5.92$  MeV,  $\Gamma_1 = 1.23 \div 0.15$  MeV 97% of the  $1h_{9/2}$  subshell strength is concentrated (or 75% of the  $1h_{9/2}$  strength and 19% of the  $2f_{7/2}$  strength). In the broad peak with the parameters  $E_{\text{max}2} = 7.63$  MeV,  $\Gamma_2 = 4.0 \div 0.45$  MeV 94% of the  $1i_{13/2}$  strength is concentrated. These structures are superimposed on a considerable background, the size and form of which were determined empirically.

An expected degree of conformity of the theory with the experiment can be concluded from the analysis of the theoretical single-particle scheme. The single-proton states of <sup>145</sup>Eu, which lie above the Fermi level are shown in fig.1. This single-proton levels are calculated in the Saxon-Woods potential with the parameters, given in table 1, by using the REDMEL program<sup>12/</sup>. The form of the Coulomb single-particle potential was assumed to be the same as for a uniformly charged sphere of the radius  $R_c = 1.2A^{1/3}$  fm. All the states above the level  $2d_{3/2}$  are quasibound, but their widths are very small, especially for the states with large  $\ell$  ( $\Gamma < 1$  eV). The states  $2f_{7/2}$ ,  $1h_{9/2}$  and  $1i_{13/2}$

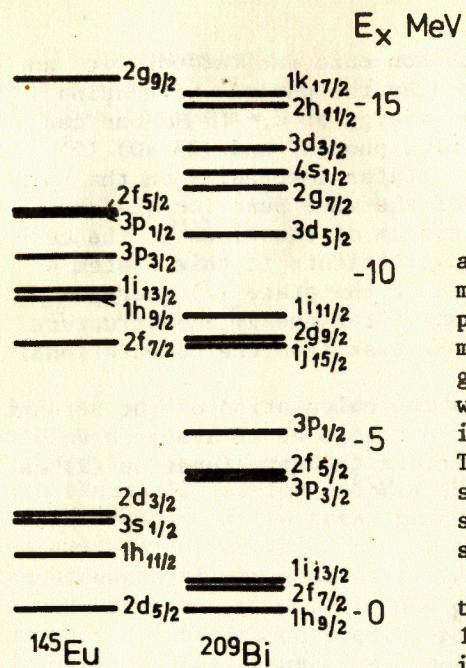


Fig.1. Single-proton states of  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$ , lying above the Fermi level ( $2d_{5/2}$  in  $^{145}\text{Eu}$  and  $1h_{9/2}$  in  $^{209}\text{Bi}$ ). Parameters of the Saxon-Woods potential are given in table 1.

are by  $1 \pm 2$  MeV higher than the maxima of the corresponding "experimental" Gaussians. Three more states lie in the same region. An average distance between proton quasibound levels is not large, of about 1 MeV. Therefore, one can expect a considerable overlapping of their strength distributions due to the spreading.

The strength functions (3) of the subshells  $2f_{7/2}$ ,  $1h_{9/2}$  and  $1i_{13/2}$  in the excitation energy interval  $1 < E_x < 12$  MeV are shown

in fig.2. Table 2 presents some general characteristics of the distributions: the centroids  $\bar{E}_x$ , the second moments  $\sigma$  and the per cent of exhaustion of the subshell strength in the interval  $\Delta E_x$  studied. Due to the interaction with complex configurations the single-proton strength turned out to be spread in a wide interval  $\Delta E_x$ . As is seen from table 2 about 90% of the subshell strength is exhausted up to an energy of  $12 \pm 14$  MeV and the remaining 10% are at higher  $E_x$ . The distribution maxima lowered by 1-1.5 MeV with respect to the position of one-quasi-particle states; as a result the maxima of the  $2f_{7/2}$  and  $1h_{9/2}$  strength functions turned out to be near maximum of the "narrow" Gaussian.

Table 1

Parameters of the Saxon-Woods potential for the proton systems in  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$

A	Z	$V_0$ , MeV	$r_0$ , fm	$a$ , fm $^{-1}$	$V_{SO}$ , MeV
141	59	57.7	1.24	1.587	11.2
209	83	60.3	1.24	1.587	10.1

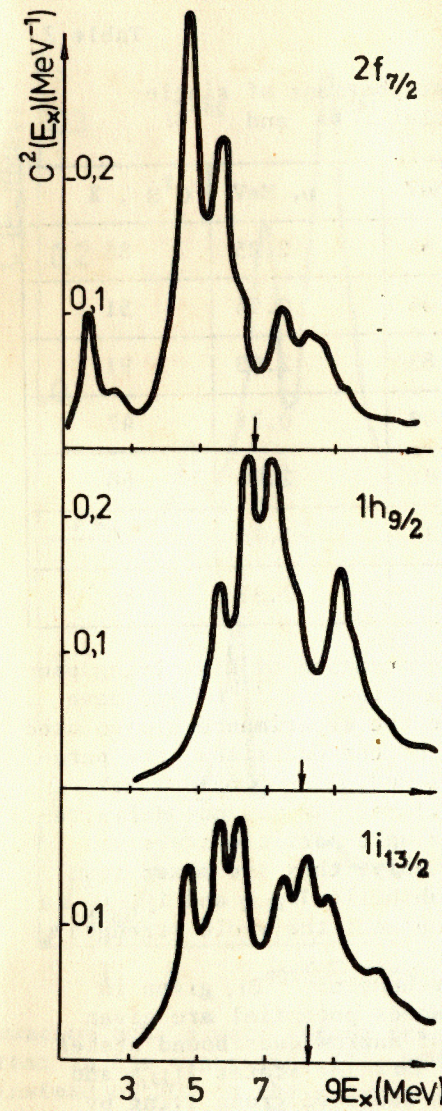


Fig.2. Strength functions of the single-proton states  $2f_{7/2}$ ,  $1h_{9/2}$  and  $1i_{13/2}$  of  $^{145}\text{Eu}$ .

The strength functions of various subshells differ noticeably from each other. Nevertheless, the values of the second moment  $\sigma$  of the three distributions are close. The values of  $\bar{E}_x$  and  $\sigma$  of the  $1i_{13/2}$  subshell strength function are close to the corresponding values of  $E_{max2}$  and  $\sigma_2 = 0.426 \Gamma_2$  of the "broad" Gaussian. The values of  $\sigma$  of the  $2f_{7/2}$  and  $1h_{9/2}$  subshell strength distributions exceed four times the values of  $\sigma_1 = 0.426 \Gamma_1$  of the "narrow" Gaussian. It is seen from fig.2 that a considerable part of the  $2f_{7/2}$  and  $1h_{9/2}$  state strength is concentrated in rather narrow intervals  $\Delta E_x$  near the distribution maxima. The values of  $\sigma$  calculated for these intervals (they are also shown in table 2) are much closer to the value of  $\sigma_1 = 0.524 + 0.063$  MeV. Only 50% of the state strength is concentrated in these intervals  $\Delta E_x$ .

However, it should be emphasized that the form of theoretical distributions of the single-particle strength is complicated and it cannot be approximated by the Gauss curves with a satisfactory accuracy. Therefore, a quantitative comparison of the values of  $\bar{E}_x$  and  $\sigma$  from table 2 with the corresponding parameters of the experimental Gaussians is unreasonable. Also, it is to be mentioned that the strength distributions of various subshells, as we have expected, overlap strongly. This makes it difficult to extract unambiguously the corresponding spectroscopic factors from the experimental cross sections.

Table 2

Characteristics of strength distributions of single-proton high-lying states in nuclei  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$

Nucleus	nj	$\Delta E_x$ , MeV	$\bar{E}_x$ , MeV	$\sigma$ , MeV	$C^2S$ , %
$^{145}\text{Eu}$	$2f_{7/2}$	$0 \div 12$	5.64	2.25	86
		$3.3 \div 6.8$	5.06	0.96	51
	$1h_{9/2}$	$0 \div 14$	7.83	2.03	91
		$5.9 \div 8.5$	7.09	0.71	47
	$1i_{13/2}$	$0 \div 14$	7.81	2.28	88
$^{209}\text{Bi}$	$1i_{11/2}$	$0 \div 12.5$	8.37	1.78	90
	$1j_{15/2}$	$0 \div 12.5$	7.1	2.33	86

The experimental data on the fragmentation of high-lying proton states of  $^{209}\text{Bi}$  are more uncertain than of  $^{145}\text{Eu}$  and have a preliminary nature<sup>4/</sup>. In this case the experimenters also used approximation of the cross sections by the Gaussians, the parameters of which were:  $E_{\max 1} = 7.2 \pm 0.2$  MeV,  $\Gamma_1 = 0.63 \pm 0.2$  MeV;  $E_{\max 2} = 8.7 \pm 0.5$  MeV and  $\Gamma_2 = 5.3 \pm 0.1$  MeV. It was not definitely concluded the strength of which single-particle state was concentrated in each peak. Most probably, they are mixed and 15% of the strength of one of the subshells  $1i_{11/2}$  and  $1j_{15/2}$  is concentrated in the narrow peak and almost the whole strength is concentrated in the broad peak.

A fragment of the single-proton scheme of  $^{209}\text{Bi}$ , given in fig.1 (the parameters of the Saxon-Woods potential are given in table 1), shows that the number of narrow quasibound states in this nucleus is larger than in  $^{145}\text{Eu}$ . The states  $1i_{11/2}$  and  $1j_{15/2}$  with the state  $2g_{9/2}$  form an isolated group lying by 2-3 MeV from other shells with much less values of  $\ell$ . Therefore, while extracting the  $1i_{11/2}$  and  $1j_{15/2}$  strengths from the experimental cross sections, one should take into account the contribution of the  $2g_{9/2}$  subshell.

The calculations show (see fig.3) that the  $1i_{11/2}$  and  $1j_{15/2}$  subshells are fragmented as strongly as the states in  $^{145}\text{Eu}$ . The values of the second moments  $\sigma$  (see table 2) of all the states are close to 2 MeV ( $\sigma = 2.0 \pm 0.3$  MeV). The value of  $\sigma(1i_{11/2})$  is on the lower bound of this interval. The values of  $\sigma(1i_{11/2})$  and  $\sigma(1j_{15/2})$  are close to  $\sigma_2 = 0.426 \Gamma_2 = 2.26 \pm 0.43$  MeV of

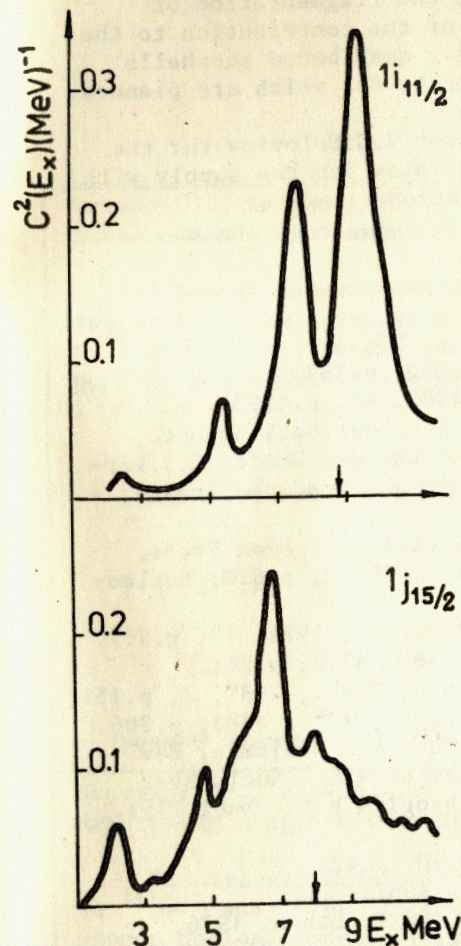


Fig.3. Strength function of the proton states  $1i_{11/2}$  and  $1j_{15/2}$  of  $^{209}\text{Bi}$ .

the broad bump. The maximum of the  $1j_{15/2}$  strength function and the local maximum of the  $1i_{11/2}$  strength function lie in the region of the "narrow" experimental Gaussian. In the interval  $\Delta E_x = \Gamma_1$  symmetric with respect to  $E_{\max 1}$ , 11% of the strength of each of the two subshells is exhausted, that is in agreement with the above-mentioned experimental estimates.

From the comparison of the experimental data on the fragmentation of high-lying single-proton states in  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$  with the theoretical calculation we can conclude about their qualitative agreement. The position of centroids of the theoretical and experimental distributions coincide with an accuracy of  $\sim 1$  MeV. The second moments of distributions are in satisfactory agreement with the experimental ones in  $^{209}\text{Bi}$  and for the  $1i_{13/2}$  state in  $^{145}\text{Eu}$ . The divergences we should like to mention are: a more strong fragmentation of the  $1h_{9/2}$  subshell than in the experiment (?); 90% of the  $2f_{7/2}$  subshell strength instead of 20% in the experiment is observed in the excitation energy studied in  $^{145}\text{Eu}$ . It is necessary to analyse the experimental data in  $^{209}\text{Bi}$  once more in order to extract the  $2g_{9/2}$  subshell contribution.

The agreement of the theoretical and experimental data confirms that the parameters of the single-particle potential we have used are correct. These parameters have been repeatedly used in other calculations, too. It is more significant that a good description of the experimental data on the fragmentation of proton-hole states in  $^{207}\text{Tl}$  has been obtained using these parameters<sup>13/</sup>, i.e., of the data on the states of the major proton shell  $50 < Z \leq 82$ .

We intend to continue the study of the fragmentation of single-proton states, in particular, of the contribution to the proton stripping cross section of other quasibound subshells and of the states in  $^{91}\text{Nb}$  the experiments for which are planned.

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Высоколежащие протонные состояния сферических ядер

В рамках квазичастично-фононной модели ядра рассчитана фрагментация высоколежащих протонных одночастичных состояний в  $^{145}\text{Eu}$  и  $^{209}\text{Bi}$ . Результаты расчетов сравниваются с данными, полученными в реакциях срыва протона  $^{144}\text{Sm}(\alpha, t)^{145}\text{Eu}$  и  $^{208}\text{Pb}(\alpha, t)^{209}\text{Bi}$ .

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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High-Lying Single-Proton States in Spherical Nuclei

The fragmentation of high-lying single-proton states in  $^{145}\text{Eu}$  and  $^{209}\text{Bi}$  is calculated within the quasiparticle-phonon nuclear model. The results of calculation are compared with the data for the proton stripping reactions  $^{144}\text{Sm}(\alpha, t)^{145}\text{Eu}$  and  $^{208}\text{Pb}(\alpha, t)^{209}\text{Bi}$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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