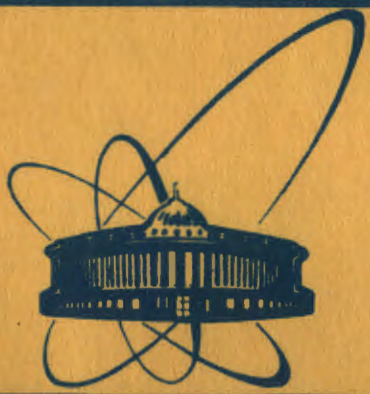

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PARTICLE-HOLE FORMALISM FOR NUCLEON EXCHANGE IN HEAVY-ION COLLISIONS

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1. INTRODUCTION

Deep inelastic heavy-ion collisions are mainly characterized by the strong energy damping and the large number of nucleons exchanged between the two ions. In the last time the latter process has been studied very successfully in the framework of diffusion models [1-5]. These models assume that after an approach and excitation process, which is rather short in time, the two ions reach a state of thermal equilibrium while forming a double nuclear system. Then, the nucleon transfer is the result of a stochastic exchange and can be described, for example, by Fokker-Planck or master equations. Many efforts have been done in order to apply transport theories to further processes, as the energy or angular momentum dissipation, and to calculate the transport coefficients microscopically [6-16].

In contrast, we are interested in a more detailed knowledge about the first stage of the collision where the main amount of the relative kinetic energy is dissipated and thermalization happens. With a similar aim Schmidt et al. [17] have investigated the energy dissipation using a particle-hole formalism. Here, this formalism is extended to the mass exchange. Like in ref. [17] we restrict ourselves to the consideration of single-particle states, although other mechanisms, as for example, the excitation of collective modes [18], may also be important.

In our model the energy dissipation as well as the nucleon exchange is described by particle-hole excitations. In the first case, the particle and the hole are in the same nucleus, whereas in the second, the particle is created in one nucleus and the hole in the other. The relative motion of the ions is treated classically. The basis set for the definition of our particle and hole creation and annihilation operators is the sum of the unperturbed bound single-particle states of the two nuclei moving along the classical trajectories $\vec{S}_1(t)$ and $\vec{S}_2(t)$, respectively. Originally such a treatment has been developed in the work of Dietrich and Hara [19], where also the nonorthogonality of this dual basis has been discussed in detail. The creation and annihilation operators defined according to this work are explicitly time-independent and fulfil the usual anticommutation relations for Fermi-operators. Note that in this way, the antisymmetrization of the whole system wave function is included.

In a quite similar treatment, but with a poor product wave function for the whole system, Bartel and Feldmeier [20] have

studied the one-body and two-body contributions to the energy dissipation. The result of this work showing that the one-body part yields the larger amount motivates us to restrict the consideration to mean field effects. Furthermore, the particle and hole rescattering is neglected in this work.

2. THE MODEL

We start with the Hamiltonian

$$\hat{H} = \hat{H}_{01} + \hat{H}_{02} + \hat{H}_t \,, \tag{1}$$

where $\hat{H}_{01}$ and $\hat{H}_{02}$ are the unperturbed intrinsic Hamiltonians of the two nuclei given by

$$\hat{H}_{01} = \sum_{p_1} \epsilon_{p_1} a^+_{p_1} a_{p_1} - \sum_{h_1} \epsilon_{h_1} \beta^+_{h_1} \beta_{h_1} \,, \tag{2}$$

$$\hat{H}_{02} = \sum_{p_2} \epsilon_{p_2} a^+_{p_2} a_{p_2} - \sum_{h_2} \epsilon_{h_2} \beta^+_{h_2} \beta_{h_2} \,.$$

As is mentioned above, the single-particle functions of $\hat{H}_{01}$ and $\hat{H}_{02}$ and the particle (hole) creation and annihilation operators $a^+_{p_i}(\beta^+_{h_i})$ and $a_{p_i}(\beta_{h_i})$ are time-independent in the system moving with the nucleus $i\,(i=1,2)$. For the time-dependent interaction $\hat{H}_t$ we write

$$\hat{H}_t = \Sigma\,(v_{p_1h_1} a^+_{p_1} \beta^+_{h_1} + v^*_{p_1h_1} \beta_{h_1} a_{p_1}) + \Sigma\,(v_{p_2h_2} a^+_{p_2} \beta^+_{h_2} + v^*_{p_2h_2} \beta_{h_2} a_{p_2}) +$$

$$+ \Sigma\,(v_{p_1h_2} a^+_{p_1} \beta^+_{h_2} + v^*_{p_1h_2} \beta_{h_2} a_{p_1}) + \Sigma\,(v_{p_2h_1} a^+_{p_2} \beta^+_{h_1} + v^*_{p_2h_1} \beta_{h_1} a_{p_2}), \tag{3}$$

where the first two terms describe the excitation in both nuclei, whereas the last two terms govern the transter. The $v_{p_ih_j}$ are the usual one-body matrix elements, and $i,j=1,2$ indicates if the particle (hole) is in nucleus one or two. The matrix elements depend on the relative distance and relative velocity of the two ions and, therefore, the interaction (3) becomes time-dependent.

In order to calculate the wave function we use Dirac's time-dependent perturbation theory. Then, the wave function which shows in general the form

$$|\Phi(t)\rangle = f_0(t)|0\rangle + \sum_{ph} f_{ph}(t)|ph\rangle + \frac{1}{2!} \sum_{php'h'} f_{php'h'}(t)|php'h'\rangle + \ldots \tag{4}$$

can be determined by

$$|\Phi(t)\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{i}{\hbar}\right)^n \int_0^t dt_1 \ldots \int_0^t dt_n \, T[H_{t_1}(t_1) \ldots H_{t_n}(t_n)]|0\rangle \,, \tag{5}$$

where T is the time-ordering operator. (Here and in the following we use the interaction picture.) The time scale is fixed in a way that at $t=0$ the two nuclei begin to interact. For $t<0$ the nuclei are in their ground states (ph -vacuum) and the interaction (3) is zero. The sums in eq. (4) include states of both nuclei 1 and 2.

Since the interaction (3) is restricted to the creation and annihilation of ph -pairs, it is sufficient to calculate the amplitudes $f_0(t)$, $f_{ph}(t)$, etc., in the lowest order. This leads to

$$f_0 = 1, \tag{6}$$

and

$$f_{ph}(t) = -\frac{i}{\hbar} \int_0^t dt_1 \, v_{ph}(t_1) e^{-\frac{i}{\hbar}(\epsilon_p - \epsilon_h)(t-t_1)} \,. \tag{7}$$

The higher order components can be obtained from the 1p1h -component according to

$$f_{php'h'\ldots p^nh^n}(t) = \begin{vmatrix} f_{ph} & f_{ph'} & \cdots & f_{phn} \\ f_{p'h} & f_{p'h'} & \cdots & f_{p'h^n} \\ \vdots & \vdots & & \vdots \\ f_{p^nh} & f_{p^nh'} & \cdots & f_{p^nh^n} \end{vmatrix} \,. \tag{8}$$

It should be noted here that the resulting wave function is not normalized.

We define the operator

$$\hat{\Delta A} = \sum_{p_1} a^+_{p_1} a_{p_1} - \sum_{h_1} \beta^+_{h_1} \beta_{h_1} \tag{9}$$

whose expectation value gives the shift in the mass number A_1 of the nucleus one

$$A_1(t) = A_1^{\text{initial}} + \langle \hat{\Delta A} \rangle \tag{10}$$

and, therefore, represents the measure for the change of the mean value of the mass distribution. Evaluating the variance $\sigma^2 = \langle \hat{\Delta A}^2 \rangle - \langle \hat{\Delta A} \rangle^2$ of the distribution we need the expectation value of the operator

$$\hat{\Delta A}^2 = \sum_{p_1} a^+_{p_1} a_{p_1} + \sum_{h_1} \beta^+_{h_1} \beta_{h_1} + \sum_{p_1 p_1'} a^+_{p_1} a^+_{p_1'} a_{p_1'} a_{p_1} +$$

$$+ \sum_{h_1 h_1'} \beta^+_{h_1} \beta^+_{h_1'} \beta_{h_1'} \beta_{h_1} - 2 \sum_{p_1 h_1} a^+_{p_1} a_{p_1} \beta^+_{h_1} \beta_{h_1} \,. \tag{11}$$

With eqs. (4), (6) and (8) the expectation values can be derived:

$$\frac{\langle\Phi|\hat{\Delta A}|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \Sigma\,|f_{p_1h_2}|^2 - \Sigma\,|f_{p_2h_1}|^2 - \tag{12}$$

$$- \Sigma\, f_{p_1h_2} f_{ph}\, f^*_{ph_2} f^*_{p_1h} + \Sigma\, f_{p_2h_1} f_{ph}\, f^*_{ph_1} f^*_{p_2h} + \cdots ,$$

$$\frac{\langle\Phi|\hat{\Delta A}^2|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \Sigma\,|f_{p_1h_2}|^2 + \Sigma\,|f_{p_2h_1}|^2 + (\Sigma\,|f_{p_1h_2}|^2)^2 +$$

$$+ (\Sigma\,|f_{p_2h_1}|^2)^2 - 2\,\Sigma\,|f_{p_1h_2}|^2 \Sigma\,|f_{p_2h_1}|^2 - \Sigma\, f_{p_1h_2} f_{p_1'h_2'} f^*_{p_1'h_2} f^*_{p_1h_2'} + \tag{13}$$

$$- \Sigma\, f_{p_2h_1} f_{p_2'h_1'} f^*_{p_2'h_1} f^*_{p_2h_1'} - \Sigma\, f_{p_1h_2} f_{ph} f^*_{ph_2} f^*_{p_1h} -$$

$$- \Sigma\, f_{p_2h_1} f_{ph} f^*_{ph_1} f^*_{p_2h} + 2\Sigma\, f_{p_1h_2} f_{p_2h_1} f^*_{p_2h_2} f^*_{p_1h_1} + \cdots$$

Let us consider only terms of the second and fourth order in v_{ph} and neglect the higher order contributions. Within this approximation we get

$$\sigma^2 = \Sigma\,|f_{p_1h_2}|^2 + \Sigma\,|f_{p_2h_1}|^2 - 2\Sigma\, f_{p_1h_2} f_{p_1'h_2'} f^*_{p_1'h_2} f^*_{p_1h_2'} -$$

$$- 2\Sigma f_{p_2h_1} f_{p_2'h_1'} f^*_{p_2'h_1} f^*_{p_2h_1'} - \Sigma\, f_{p_1h_2} f_{p_1'h_1'} f^*_{p_1'h_2} f^*_{p_1h_1'} - \tag{14}$$

$$- \Sigma\, f_{p_1h_2} f_{p_2'h_2'} f^*_{p_2'h_2} f^*_{p_1h_2'} - \Sigma\, f_{p_2h_1} f_{p_1'h_1'} f^*_{p_1'h_1} f^*_{p_2h_1'} -$$

$$- \Sigma\, f_{p_2h_1} f_{p_2'h_2'} f^*_{p_2'h_1} f^*_{p_2h_2'} .$$

For a symmetric system ($f_{p_1h_1} = f_{p_2h_2}$ and $f_{p_1h_2} = f_{p_2h_1}$) the expectation value (12) vanishes and eq. (14) becomes

$$\sigma_s^2 = 2\,\Sigma\,|f_{p_1h_2}|^2 - 4\Sigma\, f_{p_1h_2} f_{p_1'h_2'} f^*_{p_1'h_2} f^*_{p_1h_2'} - \tag{15}$$

$$- 4\Sigma\, f_{p_1h_2} f_{p_1'h_1'} f^*_{p_1'h_2} f^*_{p_1h_1'} .$$

The time derivation of this expression yields a quantity similar to the diffusion coefficient in a Fokker-Planck equation[1]:

$$D = \frac{d}{dt}(2\,\Sigma\,|f_{p_1h_2}|^2 - 4\Sigma f_{p_1h_2} f_{p_1'h_2'} f^*_{p_1'h_2} f^*_{p_1h_2'} - \tag{16}$$

$$- 4\,\Sigma\, f_{p_1h_2} f_{p_1'h_1'} f^*_{p_1'h_2} f^*_{p_1h_1'}) .$$

As can be seen below, D is a strongly time-dependent quantity and behaves very differently than the corresponding quantity in a diffusion model. The fourth-order terms in eqs. (15) and (16) contain transfer as well as excitation contributions. This is reasonable since the Pauli-principle acts independently of the fact whether a state is occupied by way of transfer or excitation.

To obtain numerical values for σ_s^2 and D a practicable way seems to determine the trajectories $\vec{S}_1(t)$ and $\vec{S}_2(t)$ of the ions by means of a classical friction model[21]. Then, the matrix elements which are time-dependent via the relative distance and velocity can be computed using a suitable single-particle set. Furthermore, the formalism presented here also allows one to determine the dissipative force acting on the relative motion due to transfer and excitation. This force can be used in the classical equation of the relative motion instead of an ordinary friction force. Since the dissipative force again depends on the relative motion this leads to a self-consistent calculation. Such a treatment which needs no more phenomenological inputs has been suggested in ref.[17].

3. A SIMPLE ESTIMATION WITH CONSTANT MATRIX ELEMENTS

In order to get some qualitative insights we perform a simple estimation for the quantities derived in the last section. First, we neglect the relative motion and assume that the interaction $\hat{H}_t$ is switched on at $t = 0$ and does not change its amplitude. Furthermore, we assume the excitation probability to be the same for all states. These idealizations lead to constant matrix elements. We replace the summations by integrations over the particle-hole energies ϵ_{ph}. For equidistant single-particle states the particle-hole density $\rho(\epsilon_{ph})$ can nearly be expressed by

$$\rho(\epsilon_{ph}) = \rho_N^2 \epsilon_{ph} e^{-\epsilon_{ph}/\Delta} , \tag{17}$$

where ρ_N is the single-particle density. The exponential function in eq. (17) ensures that only bound states can be excited.

The advantage of all this handling is that one obtains an analytical expression for the second-order term

$$\sum_{ph} |f_{ph}|^2 = |\bar{v}_{ph}|^2 \rho_N^2 \ln(1 + \frac{t^2\Delta^2}{\hbar^2}) \tag{18}$$

and can evaluate the exchange term approximately:

$$\sum_{ph\,p'h'} f_{ph}\, f_{p'h'}\, f^*_{p'h}\, f^*_{ph'} \approx |\bar{v}_{ph}|^4 \rho_N^4 \{2\ln(1+\frac{t^2\Delta^2}{\hbar^2}) - \quad (19)$$

$$- \ln(1+\frac{t^2\Delta^2}{4\hbar^2}) - \frac{4\hbar}{t\Delta}\arctan\frac{t\Delta}{\hbar} - \frac{8\hbar}{t\Delta}\arctan\frac{t\Delta}{2\hbar}\}.$$

Here, $\bar{v}_{ph}$ denotes an averaged matrix element. The time derivation of the expressions (18) and (19) gives

$$\frac{d}{dt}\sum_{ph}|f_{ph}|^2 = |\bar{v}_{ph}|^2 \rho_N^2 \frac{2t\frac{\Delta^2}{h^2}}{1+\frac{t^2\Delta^2}{h^2}} \quad (20)$$

and

$$\frac{d}{dt}\sum_{ph\,p'h'} f_{ph}\, f_{p'h'}\, f^*_{p'h}\, f^*_{ph'} = \quad (21)$$

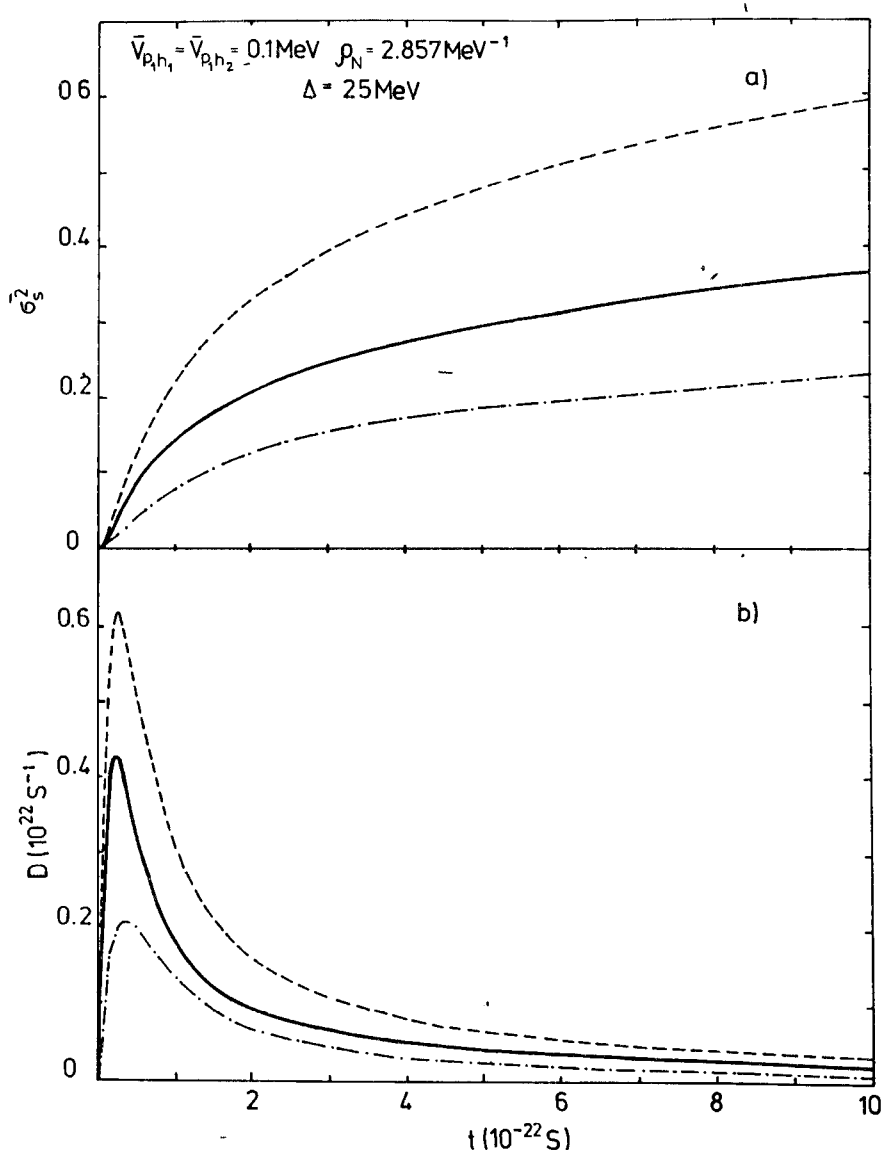
$$\approx |\bar{v}_{ph}|^4 \rho_N^4 \{\frac{\frac{t\Delta^2}{2h^2}}{1+\frac{t^2\Delta^2}{4h^2}} - \frac{4h}{t^2\Delta}\arctan\frac{t\Delta}{h} + \frac{8h}{t^2\Delta}\arctan\frac{t\Delta}{2h}\}.$$

Using eqs. (18)-(21) the mass variance σ_s^2 and its time-derivative D have been calculated. The results are shown in the figure. In order to discuss the time evolution the results for D are more illustrative. Considering a typical reaction time in the order of 10^{-21} s, D increases rapidly and reaches its maximum at a very short time. Also the width of this peak is rather small. This behaviour shows that the system reacts very fast on the switching on of the interaction. This response time is small compared with the collision time or a time typical for the change of the collective degrees of freedom.

4. CONCLUSIONS

A time-dependent particle-hole formalism has been applied to heavy-ion collisions. The interaction has been restricted to the excitation of particle-hole pairs due to the effect of the nuclear mean field. In order to obtain the wave function Dirac's time-dependent perturbation theory has been used. Within this formalism the dissipation of relative kinetic energy as well as the mass exchange can be described.

As an example, we have studied the time-evolution of the mass variance σ_s^2 for symmetric systems. The operator of this quantity has been derived, and it has been shown that its



Variance of the mass distribution (a) according to eqs. (20) and (21) and its time-derivative (b) according to eqs. (22) and (23) as a function of time (full lines). The broken curves show the 2nd-order and the chain curves the 4th-order contributions to the quantities. Note that the 4th-order contributions have a negative sign.

expectation value can be expressed in terms of the single-particle amplitude $f_{ph}(t)$.

Within a simple estimation analytical expressions for the mass variance and its time-derivative could be evaluated. The results show a time behaviour of the system considered which differs essentially from that in a transport approach.

The calculation of useful numerical data requires a detailed knowledge of the time-dependent excitation and transfer matrix elements, and therefore, the inclusion of the relative motion. Such calculations and an extension of the model to further reaction quantities which are not considered in this work will be performed in a subsequent paper /22/.

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Частично-дырочный механизм обмена нуклонами в столкновениях тяжелых ионов

Изучается влияние переноса нуклонов на динамику столкновений тяжелых ионов. Методы, развитые в транспортной теории, неприменимы, это относится прежде всего к начальной фазе столкновения, когда распределяется большая доля кинетической энергии относительного движения, а термализация еще не наступила. Показано, что частично-дырочный формализм позволяет описать единым образом как одночастичные возбуждения, так и процесс обмена нуклонами. Для построения волновой функции системы, состоящей из двух ядер, была использована зависящая от времени теория возмущений. В рамках модели с постоянным взаимодействием ионов оценивалась дисперсия массового распределения. Результаты применялись для изучения поведения системы во времени и было показано, что последнее определяется главным образом матричными элементами зависящих от времени передач и возбуждений.

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Сообщение Объединенного института ядерных исследований. Дубна 1982

Teichert J. E4-82-848

Particle-Hole Formalism for Nucleon Exchange in Heavy-Ion Collisions

The purpose of the paper is to study the nucleon transfer influence on the dynamics of heavy-ion collisions, in order to describe especially the first stage of the collision, where the main part of the relative kinetic energy dissipates but the thermalization has not happen, transport theoretical methods are not applied.The paper shows that single-particle excitations as well as the nucleon exchange can be treated within the particle-hole formalism in a unified way. Thereby the consideration is restricted to mean-field effects. To determine the wave function for the system of two nuclei a time-dependent perturbation expansion has been used.An estimation for the mass dispersion has been performed within a model with constant interaction between the ions. The result has been used to study the time-behaviour of the system and it has been shown that it is essentially governed by the time-dependent transfer and excitation matrix elements.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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