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ON THE CALCULATION OF THE ELASTIC AND INELASTIC PROTON-DEUTERON SCATTERING

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ON THE CALCULATION OF THE ELASTIC AND INELASTIC PROTON-DEUTERON SCATTERING

Мэллер К.

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Расчеты упругого и неупругого рассеяния протонов дейтронами

На основе данного двухчастичного локального потенциала дается схема вычисления поперечного сечения упругого и неупругого рассеяния протонов дейтронами. Интегральные уравнения для трехчастичной амплитуды представлены в виде диаграмм . Данная схема используется для вычисления первых двух членов ряда многократного рассеяния (полюсная и прямоугольная диаграммы) в линейном приближении по эффективному радиусу.

Сообщение Объединенного института ядерных исследований
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Möller K.

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On the Calculation of the Elastic and Inelastic Proton-Deuteron Scattering

A concise scheme is given for the calculation of the cross sections of the elastic and inelastic proton-deuteron scattering starting from a given two-body local potential. The integral equations for the three-body amplitudes are represented in terms of graphs. The given scheme is applied to calculate the first two terms of the multiple scattering series (pole and square graph) in linear effective range approximation.

Communications of the Joint Institute for Nuclear Research.
Dubna, 1974

1. Introduction

The basis for the treatment of the elastic and inelastic proton-deuteron scattering is given by the Faddeev equations [1]. In actual calculations a lot of authors used the formulation of the Faddeev equations which was given by Alt et al. [2]. In 1963 Sitenko et al. [3] first formulated the Faddeev equations with spin and isospin. In the following years suitable mathematical approximations have been developed to solve the Faddeev integral equation system. To calculate three particle scattering states a number of separable potential models have been developed, the best known of them being the Amado-Lovelace model [4,5]. The numerical results of this model have later been improved by Cahill [6]. In the last time there has also been remarkable progress in solving the Faddeev equations with local potentials [7].

But despite of the great progress of the three particle theory even today quite a lot of three particle scattering experiments is interpreted in terms of very simple models. Certainly, this is partly due to the fact that it is difficult for experimentalists to keep up with the fast development of three-body theory. Moreover most theoretical papers cannot directly be used to perform numerical calculations since they do not contain details on spin-factors, antisymmetrization and so on.

It is not the purpose of the present paper to compete with the theoretical papers mentioned above but only to give a concise representation of the $p-d$ scattering which can directly be used to perform numerical calculations without the necessity to consult other papers.

This is achieved by giving explicitly all the factors arising from spin, isospin and antisymmetrization which are necessary to calculate the cross sections observed experimentally starting from a given two-body local potential. The paper is intended to be a continuation of the paper [8]. Similar representations are contained for example in papers published by Cahill [6] and Ebenhöh [9].

In a first step the Faddeev equations for the three-particle wave function are transformed into integral equations for the three-body amplitudes. This has proved very useful since the wave function is not explicitly needed in the calculations. Furthermore the integral equations for the amplitudes can very easily be represented in terms of graphs. The iteration of the integral equations for the amplitudes gives the so-called multiple scattering series, which can be interpreted as a decomposition of the total amplitude into partial amplitudes with increasing order of rescattering. Because of its simple physical meaning this series can serve as a starting point for different models and the investigation of its properties (convergence, analyticity) is of special interest. A detailed investigation of the multiple scattering series for two body separable potentials was performed by Sloan [10]. It turned out that in general we cannot expect the ordinary multiple scattering series to converge at incident proton energies less than 100 MeV. Improvement of the convergence can be achieved by cutting off the low partial waves (peripheral model [11]), unitarizing the multiple scattering series [10] or by converting the ordinary series into a convergent series by means of Pade-technique [7].

Not in every case it is necessary to perform protracted calculations. In recent years it turned out that in many cases the single scattering term of the ordinary multiple scattering series is sufficient to describe the shape of the peaks appearing in the experimental break-up spectra (quasi free scattering (QFS) and final state interaction (FSI) peak). For example it has been used with some success to extract the $n-n$ scattering length despite the fact that below 100 MeV the absolute magnitude of the peaks is reproduced very badly.

In the present paper we calculate the next term (square graph) of the scattering series to see how it changes the shape and the absolute magnitude of the spectra. The two body interaction has been choosen in the form of the linear approximation of the effective range theory. The results show that no improvement of the theoretical description can be achieved as compared with the single scattering term. This is in contrast to results obtained for the Tabakin potential [12].

2. Faddeev Equations for the Three-Body Transition Matrix

2.1. Transformation of the Faddeev equations for the wave function into Faddeev equations for the scattering amplitudes

At the beginning we repeat some formulae given in paper [8]. The connection between the cross section and the matrix elements for the elastic and inelastic $p-d$ scattering is given by

Elastic scattering: $$\left(\frac{d\sigma}{d\Omega}\right)_{c.m.} = \frac{4\pi^2}{h^4}\mu^2 \,|\,T_{el}\,|^2, \qquad (1a)$$

Break-up:
(deuteron scattered on protons) $$\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_s}\right)_{lab} = \frac{4\pi^2 m_d}{h^7 p_d}\rho_s |T_{inel}|^2. \qquad (1b)$$

Here, m_d is the mass of the deuteron and μ the reduced mass in the $p-d$ system. The quantity p_d denotes the laboratory momentum of the deuteron in the entrance channel and ρ_s the phase space factor. The matrix elements T_{el} and T_{inel} must be taken in the cms-system. The cross section given for the break-up refers to the so-called complete experiments.

As concerns the potential we assume in the further considerations that Coulomb and three body forces are neglected and that the two particle interaction is given by

$$V(ik) = V_1(ik)P_\sigma^{(t)}(ik)P_\tau^{(s)}(ik) + V_2(ik)P_\sigma^{(s)}(ik)P_\tau^{(t)}(ik) + V_3(ik)P_\sigma^{(t)}(ik)P_\tau^{(t)}(ik) + V_4(ik)P_\sigma^{(s)}(ik)P_\tau^{(s)}(ik). \quad (2)$$

The operators $P_{\sigma/\tau}^{(t/s)}$ project on singlet and triplet spin and isospin states correspondingly.

Using the potential (2) the matrix elements in eq. (1) can be expressed by matrix elements for different spin states [8]

$$|T_{el}|^2 = \frac{2}{3}|{}^eT_{3/2}|^2 + \frac{1}{3}|{}^eT_{1/2}|^2, \quad (3a)$$

$$|T_{inel}|^2 = \frac{2}{3}|{}^iT_{3/2}|^2 + \frac{1}{3}|{}^i_tT_{1/2}| + \frac{1}{3}|{}^i_sT_{1/2}|^2. \quad (3b)$$

As has been shown in [8] these matrix elements can be expressed by the following quantities (some indices omitted to simplify the notation)

$${}^eT_{3/2} = \frac{1}{2}\tau_{1,12} + \frac{\sqrt{3}}{2}\tau_{3,12} - \tau_{1,23} + \frac{1}{2}\tau_{1,31} - \frac{\sqrt{3}}{2}\tau_{3,31},$$

$${}^eT_{1/2} = -\frac{1}{4}\tau_{1,12} + \frac{3}{4}\tau_{2,12} - \frac{\sqrt{3}}{4}\tau_{3,12} + \frac{\sqrt{3}}{4}\tau_{4,12} - \tau_{1,23} - \frac{1}{4}\tau_{1,31} + \frac{3}{4}\tau_{2,31} + \frac{\sqrt{3}}{4}\tau_{3,31} - \frac{\sqrt{3}}{4}\tau_{4,31}, \quad (4a)$$

$${}^iT_{3/2} = -\sqrt{\frac{2}{3}}\tau_{3,12} + \frac{1}{\sqrt{2}}\tau_{1,23} + \frac{1}{\sqrt{6}}\tau_{3,23} - \frac{1}{\sqrt{2}}\tau_{1,31} + \frac{1}{\sqrt{6}}\tau_{3,31},$$

$${}^i_tT_{1/2} = -\sqrt{\frac{2}{3}}\tau_{3,12} - \frac{1}{2\sqrt{2}}\tau_{1,23} - \frac{1}{2\sqrt{2}}\tau_{2,23} - \frac{1}{2\sqrt{6}}\tau_{3,23} - \frac{1}{2}\sqrt{\frac{3}{2}}\tau_{4,23} + \frac{1}{2\sqrt{2}}\tau_{1,31} + \frac{1}{2\sqrt{2}}\tau_{2,31} - \frac{1}{2\sqrt{6}}\tau_{3,31} - \frac{1}{2}\sqrt{\frac{3}{2}}\tau_{4,31} \quad (4b)$$

$${}^i_sT_{1/2} = -\sqrt{\frac{2}{3}}\tau_{2,12} + \frac{1}{2}\sqrt{\frac{3}{2}}\tau_{1,23} - \frac{1}{2\sqrt{6}}\tau_{2,23} + \frac{1}{2\sqrt{6}}\tau_{3,23} - \frac{1}{2\sqrt{2}}\tau_{4,23} + \frac{1}{2}\sqrt{\frac{3}{2}}\tau_{1,31} - \frac{1}{2\sqrt{6}}\tau_{2,31} - \frac{1}{2\sqrt{2}}\tau_{3,31} + \frac{1}{2\sqrt{2}}\tau_{4,31}.$$

The quantities $\tau_{i,mn}$ are defined by

$${}^e\tau^S_{i,mn} \equiv {}^e\tau^S_i(\hat{k}_\ell) = \frac{3}{4}\frac{\hbar^2}{m}\lim_{k^2\to k_0^2}(k_0^2 - k^2)\int d\vec{\rho}_1 d\vec{\rho}_{23}\phi_d(\vec{\rho}_{23}) \times e^{-i\vec{k}\vec{\rho}_1}\psi_i^S(\vec{\rho}_{mn}, \vec{\rho}_\ell), \quad (5a)$$

$${}^{inel}\tau^S_{i,mn} \equiv {}^{inel}\tau^S_i(\vec{k}_{mn}, \vec{k}_\ell) = \frac{3}{4}\frac{\hbar^2}{m}\lim_{k^2\to k_\ell^2}(k_\ell^2 - k^2)\psi_i^S(\vec{k}_{mn}, \vec{k}) \quad (5b)$$

with $i=1,2,3,4$ and $(\ell,m,n)=(1,2,3),(2,3,1),(3,1,2)$. Here, ϕ_d denotes the deuteron wave function. The index i runs from 1 to 4 according to the 4 terms in the potential (2). The indices (ℓ,m,n) denote the three possibilities to introduce a system of Jacobi coordinates in a three-body system. The symbol $\hat{k}$ denotes the direction of the vector $\vec{k}$. This quantity and the vectors $\vec{k}_\ell$ and $\vec{k}_{mn}$ are defined by the experimental situation in the exit channel. The wave functions $\psi_i^s(\vec{f},\vec{k})$ are given by the Faddeev equations [8]*

$$\psi_i^S(\vec{f},\vec{k}) = (2\pi)^3 \phi_d(\vec{f})\,\delta(\vec{k}-\vec{k}_0)\,\delta_{1i}$$

$$-\frac{m}{\hbar^2(f^2-\bar{\eta})}\sum_{j=1}^{4}\int\frac{d\vec{k}'}{(2\pi)^3}\left[\alpha_{ij}^S t_i(\vec{f},-\vec{p}_2,\bar{\eta})+\beta_{ij}^S t_i(\vec{f},\vec{p}_2,\bar{\eta})\right]\times \tag{6}$$

$$\times\,\psi_j^S(\vec{p}_1,\vec{k}'),$$

where we introduced the following definitions: $\vec{k}_0$ = relative momentum between proton and deuteron in the entrance channel, E = total energy in the three particle system,

$$\vec{p}_1=\vec{k}+\vec{k}'/2,\quad \vec{p}_2=\vec{k}/2+\vec{k}',\quad \eta=\frac{m}{\hbar^2}(E+i0),\quad \bar{\eta}=(\eta-\frac{3}{4}k^2),$$

$S=(3/2,1/2)$, $i=1,2,3,4$

$$\alpha_{ij}^{3/2}=\frac{1}{2}\begin{pmatrix}-1 & 0 & -\sqrt{3} & 0\\ 0 & 0 & 0 & 0\\ \sqrt{3} & 0 & -1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix},\quad \beta_{ij}^{3/2}=\frac{1}{2}\begin{pmatrix}-1 & 0 & \sqrt{3} & 0\\ 0 & 0 & 0 & 0\\ -\sqrt{3} & 0 & -1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix},$$

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* Eq. (6) does not coincide completely with eq. (71) of [8]. In the present paper we used additionally the equality $\psi(\vec{p}_1,\vec{k})=\psi(-\vec{p}_1,\vec{k})$ which follows if we impose on the two-body potential the condition $V(\vec{r})=V(-\vec{r})$.

$$\alpha_{ij}^{1/2}=\frac{1}{4}\begin{pmatrix}1 & -3 & \sqrt{3} & -\sqrt{3}\\ -3 & 1 & \sqrt{3} & -\sqrt{3}\\ -\sqrt{3} & -\sqrt{3} & 1 & 3\\ \sqrt{3} & \sqrt{3} & 3 & 1\end{pmatrix},\quad \beta_{ij}^{1/2}=\frac{1}{4}\begin{pmatrix}1 & -3 & -\sqrt{3} & \sqrt{3}\\ -3 & 1 & -\sqrt{3} & \sqrt{3}\\ \sqrt{3} & \sqrt{3} & 1 & 3\\ -\sqrt{3} & -\sqrt{3} & 3 & 1\end{pmatrix}$$

The two-particle off-shell matrix is defined by the equation

$$t_i(\vec{k}',\vec{k},\eta)=V_i(\vec{k}'-\vec{k})-\frac{m}{\hbar^2}\int\frac{d\vec{k}''}{(2\pi)^3}\frac{V_i(\vec{k}'-\vec{k}'')}{(k''^2-\eta)}\,t_i(\vec{k}'',\vec{k},\eta) \tag{7}$$

with

$$V_i(\vec{k})=\int d\vec{r}\,e^{-i\vec{k}\vec{r}}\,V_i(\vec{r}).$$

The terms V_i are given in (2). Since the three particle wave function $\psi_i^S(\vec{f},\vec{k})$ is not needed explicitly to solve the problem but only the quantities $r_i(\vec{k}_{mn},\vec{k}_\ell)$ we will find an integral equation for these quantities. Let us first consider the inelastic scattering. We apply the limiting procedure (5b) to the integral equation (6) and get

$$r_i^S(\vec{k}_{mn},\vec{k}_\ell)=\sum_{j=1}^{4}\int\frac{d\vec{k}'}{(2\pi)^3}\left[\alpha_{ij}^S t_i(\vec{f},-\vec{p}_2,\bar{\eta})+\beta_{ij}^S t_i(\vec{f},\vec{p}_2,\bar{\eta})\right]\times \tag{8}$$

$$\times\,\psi_j^S(\vec{p}_1,\vec{k}')\Big|_{\substack{\vec{k}=\vec{k}_\ell\\ \vec{f}=\vec{k}_{mn}}}.$$

Here we used the relations

$$\bar{\eta}=\eta-\frac{3}{4}k^2,\quad \eta=\frac{3}{4}k_1^2+k_{23}^2=\frac{3}{4}k_2^2+k_{31}^2=\frac{3}{4}k_3^2+k_{12}^2. \tag{9}$$

Since all the quantities on the right-hand side of eq. (8) are defined for arbitrary values $\vec{k}$ and $\vec{f}$ we can replace $\tau_i^s(\vec{k}_{mn},\vec{k}_\ell)$ by $\tau_i^s(\vec{f},\vec{k})$.

Then the wave function $\psi_i^s(\vec{f},\vec{k})$ can be written in the form

$$\psi_i^S(\vec{f},\vec{k})=(2\pi)^3\phi_d(\vec{f})\delta(\vec{k}-\vec{k}_0)\delta_{1i}-\frac{\tau_i^S(\vec{f},\vec{k})}{(f^2+\frac{3}{4}k^2-\eta)}. \tag{10}$$

Inserting (10) into the right-hand side of eq. (8) we get an integral equation for

$$\tau_i^S(\vec{f},\vec{k})=-\frac{G(\vec{p}_{10})t_{i1}^S(\vec{f},\vec{p}_{20},\eta)}{(\hbar^2/m)(p_{10}^2+\alpha_t^2)}-\sum_{j=1}^{4}\int\frac{d\vec{k}'}{(2\pi)^3}\frac{t_{ij}^S(\vec{f},\vec{p}_2,\eta)\tau_j^S(\vec{p}_1,\vec{k}')}{(\hbar^2/m)(k^2+\vec{k}\vec{k}'+k'^2-\eta)} \tag{11}$$

with $\vec{p}_{10}=(\vec{k}+\vec{k}_0/2)$, $\vec{p}_{20}=(\vec{k}/2+\vec{k}_0)$ and an effective two-body t-matrix defined by

$$t_{ij}^S(\vec{k},\vec{k}',\eta)=\alpha_{ij}^S t_i(\vec{k},-\vec{k}',\eta)+\beta_{ij}^S t_i(\vec{k},\vec{k}',\eta). \tag{12}$$

Moreover we introduced a form factor $G(\vec{k})$ by

$$G(\vec{k})=(-\hbar^2/m)(k^2+\alpha_t^2)\phi_d(\vec{k}). \tag{13}$$

Now we turn to the corresponding equation for the elastic scattering. First of all we replace the space dependent wave function in eq. (5a) by its Fourier transform

$$\psi_i^S(\vec{\rho}_{mn},\vec{\rho}_\ell)=\int\frac{d\vec{k}_1 d\vec{k}_{23}}{(2\pi)^6}e^{i(\vec{k}_1\vec{\rho}_\ell+\vec{k}_{23}\vec{\rho}_{mn})}\psi_i^S(\vec{k}_{23},\vec{k}_1). \tag{14}$$

According to the transformation formulae for Jacobi coordinates and momenta (see eq. (7) in /8/) we can write

$$(\vec{k}_1\vec{\rho}_\ell+\vec{k}_{23}\vec{\rho}_{mn})=(\vec{k}'\vec{\rho}_1+\vec{f}'\vec{\rho}_{23}),\quad(\vec{k}',\vec{f}')=\begin{matrix}(\vec{k}_1,\vec{k}_{23})\\(\vec{k}_2,\vec{k}_{31})\\(\vec{k}_3,\vec{k}_{12})\end{matrix} \tag{15}$$

$$\text{for } (\ell,m,n)=\begin{matrix}(1,2,3)\\(3,1,2)\\(2,3,1)\end{matrix}$$

Then eq. (5a) can be written

$$^{e}\tau_i(\hat{k}_\ell)=\frac{3}{4}\frac{\hbar^2}{m}\lim_{k^2\to k_0^2}(k_0^2-k^2)\int\frac{d\vec{k}_{23}}{(2\pi)^3}\phi_d(-\vec{f}')\delta(\vec{k}-\vec{k}')\psi_i^S(\vec{k}_{23},\vec{k}_1). \tag{16}$$

At first we investigate the case $(\ell,m,n)=(1,2,3)$. We get

$$^{e}\tau_i(\hat{k}_1)=\int\frac{d\vec{k}_{23}}{(2\pi)^3}\phi_d(-\vec{k}_{23})\Big[\frac{3}{4}\frac{\hbar^2}{m}\lim_{k^2\to k_0^2}(k_0^2-k^2)\psi_i^S(\vec{k}_{23},\vec{k})\Big]. \tag{17}$$

According to eq. (10) we can replace the wave function in eq. (17) by $\tau_i^S(\vec{k}_{23},\vec{k})$. Then we apply the limiting procedure to $\tau_i^S(\vec{k}_{23},\vec{k})$ using eq. (11). The amplitude $\tau_i^S(\vec{k}_{23},\vec{k})$ has a pole singularity at $k=k_0$ which is caused by a pole in the two particle transition matrix t_1. We have

$$\frac{3}{4}\frac{\hbar^2}{m}\lim_{k^2\to k_0^2}(k_0^2-k^2)\,t_1(\vec{k}',\vec{k},\eta)=G(\vec{k}')G(\vec{k}), \tag{18}$$

where we used the relations (g_0, g - two particle Green functions without and with potential correpondingly)

$$t=g_0^{-1}g\,V=g_0^{-1}\sum_n\frac{|\phi_n\rangle\langle\phi_n|V}{(E-E_n)}\approx\frac{V|\phi_d\rangle\langle\phi_d|V}{(E-E_d)},\quad E\approx E_d \tag{19}$$

$$t(\vec{k}', \vec{k}, \eta) = \langle \vec{k}' | t | \vec{k} \rangle = \frac{\langle \vec{k}' | V | \phi_d \rangle \langle \phi_d | V | \vec{k} \rangle}{(\hbar^2/m)(\eta + \alpha_t^2)} = \frac{G(\vec{k}')G(\vec{k})}{(\hbar^2/m)(\eta + \alpha_t^2)} .$$

It should be noticed that only the quantity ${}^{e}\tau^{S}_{1,23}$ gives a contribution different from zero. For all other index combinations the integral in eq. (16) has no singularity at $k = k_0$.

Using eq. (18) formula (12) reads

$$t^S_{1j}(\vec{k}', \vec{k}, \eta) = \gamma^S_{1j} \frac{G(\vec{k}')G(\vec{k})}{(\hbar^2/m)(\eta + \alpha_t^2)}, \quad \gamma^S_{ij} = \alpha^S_{ij} + \beta^S_{ij}, \quad \eta \approx -\alpha_t^2 \tag{20}$$

Regarding eqs. (17), (11), (18) and the normalization

$$\int \frac{d\vec{f}}{(2\pi)^3} \phi_d(-\vec{f}) \phi_d(\vec{f}) = 1 \tag{21}$$

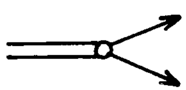
for the elastic scattering we get finally

$$ {}^{e}\tau^{S}_{1}(\hat{k}_1) = \Big[-\gamma^S_{11} \frac{G(\vec{p}_{10})G(\vec{p}_{20})}{(\hbar^2/m)(p_{10}^2 + \alpha_t^2)} - \sum_{j=1}^{4} \gamma^S_{1j} \int \frac{d\vec{k}'}{(2\pi)^3} \times \tag{22}$$

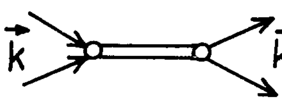
$$\times \frac{G(\vec{p}_2)\tau^S_j(\vec{p}_1, \vec{k}')}{(\hbar^2/m)(k^2 + \vec{k}\vec{k}' + \vec{k}'^2 - \eta)} \Big]_{k = k_0}$$

2.2. Graph representation of the Faddeev equations

We introduce the following graph elements [8]

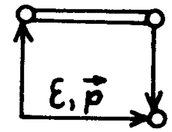
a)  $\vec{k}$ $\hat{=}$ $G(\vec{k})$ vertex

effective two-body matrix (s.eq. (12)) (23)

b) $\vec{k}$  $\vec{k}'$ $\hat{=}$ $t^S_{1j}(\vec{k}', \vec{k}_0, \eta)$

c)  $\hat{=}$ $(E(\vec{p}) - E - i0)^{-1}$, $E(\vec{p}) = p^2/2m$ propagator

(23)

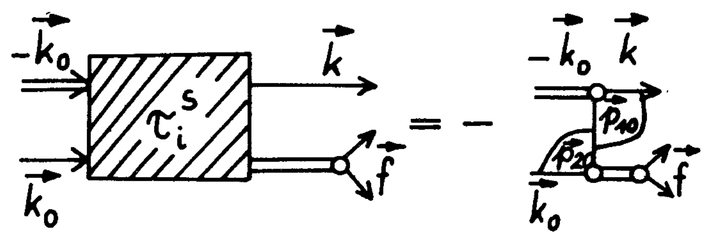
d)  $\hat{=}$ $\int \frac{d\varepsilon\, d\vec{p}}{(2\pi)^3} F(\varepsilon, p)$ closed loop

Additionally we have the conditions:

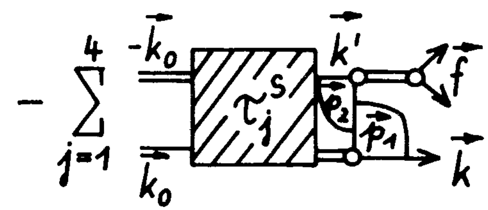
e) Energy and momentum conservation must be valid for each node of the graph.

f) The total mathematical expression corresponding to a given graph is obtained by multiplying the expressions for the different graph elements.

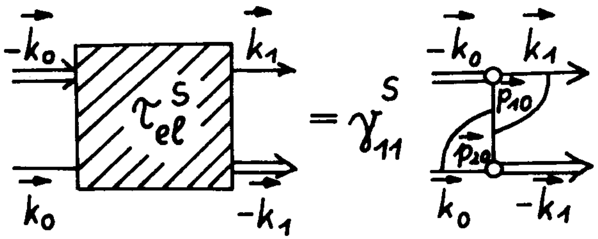
Then the integral equation system for the quantities $\tau_i(\vec{f}, \vec{k})$ may be represented in the form

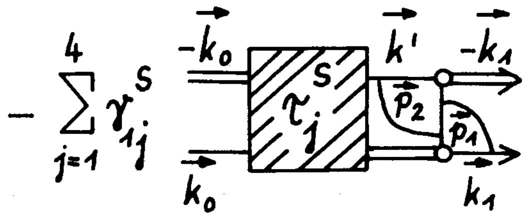


(24a)



For the elastic scattering we get





(24b)

As can be seen from (24b) to find the elastic amplitude we need not solve an integral equation if the inelastic amplitude is already known. In this case the elastic amplitude can be found simply by integration.

3. Faddeev Equations in Zero Range Approximation

If the two particle interaction is choosen in zero range approximation of the effective range theory the graph elements defined in the preceding chapter are given by /8/

$$G(\vec k) = -\frac{\hbar^2}{m}\sqrt{8\pi\alpha_t},\quad t_1(\vec k',\vec k,\eta) = \frac{4\pi\hbar^2/m}{(\alpha_t + i\sqrt{\eta})},\quad t_2(\vec k',\vec k,\eta) = \frac{4\pi\hbar^2/m}{(\alpha_s + i\sqrt{\eta})}, \tag{25}$$

$t_3 = t_4 = 0.$

with $\sqrt{\eta} = i\sqrt{|\eta|}$ for $\eta < 0$.

We define the following quantities

$$\begin{aligned}
\tau_1^{3/2}(\vec f,\vec k) &= 4\pi G\bar a_{3/2}(\vec k) = 4\pi G a_{3/2}(\vec k)/(\vec k^2 - k_0^2),\\
\tau_1^{1/2}(\vec f,\vec k) &= 4\pi G\bar a_t(\vec k) = 4\pi G a_{1/2}(\vec k)/(k^2 - k_0^2),\\
\tau_2^{1/2}(\vec f,\vec k) &= -4\pi G\bar a_s(\vec k) = -4\pi G b_{1/2}(\vec k)/(k^2 - k_0^2).
\end{aligned} \tag{26}$$

Inserting the expressions (25) and (26) into eq. (11) we get

$$\begin{aligned}
\bar a_{3/2}(\vec k) &= -\frac{1}{(\gamma_k - \alpha_t)}\left[\frac{1}{N(\vec k,\vec k_0)} + \int\frac{d\vec k'}{2\pi^2}\frac{\bar a_{3/2}(\vec k')}{N(\vec k,\vec k')}\right],\\
\bar a_t(\vec k) &= \frac{1}{(\gamma_k - \alpha_t)}\left[\frac{1/2}{N(\vec k,\vec k_0)} + \int\frac{d\vec k'}{2\pi^2}\frac{[1/2\bar a_t(\vec k') + 3/2\bar a_s(\vec k')]}{N(\vec k,\vec k')}\right],\\
\bar a_s(\vec k) &= \frac{1}{(\gamma_k - \alpha_s)}\left[\frac{3/2}{N(\vec k,\vec k_0)} + \int\frac{d\vec k'}{2\pi^2}\frac{[3/2\bar a_t(\vec k') + 1/2\bar a_s(\vec k')]}{N(\vec k,\vec k')}\right].
\end{aligned} \tag{27a}$$

with

$$N(\vec k,\vec k_0) = (k^2 + \vec k\vec k_0 + k_0^2 - \eta),$$

$$\gamma_k = \begin{cases} \sqrt{3k^2/4 - \eta} \\ -i\sqrt{\eta - 3k^2/4} \end{cases} \quad \text{for } (3k^2/4 - \eta) \gtrless 0$$

The equations (27a) are identical with the well known equations of Skornyakov and Ter-Martirosyan /14/. For the elastic scattering we get from eqs. (22) and (25)

$$\tau_1^{3/2}(\hat k_1) = \frac{mG^2}{\hbar^2}\left[\frac{1}{N(\vec k,\vec k_0)} + \int\frac{d\vec k'}{2\pi^2}\frac{\bar a_{3/2}(\vec k')}{N(\vec k,\vec k')}\right]_{k=k_0} = \frac{3\pi\hbar^2}{m}a_{3/2}(\vec k)_{k=k_0}.$$

$$r_1^{1/2}(\hat{k}_1) = -\frac{mG}{\hbar^2}\left[\frac{1/2}{N(\vec{k},\vec{k}_0)} + \int \frac{d\vec{k}'}{2\pi^2}\,\frac{[1/2\,\bar{a}_t(\vec{k}') + 3/2\,\bar{a}_s(\vec{k}')]}{N(\vec{k},\vec{k}')}\right]_{k=k_0} =$$

$$= \frac{3\pi\hbar^2}{m}\, a_t(\vec{k})_{k=k_0} \qquad \text{(27b)}$$

To get the last equality on the right-hand side of eq. (27b) we used the definitions (26) and the fact that $\lim_{k\to k_0} (\gamma_k - \alpha_t)/(k^2 - k_0^2) = 3/8\alpha_t$.

4. Calculation of the Pole Graph and the Square Graph

4.1. Method of calculation

In chapter 2 we have shown systematically how to calculate the elastic and inelastic p–d scattering starting from a given two-body local potential. It is not the aim of this paper to deal with mathematical methods to solve these equations. We only investigate the physical meaning of the inhomogeneous term (pole graph) and of the first iteration (square graph) of the integral equation (27). In this case the amplitude is given by

$$\bar{a}_{3/2}(\vec{k}) = -f_t(k)\left[\frac{1}{N(\vec{k},\vec{k}_0)} - J_t(\vec{k})\right],$$

$$\bar{a}_t(\vec{k}) = f_t(k)\left[\frac{1/2}{N(\vec{k},\vec{k}_0)} + \frac{1}{4}J_t(\vec{k}) + \frac{9}{4}J_s(\vec{k})\right], \qquad \text{(28a)}$$

$$\bar{a}_s(\vec{k}) = f_s(k)\left[\frac{3/2}{N(\vec{k},\vec{k}_0)} + \frac{3}{4}J_t(\vec{k}) + \frac{3}{4}J_s(\vec{k})\right].$$

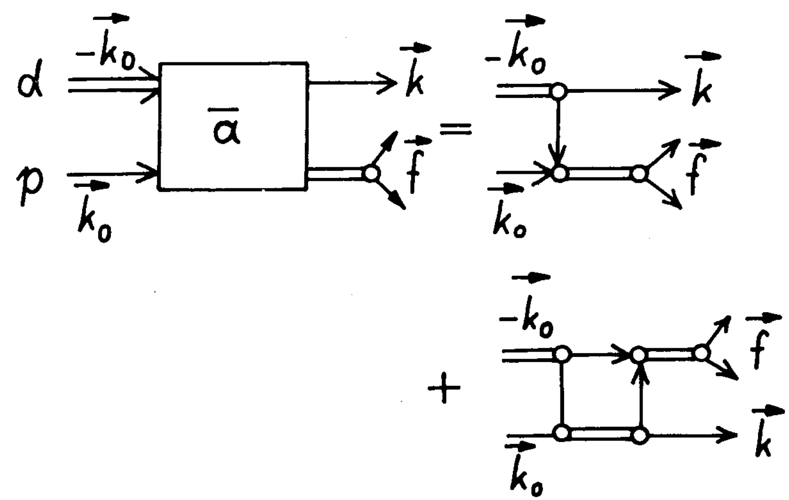
with

$$J_\beta(\vec{k}) = \frac{1}{2\pi^2}\int d\vec{k}'\,\frac{f_\beta(k')}{N(\vec{k}',\vec{k}_0)\,N(\vec{k}',\vec{k}_0)}, \quad \beta = \begin{matrix}\text{singlet}\\ \text{triplet}\end{matrix} \qquad \text{(28b)}$$

In actual calculations instead of the zero range approximation we used the linear approximation for the two-body matrix elements. It is given by

$$f_\beta(k) = \frac{1}{\left(-a_\beta^{-1} + \gamma_k - \frac{r_{0\beta}}{2}\gamma_k^2\right)},$$

a_β = scattering length, $r_{0\beta}$ = effective radius. In simplified graphical representation we have for eq. (28)



(29)

The first graph has been investigated in the papers [8,13]. It turned out that this graph contains both the spectator model (impuls approximation) and the final state interaction model (Watson-Migdal approximation).

The calculations at 7 MeV proton energy have shown that the general features of the experimental spectra are reproduced by this graph but that the theory overestimates the absolute value by about an order of magnitude.

In the present paper we calculated the pole graph for energies up to 100 MeV. Additionally the second graph (square graph) was calculated. Here the main problem consists in calculating the integral (28b). To do this the functions $N(\vec{k},\vec{k}_0)$ and $N(\vec{k},\vec{k}')$ in the dominator of eq. (28b) were decomposited into a series of Legendre polynomials. After some simple calculations we end up with

$$J_\beta(\vec{k}) = \frac{2}{\pi k k_0} \sum_{\ell=0}^{\infty} (2\ell+1) A_{\beta\ell}(k) P_\ell\left(\frac{\vec{k}_0 \vec{k}}{k_0 k}\right),$$

$$A_{\beta_\ell}(k) = \int_0^\infty dk' f_\beta(k') Q_\ell\left(\frac{-A(k')}{kk'}\right) Q_\ell\left(\frac{-B(k')}{k_0 k'}\right), \tag{30}$$

where

$$A(k') = (k'^2 + k^2 - \eta), \quad B(k') = (k'^2 + k_0^2 - \eta).$$

The functions P_l and Q_l are the Legendre polynomials of the first and second kind correspondingly. In the region $0 \le k' < \infty$ the integrand in (30) has two pole singularities caused by $f_t(k')$ and two logarithmic branch points which are contained in Q_ℓ. Thus to calculate the integral we have deformed the integration contour into the complex k'-plane (compare /9/). To check the numerical results independently of the described method the integral (28b) was additionally solved by another method. In this case the integration over the angles $\vec{k}'/k'$ was done analytically by standard methods and the integration over k' was performed numerically. No contour deformation was used in this case. Instead of this in the vicinity of the poles of $f_t(k')$ the integral has been solved analytically by decomposing the integrand into a series. The logarithmic singularities are so weak that we can leave out small regions of the integration contour containing the singularities without causing great error in the results.

4.2. Results and conclusions

The results of the calculations are given in figures 1 to 5. We can make the following conclusions:

For the break-up case the pole graph describes the main features of the experimental spectra (FSI and QFS mechanism). The agreement of the theoretical and experimental absolute values is improving with increasing energy. For the p−p quasi free scattering we have an approximate agreement of the theoretical and experimental absolute values at 100 MeV (fig. 3). It should be noted that the experimental n−p quasi free scattering cross section in the investigated kinematic region (fig. 1) is about twice the value of the p−p quasi free scattering. This difference cannot be explained by the antisymmetrization effect as can be seen comparing the theoretical curves for the n−p and p−p quasi free scattering in fig. 1. To explain this we must introduce either Coulomb forces or assume the p−p and n−p nuclear potentials to be different. As concerns the square graph it describes qualitatively the FSI mechanism whereas it has a minimum in the region of the QFS mechanism. From the curves representing the sum of the two graphs we see that the square graph in general gives a correction in the wrong direction. The calculations for the elastic scattering show that the inclusion of the square graph gives an improvement of the angular distribution in forward direction as compared with the pole graph alone. The absolute magnitude of the spectra however cannot be reproduced.

In general we can conclude that under the given assumptions for the two particle interaction and in the discussed energy region the investigated models can be used to describe some qualitative features of the spectra, but for a quantitative description a more accurate solution of the integral equation is necessary.

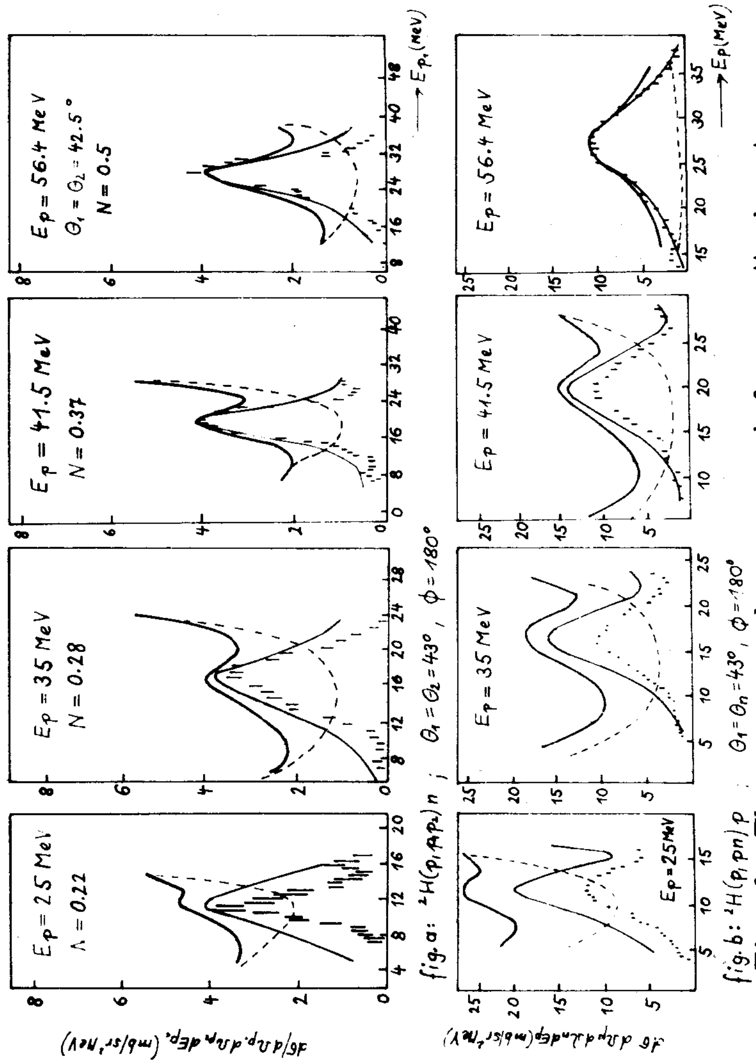


Fig. 1. The p–p and n–p quasi free scattering at different incident energies. Experimental points taken from /12/ ——— pole graph, - - - - square graph, ▬▬▬ pole + square graph. The theoretical curves are normalized by the factor N.

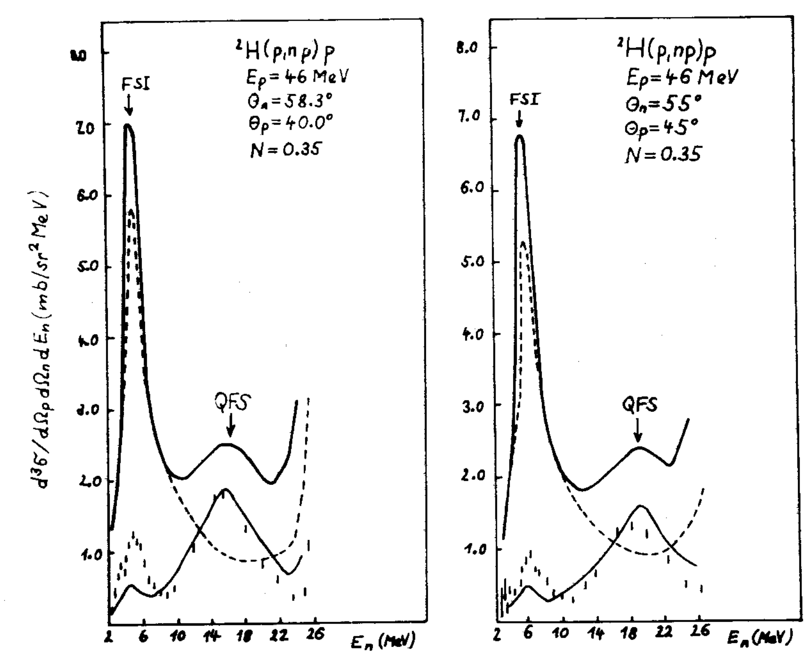


Fig. 2. Pole and square graph for n–p quasi free scattering (QFS) and for final state interaction (FSI). Experimental points are taken from /15/ . See caption of fig. 1.

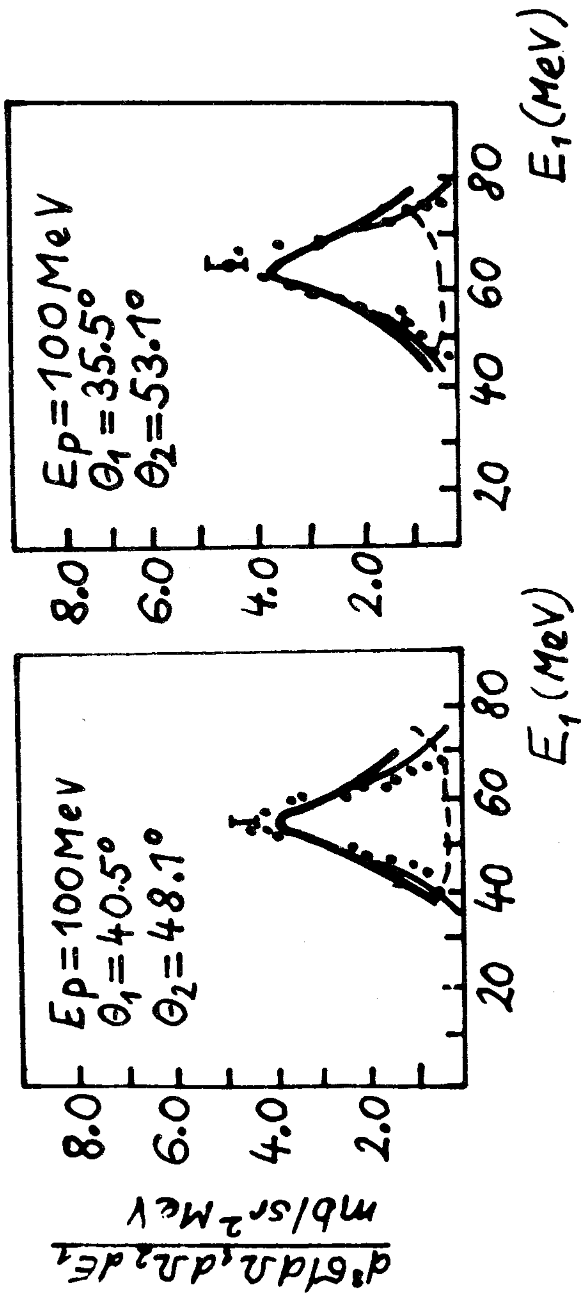


Fig. 3. Quasi free scattering in the reaction $^2H(p,pp)n$. Experimental points taken from /16/. See caption of fig. 1.

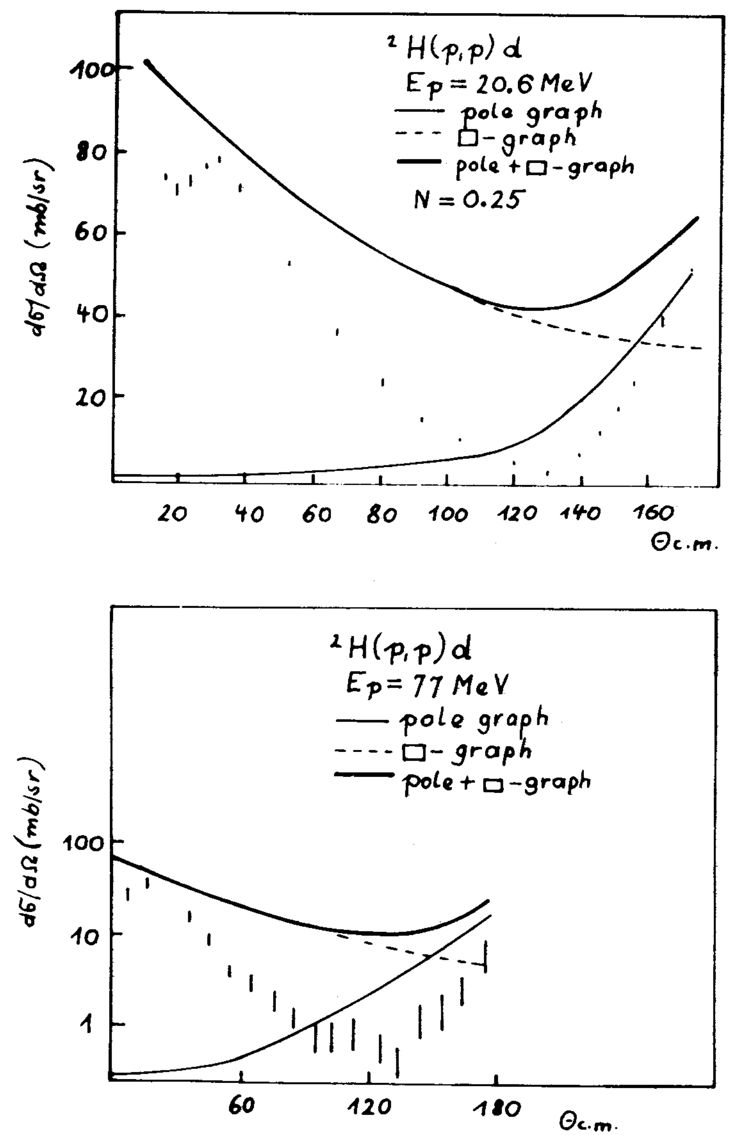


Fig. 4. Proton-deuteron elastic scattering. Experimental points taken from /17,18/.

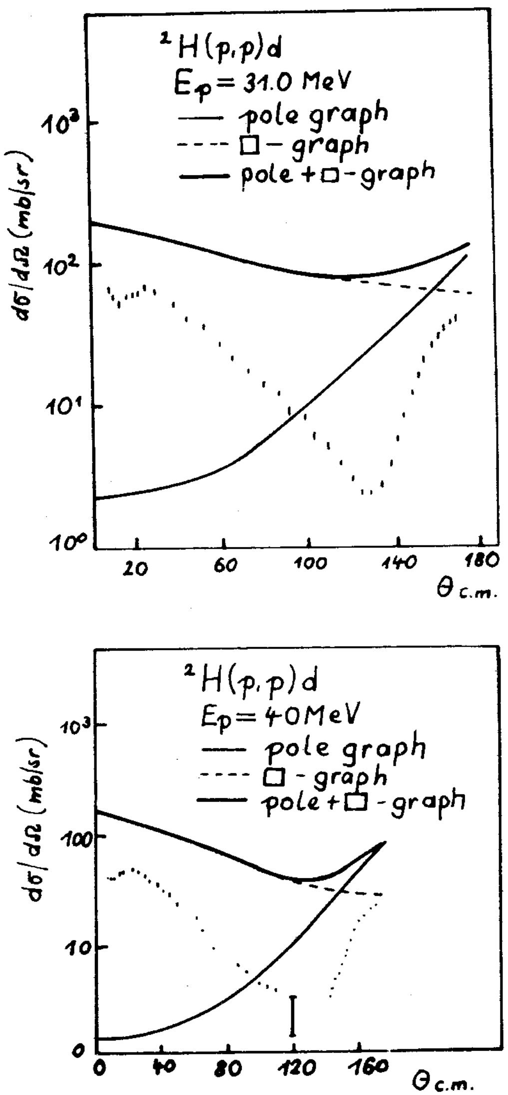


Fig. 5. Proton-deuteron elastic scattering. Experimental points taken from /19,20/.

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