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OPTICAL MODEL ANALYSIS OF PION ELASTIC SCATTERING BY ^{3}He

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Introduction

In the last years, several measurements have been performed of pion elastic scattering cross sections for ^4He, ^{12}C and ^{16}O nuclei in the energy region 60 - 250 MeV. Good qualitative description of elastic and total cross sections for pions scattered by zero spin and isospin (J = T = 0) nuclei can be given in terms of both the optical model and also the Glauber model at somewhat higher energies. (For a general review see /1/). The mentioned models are constructed from the spin and isospin independent part of the pion-nucleon amplitude and the nuclear structure enters the calculations solely due to the nuclear density.

Recently, differential cross sections of elastic $\pi^+ - ^3He$ scattering have been measured at 97 and 154 MeV /2,3/. It arises an interesting question, whether it is possible to obtain, also for $J \neq 0$, $T \neq 0$ light nuclei, a good description of elastic cross sections in terms of the first order optical potential. For non-zero spin and isospin nuclei, the optical potential is constructed from the complete pion-nucleon amplitude and the nuclear structure will enter the potential in a more complicated way. It has been shown /1/ that the terms proportional to the nuclear spin and isospin operators, which will appear now in the potential, are smaller by a factor $1/A$ (A is the number of nucleons in nucleus) compared to the spin and isospin independent ones. Therefore, the influence of the spin and isospin part of the $\pi - ^3He$ optical potential on the elastic cross sections should be rather strong.

Some time ago, elastic cross sections of $\pi^{\pm}-{}^3He$ reactions were calculated in the simple impulse approximation at 250 MeV [5]. It was shown that a ~ 4% admixture of the mixed symmetry S′ state in the ^3He wave function affects almost negligibly the differential cross sections.

In the following part of this paper, the spin and isospin dependent optical potential is presented and some of its interesting features are discussed. In the next part the experimental and calculated $\pi^{\pm}-{}^3He$ differential cross sections are compared and the importance of different parts of the optical potential is studied.

Optical model

In the framework of the optical model, pion-nucleus phase-shifts can be found matching the solutions of the following equation

$$(\nabla^2 + p^2)\Phi = V\Phi \tag{1}$$

to the Coulomb functions outside the nuclear force region. Here p is the pion momentum in the pion-nucleus centre-of-mass system. If we denote the channel isospin and its projection as τ, τ_z and let M be the projection of the nuclear spin, then the wave function can be labelled as $\Phi \equiv \langle \vec{r}, \tau, \tau_z, J, M | \Phi \rangle$. For nuclei with $J = 1/2$, the optical potential has the following form [4,6]

$$V\Phi = (A-1)\{2\mu Q^{\tau}_{\tau_z} V_c(r) + (B_0 + 2\frac{\vec{t}\cdot\vec{T}}{A} B_1)\rho(r) +$$

$$+ (C_0 + 2\frac{\vec{t}\vec{T}}{A} C_1)(-\vec{\nabla}\rho(r)\vec{\nabla} + \frac{A-1}{A}\frac{\mu}{2m}\nabla^2\rho(r)) + \tag{2}$$

$$+2\frac{\vec{J}\vec{\ell}}{A}(1 + \frac{A-1}{A}\frac{\mu}{m})(\alpha_1 D_0 + 2\alpha_2 \vec{t}\vec{T} D_1)\frac{1}{r}\frac{\partial\rho(r)}{\partial r}\}\Phi,$$

where Q^{τ}_{z} is the effective charge in the τ channel, $V_c(r)$ is the Coulomb potential, the coefficients B_i /7/, C_i and D_i, $i = 0, 1$ are related in the usual way to the free pion-nucleon phase-shifts and $\vec{t}$ is the pion isospin operator. The appearance of two terms proportional to $\frac{A-1}{A}\frac{\mu}{m}$ in eq. (2) is a consequence of the inclusion of the Fermi motion /6,8/. Further, μ is the reduced pion-nucleus mass and m the mass of the nucleon. Finally, the parameters α_1 and α_2 are defined as

$$\alpha_1 = \frac{\langle 0|\vec{J}\cdot\vec{S}|0\rangle}{J(J+1)}, \qquad \alpha_2 = \frac{\sum_{i=1}^{A}\langle 0|(\vec{J}\cdot\vec{S}_i)(\vec{T}\cdot\vec{t}_i)|0\rangle}{J(J+1)\,T(T+1)}, \quad (3)$$

where $\vec{S} = \sum_{i=1}^{A}\vec{s}_i$ and $\vec{s}_i$, $\vec{t}_i$ are the spin and isospin operators of i-th nucleon, respectively.

We described the ^3He nucleus by a simple harmonic-oscillator model, supposing that the ground state $|0\rangle$ is the pure symmetric S-state. Thus the nuclear density has the form

$$\rho(r) = \frac{1}{(\pi a^2)^{3/2}}\, e^{-r^2/a^2}, \quad (4)$$

where parameter a is determined by the total charge radius R via the relation $a = \sqrt{\frac{2}{3}(R^2 - r_p^2)}$. Here, $r_p = 0.8$ fm is the proton radius. In our model the parameters α_1 and α_2 take the following values

$$\alpha_1 = -\alpha_2 = 1. \quad (5)$$

For $A=3$ nuclei, potential (2) exhibits some interesting features. In contradiction to the pion scattering by heavier nuclei, where $\alpha_2 \approx \frac{\alpha_1}{A}$ /4/, the mixed spin-isospin term plays an important role in the optical potential for $\pi - {}^3He$ reactions. It is interesting to note, that this terms has the opposite sign compared to the corresponding one in the $\pi - N$ amplitude. Let us compare now qualitative features of the π^+ and π^- elastic cross sections. Since $2\vec{t}\cdot\vec{T} = -2$ for $\tau = 1/2$ and $2\vec{t}\vec{T} = 1$ for $\tau = 3/2$, it can be immediately seen from eqs. (2,5) that the influence of the spin-orbital term in the optical potential will be much smaller in the $\tau = 3/2$ channel compared to the $\tau = 1/2$ one. Further, in terms of spin-flip and spin-nonflip amplitudes g_τ and f_τ, the differential cross section for $\pi^\pm$ -reactions can be written as

$$\frac{d\sigma^+}{d\Omega} = |f_{3/2}|^2 + \sin^2\theta |g_{3/2}|$$

$$\frac{d\sigma^-}{d\Omega} = |\frac{2}{3} f_{1/2} + \frac{1}{3} f_{3/2}|^2 + \sin^2\theta |\frac{2}{3} g_{1/2} + \frac{1}{3} g_{3/2}|^2, \qquad (6)$$

where θ is the scattering angle. Therefore, we can expect that the minimum in the $\pi^- - {}^3He$ differential cross section will be less deep than that for $\pi^+ - {}^3He$ reactions. Finally, due to the strong cancellations in optical potential (2) for the $\tau = 1/2$ channel, the $\pi^- - {}^3He$ reaction should be more sensitive to details of the ${}^3He$ ground state wave function in comparison to the π^+ meson processes. Hence, the differential cross sections calculated in our simple model should explain better the $\pi^+ - {}^3He$ experimental data than those for $\pi^- - {}^3He$ reactions.

Discussion and Conclusions

We calculated $\pi^\pm - {}^3He$ elastic scattering cross sections at 97 and 154 MeV using CERN phase-shifts [10] in the expressions for elementary $\pi - N$ amplitudes. Further, we used a charge radius R = 1.88 fm [9], which yields the value a = 1.38 fm for the nuclear density parameter. There are no free parameters in our calcu-

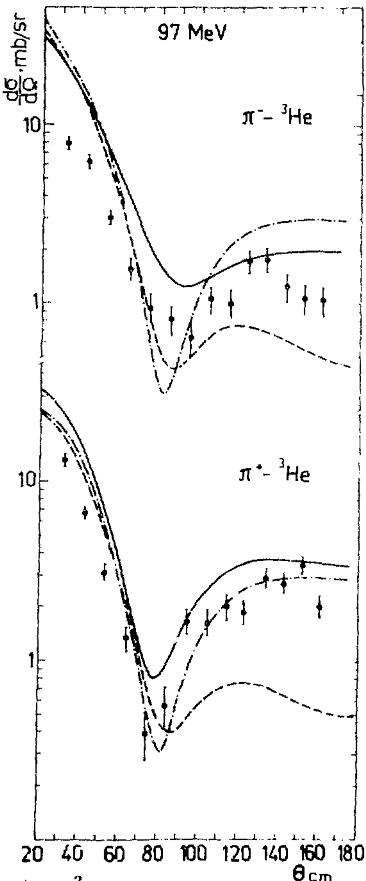

Fig. 1. The $\pi^\pm - {}^3He$ elastic differential cross sections at 97 MeV. The experimental data are taken from ref. /2/. The dashed line shows the result obtained with the original Kisslinger model. The dot-dashed line is obtained if the Fermi motion is taken into account and the $1/A$ corrections are included. The full line was calculated from the complete potential (2).

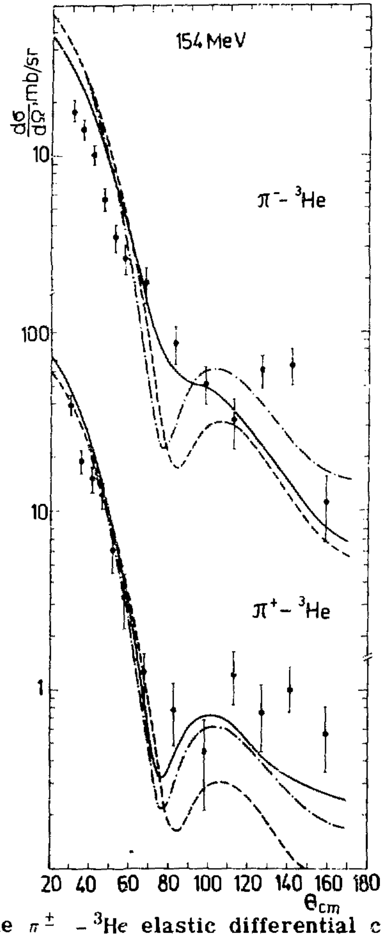

Fig. 2. The $\pi^{\pm}$ - ^{3}He elastic differential cross section at 154 MeV. The experimental data are taken from ref. /3/ The dashed line shows the result obtained with the original Kisslinger model including the 1/A corrections. The meaning of the remaining curves is the same as in Fig. 1.

lations. The measured [2,3] and calculated differential cross sections are compared in Figs. 1 and 2. The calculation with the original Kisslinger model is shown in Fig. 1 by the dashed line. The inclusion of different 1/A and the Fermi corrections [8] is represented by the dot-dashed line (the spin and isospin terms in eq. (2) are still neglected). The result obtained with full optical potential (2) is shown by the solid line. The dashed line in Fig. 2 is calculated with the original Kisslinger model including 1/A corrections. The meaning of the remaining curves is the same as in Fig. 1. Therefore, the difference between dashed and dot-dashed lines in Fig. 2 shows the net effect of inclusion of the Fermi motion correction.

The following conclusions can be drawn from the comparison shown in Figs. 1 and 2.

(i) The inclusion of the Fermi motion and 1/A correction terms improves the behaviour of calculated cross sections remarkably at large scattering angles.

(ii) The presence of spin and isospin dependent terms in eq. (2) affects strongly especially the $\pi^- - {}^3He$ cross sections. If we include them, the position of the minimum is shifted in the correct direction (at 97 MeV) and a better description of cross sections is obtained at large scattering angles (at 154 MeV). Both the experimental and the theoretical minima are less deep for $\pi^- - {}^3He$ reactions.

(iii) Nevertheless, the agreement between experimental data and theoretical curves is (namely for π^--reactions) only qualitative. There remain large discrepancies at small scattering angles at 97 MeV, similar to those observed in $\pi - {}^4He$ cross sections [7,8] in the corresponding energy region. These discrepancies are probably inherent to the first order optical model calculations. The theoretical and experimental results contained in this paper have a somewhat preliminary character. Both experiments at more energies and a more elaborate description of the ${}^3He$ ground state wave function will be needed (e.g., the terms proportional to the first power of D-state admixture parameter should enter into the spin-orbital part of the optical potential [6] in order to conclude to what degree of accuracy the spin and isospin dependent first order optical potential can describe the scattering of pions on light $J \neq 0$, $T \neq 0$ nuclei.

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