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1. Introduction

The process of muon capture by deuteron

$$\mu^- + d \rightarrow n + n + \nu \qquad (1)$$

has been investigated both theoretically [1-5] and experimentally [6,7]. Studying of this process is of great interest by the following two main reasons. The first one is due to the possibility of extracting an information on the weak interaction coupling constants and, especially, on the constant of the induced pseudoscalar which is known presently with a rather large error. The second one is due to the possibility of obtaining the independent information on parameters of nucleon-nucleon interaction at low energies. In considering process (1) it appears to be possible also to study some effects of particular features of the nuclear structure on characteristics of muon capture which are to be neglected due to difficulties of their consideration in heavier nuclei.One of these moments is the investigation of influence of small components of the ground state wave function of an initial nucleus on various characteristics of muon capture.

Though the basis feature of process (1) has been well established, nevertheless the obtained theoretical results considerably differ from each other for some cases (see, e.g., ref. [4]). This is connected not only with the different choice of values of coupling constants but also with a type of approximations used in concrete calculations: taking into account the D_2-component for the deuteron wave function and final state interaction, correct considering of the so-called velocity terms of muon-nucleon interaction, etc. As the results of recent investigations for heavier nuclei have revealed, the latter can appear to dominate in some cases. Thus, to obtain the reliable quantitative information on characteristics of process (1) it seems to be important to study their dependence of applied approximations and to find rigorous quantitative results.

Most of papers dealt only with the integral characteristic - capture rate. The use of the integral characteristic only does not allow one to exploit in full the advantages of the two-body problem. Studying of the differential characteristics: energy spectrum, energy dependence of asymmetry of the neutron angular distribution., etc.,will make it possible to obtain a comprehensive information both on the deuteron structure and on the two-neutron interaction.

The present paper is devoted to some problems of the muon-deuteron interaction. We investigate the following characteristics of process (1): the capture rate, energy spectrum and asymmetry of the neutron angular distribution in the capture of polarized muons and their dependence on a character of nucleon-nucleon and muon-nucleon interactions. The values for muon-nucleon interaction constants are taken to be conventional ones. Three possible cases are treated for the initial state of a mesoatom: doublet $F_- = J_i - 1/2$, quartet $F_+ = J_i + 1/2$ and statistical population. Here F is the mesoatom spin and J_i is the deuteron spin.

A special attention is paid to asymmetry of the neutron angular distribution. The obtained recently theoretical results on the behaviour of this characteristics have revealed its large sensitivity both to the details of muon-nucleon interaction and to those of nucleon-nucleon one. The investigation of asymmetry for the capture by deuteron makes it possible to avoid many model-dependent approximations and to check very accurately the main regularities specific for this quantity.

II. Basic Statements

1. For description of the processes of muon capture by atomic nuclei the use is made of the impulse approximation. In this approximation the muon-nucleon interaction Hamiltonian is as follows:

$$H_\mu = \frac{1}{2}(1 + \vec{\sigma}\vec{\nu}) \sum_{i=1}^{A} \tau_-(i)\{G_V 1 \cdot 1_i + G_A \vec{\sigma}\vec{\sigma}_i + G_P \vec{\sigma}_i \vec{\nu} + \tag{2}$$

$$+ h_{vel}(i)\}.$$

Here $\vec{\nu}$ is the unit vector along the neutrino momentum. The so-called velocity terms (they are proportional to the nucleon momentum $\vec{p}_i$ in a nucleus) are collected into one group:

$$h_{vel}(i) = g_A \vec{\sigma} \frac{\vec{p}_i}{M_N} + g_V \vec{\sigma} \frac{\vec{p}_i}{M_N}. \tag{3}$$

The muon-nucleon interaction constants are defined in the standard manner:

$$G_V = g_V\left(1 + \frac{E_\nu}{2M_N}\right)$$

$$G_A = g_A - (g_V + g_M)\frac{E_\nu}{2M_N}$$

$$G_P = [(g_P - g_A) - (g_V + g_M)]\frac{E_\nu}{2M_N},$$

and are taken to be conventional ones:

$$G \cos\theta_c = 1,42 \cdot 10^{-49} \text{ erg. cm}^3$$

$$g_A(0)/g_V(0) = -1.23$$

$$g_P = \frac{2m_\mu M_N}{q^2 + m_\pi^2} g_A(0)$$

$$g_M = 3.7 g_V,$$

where $q^2 = m_\mu(2E_\nu - m_\mu)$ is the four-momentum transfer, E_ν is the neutrino energy. The dependence of the constants on a four-momentum transfer was taken in the dipole approximation:

$$f(q^2) = \frac{1}{[1 + (\frac{q}{M})^2]^2} ,$$

where

$$M_V^2 = 0.71\ \mathrm{GeV}^2/c^2 \quad \text{and} \quad M_A^2 = 0.79\ \mathrm{GeV}^2/c^2 .$$

2. For describing the two-nucleon system we used the Reid potential /8/ both with the soft and with the hard core. The wave functions for the bound state and continuous spectrum have been found by numerical solving of equations by the phase-function method. Here we note at once that both of the variants of the potential have practically resulted in the same values of the investigated characteristics.

To the used potential with hard core there correspond the scattering lengths

$$a_{np}(^{13}S) = 5.4F \qquad a_{nn}(^{31}S) = -16.7F$$

and effective ranges

$$r_0(^{13}S) = 1.724F \qquad r_0(^{31}S) = 2.87F .$$

The appropriate wave function contains 6.50% of the D-component admixture.

3. The differential rate of process (1) in the absorption of partially polarized muons from a state F of a mesoatom has the form

$$\frac{d\Lambda_F}{dE_n\, d\Omega} = A(E_n)\{1 + \langle F\rangle \frac{B(E_n)}{A(E_n)} \cos\theta\} . \qquad (4)$$

Here $\langle F\rangle / F = P_F$ is the degree of polarization of the mesoatom in the state F. The quantities $A(E_n)$ and $B(E_n)$ are a combination of nuclear matrix elements and interaction constants (see, e.g., ref. /9/), θ is the angle between the direction of emitted neutron and muon spin, E_n is the energy of one of the neutrons. The integration over variables of the second neutron has been performed. The quantity P_F is connected with the residual polarization P_μ of muons on K-orbit in the following way:

$$P_\mu = \begin{cases} P_F = \frac{2}{3}\langle F_+\rangle & F_+ = J_i + 1/2 \\ -\frac{2J_i - 1}{2J_i + 1} P_F = -\frac{2}{3}\langle F_-\rangle & F_- = J_i - 1/2 . \end{cases}$$

The asymmetry coefficient for the neutron angular distribution is defined by the relation

$$a(E_n) = \frac{B(E_n)}{A(E_n)} . \qquad (5)$$

III. Calculation Results and Discussions

1. The capture rate.

Values of the capture rates Λ_F from various states of mesodeuterium are listed* in Tables 1-3 as functions of the structure both of nucleon-nucleon interaction and of muon-nucleon one.

A. As follows from the results listed in Table 1, allowing for the interaction in S-wave of the relative motion of emitted neutrons (the spin of two neutrons is $S_f = 0$) gives rise to increasing capture rate which is due to the attractive nature of the potential. The interaction

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*We do not consider an effect of finite sizes of the deuterium nucleus. Taking into account of this effect results in the appearance of a factor of the order $R_d \approx 0.98$ by which one should multiply all the listed results.

Table 1
The effect of neutron interaction on the partial and total muon capture rates in deuteron: 1 - the plane-wave approximation, 2 - the neutron interaction is taken into account.

Spin of two neutrons in the final state	$\Lambda_{3/2}$, sec^{-1}		$\Lambda_{1/2}$, sec^{-1}	
	1	2	1	2
$S_f = 0$	6.6	8.3	198	246
$S_f = 1$	2.4	2.6	129	134
Σ	9.0	10.9	327	380

Table 2
The D-component contribution and that of the velocity terms to the rate of muon capture by deuterons: 1 - without D-component, 2 - with D-component.

	The velocity terms are neglected		The velocity terms are taken into account	
	1	2	1	2
$\Lambda_{3/2}$, sec^{-1}	9.9	10.3	10.3	10.9
$\Lambda_{1/2}$, sec^{-1}	402	374	408	380

Table 3
Comparison of the results of calcultaions of the muon capture rate performed by various approaches.

Paper	The capture rate			The coupling constant	
	$\Lambda_{1/2}$, sec^{-1}	$\Lambda_{3/2}$, sec^{-1}	Λ_{STAT} sec^{-1}	g_P/g_A	g_A/g_V
/2/	334	15	121	7.2	-1.19
/3/	450	30	170	6.5	-1.15
/4/	313	12	112	8.35	-1.13
/5/	381	10.7	134	7	-1.226
Present paper	380	11	134	7*	-1.23
Experiment	$365 \pm 96^{/6/}$	$445 \pm 60^{/7/}$			

*At the maximum point of energy spectrum.

in P-wave (the spin of two neutrons is $S_f = 1$) results in a small change of the capture rate only.

B. Taking into account of the D -component for the wave function leads to a noticeable change of the rate: it decreases nearly by 7% in the case of absorption from the doublet state and increases by the same value in the case of capture from the quartet state (see Table 2).

C. The influence of the velocity terms of muon-nucleon interaction on the value of Λ_F proves to be of the same order: $\Lambda_{1/2}$ increases by 2% and $\Lambda_{1/2}$ by 6%. The relatively small change of Λ_F as a function of the velocity terms is explained by that in the given process the main contribution comes to the allowed Gamov-Teller ^{31}S - ^{13}S transition whereas their influence is most revealed in the first forbidden transitions.

D. In Table 3 the results of this paper are compared with those of other papers and with the presently available experimental data. The difference of theoretical results is not only due to the different choice of interaction constants (it is removed by reducing then to the common constants) but also due to the character of the above discussed approximations used in the cited papers. In particular, in paper /4/ the interaction of emitted neutron is not allowed for, and in paper /2/ the D-component is taken into account via the normalization factor only. Too large value of the rate from paper /3/ is connected with some inaccuracies of calculations.

The results obtained evidence that in describing the capture rate in deuteron it is of practical importance to consider all the indicated above factors. Though the existing experimental data are consistent with all the calculation results, however, this is due to the large experimental errors. Increasing the measurement accuracy will allow one to discriminate the results obtained under various assumptions about the nature of nucleon-nucleon interaction.

2. Neutron Energy Spectra

The neutron spectra for muon capture from the doublet and quartet states are given in Figs. 1 and 2. The spectrum from doublet state (Fig. 1) is given with the deuteron D-component. For the capture from quartet state the spectrum is given for the two variants: with and without D-component. Including the D-component for both the cases changes the spectrum shape: the yield of slow neutrons decreases and that of fast neutrons increases.

3. Asymmetry of the Neutron Angular Distribution in the Capture of Polarized Muons

The recent theoretical investigations have shown that coefficient of asymmetry of the neutron angular distribution has a rather complicated dependence both on the character of final state interaction and on the structure of muon-nuclear Hamiltonian. All these conclusions can be verified very reliably in considering the process on deuteron. In this case it is possible to avoid many approximations inevitable for describing more complex nuclei.

Asymmetry of the neutron angular distribution without the velocity terms is presented in Fig. 3. Two cases are considered: a) the neutrons in final state are free and b) their interaction is taken into account. As in the case of more complex nuclei, allowing for the interaction results in a sharp change of energy dependence of the quantity $a(E_n)$.

The velocity terms for deuteron are not large. However, even this value considerably influences the asymmetry of angular distribution. The corresponding results are shown in Fig. 4. Both in the plane-wave approximation and in allowing for the interaction the energy dependence of asymmetry proves to be different. Thus, for the theoretical analysis of this quantity it is of principal importance to take into account both the velocity terms and the final state interaction.

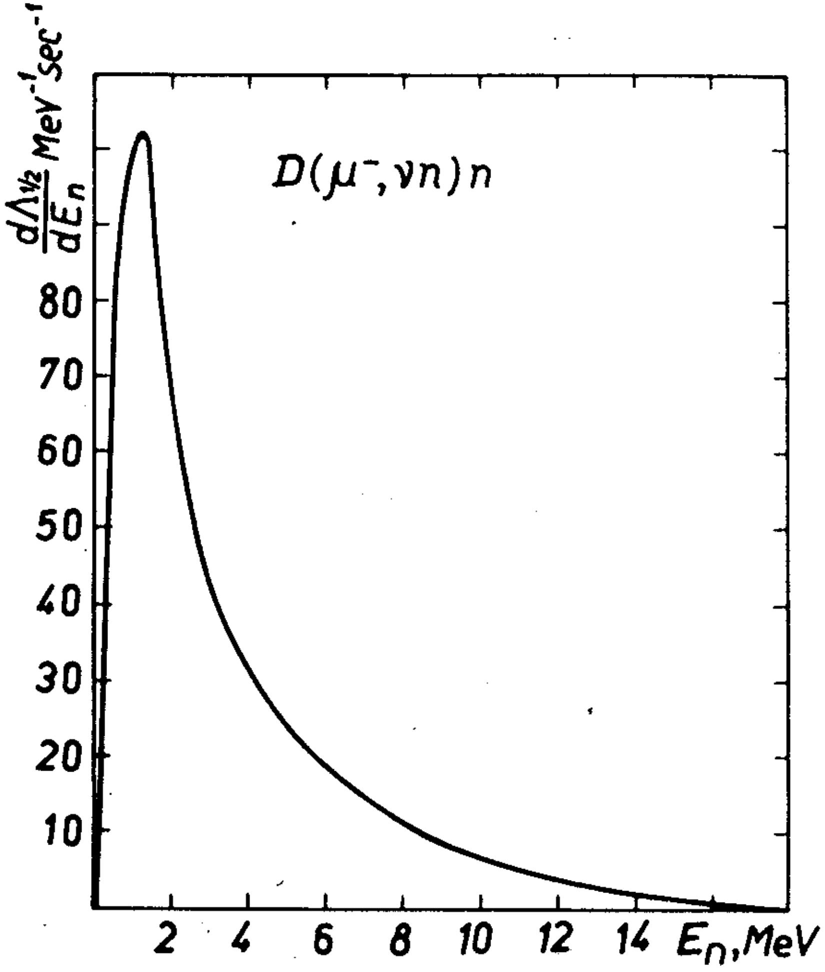

Fig. 1. The neutron energy spectrum in process $\mu^-(d,2n)\nu$. The capture from doublet state.

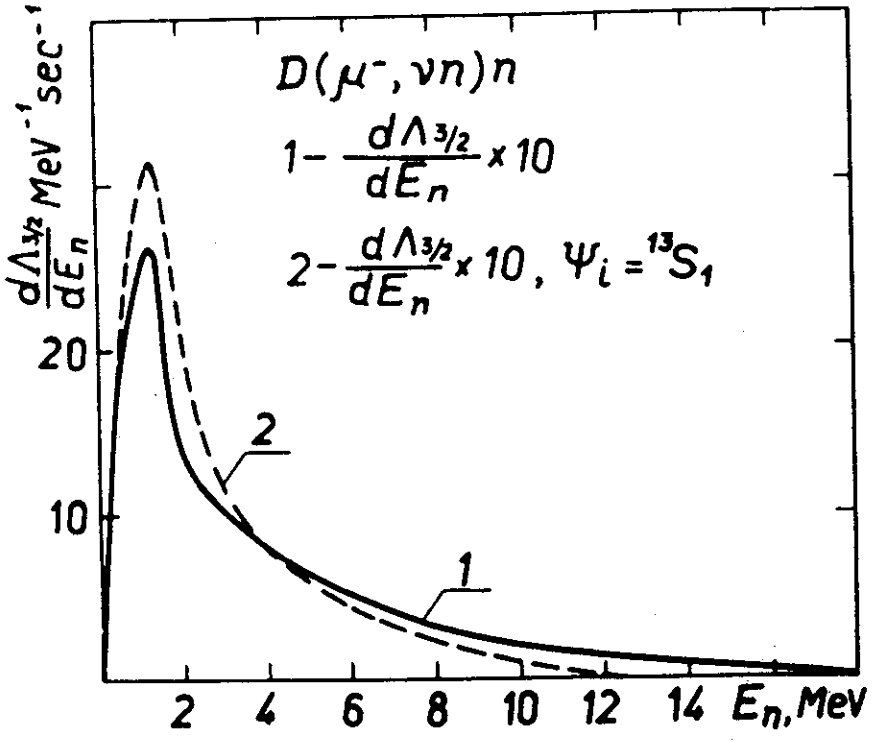

Fig. 2. The neutron energy spectrum in process $\mu^-(d,2n)\nu$. The capture from quartet state. Curve 1 is obtained with D-component in the deuteron wave function, curve 2 - without this component.

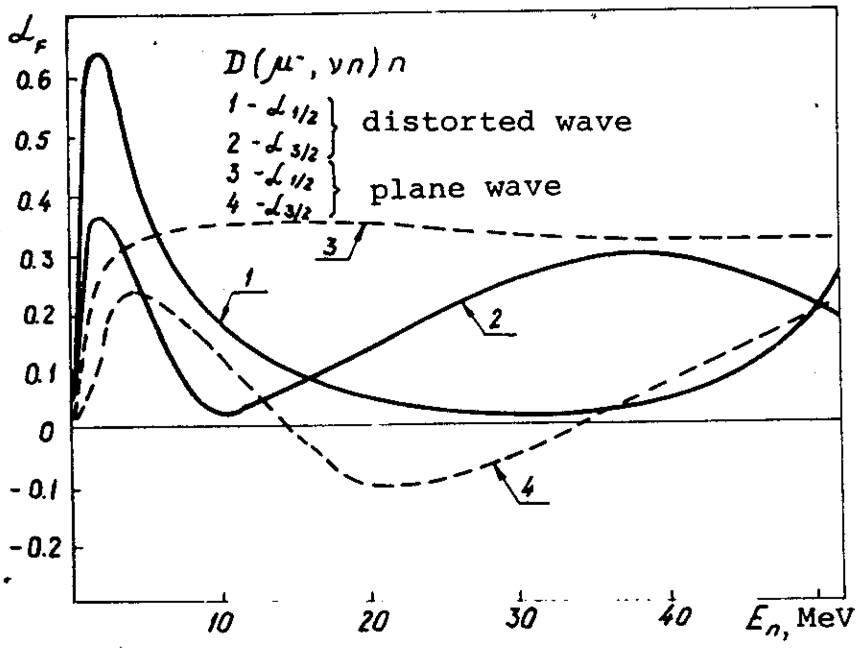

Fig. 3. Asymmetry of the neutron angular distribution in process $\mu^-(d,2n)\nu$. The velocity terms are omitted.

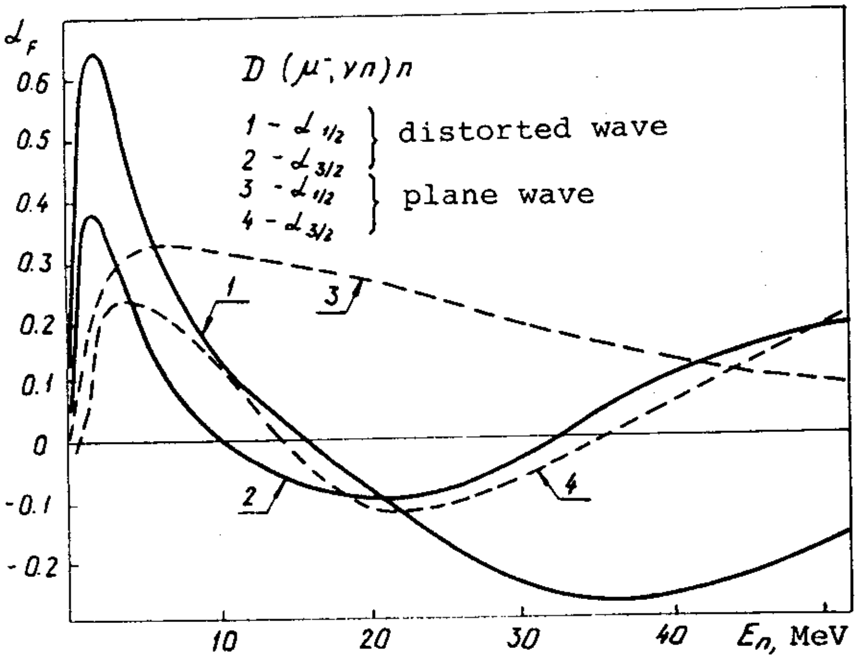

Fig. 4. Asymmetry of the neutron angular distribution in process $\mu^-(d, 2n)\nu$. The velocity terms are taken into account.

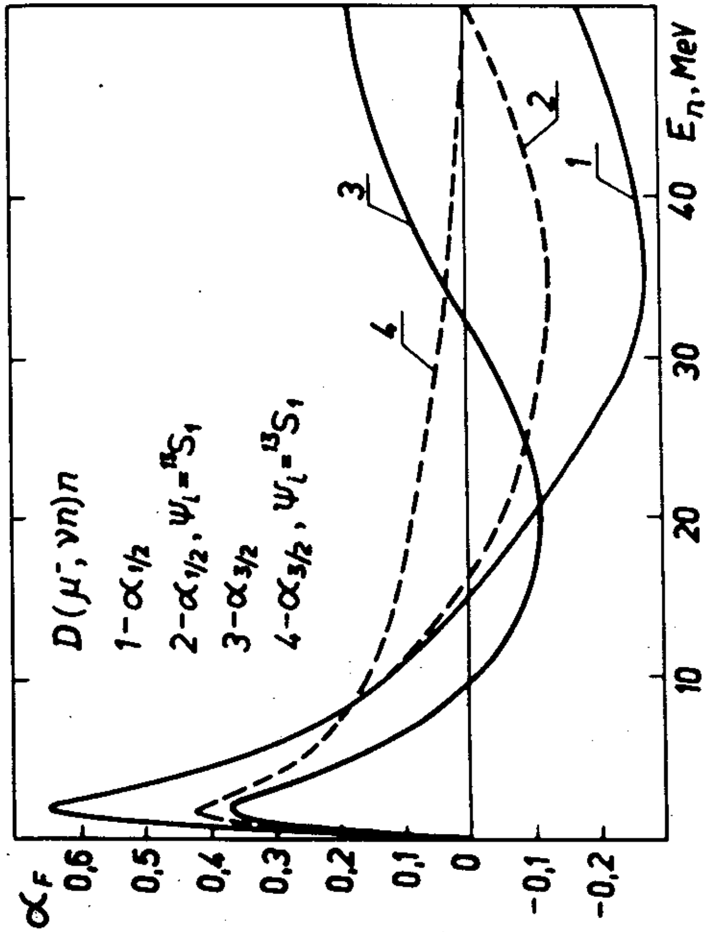

Fig. 5. Asymmetry of the neutron angular distribution in process $\mu^-(d, 2n)\nu$. The velocity terms are taken into account. Curves 1 and 3 are obtaines with D-component in the deuteron wave function, curves 2 and 4 - without this component.

The D-component dependence of asymmetry also turns out to be significant. The results are given in Fig. 5. In the capture from doublet state the energy dependence of the asymmetry coefficient is changed within the energy range above 10 MeV. In the capture from quartet state the change is observed throughout the whole region.

Conclusion

The preformed analysis of the muon capture by deuteron has shown the following.

a. The final state interaction is of principal importance for description of the characteristics of muon absorption.

b. The velocity terms of muon-nucleon Hamiltonian significantly determine the nature of energy dependence of the asymmetry coefficient of the neutron angular distribution. Their effect on the energy spectra is not very large.

c. The small component in the wave function of ground state also results in a quantitative change of the characteristics of the capture.

d. The measurement of the muon capture rate from quartet state, and also of the neutron energy spectra, will enable one to obtain an additional information on the nature of two nucleon interaction both in the bound state (the D - component contribution) and in the continuous spectrum.

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