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TUNNEL SUPERPENETRABILITY OF POTENTIAL BARRIERS

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According to the quantum mechanics, particles can pass with finite probability through the regions where their energy is negative. The quasiclassical estimate for the tunnel penetrability coefficient is determined by the expression

$$D \sim \exp[-\frac{1}{\hbar}\int_a^b \sqrt{2m(V(x)-E)}dx], \qquad (1)$$

where $V(x)$ is the potential, m and E are mass and energy of the particle, a and b are classical turning points.

The transition of two particles through the same barrier has some unexpected features. At first, we shall compare the limiting cases when the particles are joined together in a single particle with double m, V, E and when they pass the barrier independently. In the last case the probability for both the particles to penetrate through the barrier is equal to the product of expressions (1). So, the powers of exponential factors are added, and the coefficient D for the uncorrelated pair turns out to be equal to the one for the united particles (a factor 2 appears in both the cases in front of the integral).

As an intermediate case, we shall consider the pair of particles bo d in a quasideuteron of a finite size. Let the upper energy levels of this quasideuteron are much higher than its ground state energy, so that their excitation in the external field V can be neglected. Then, in the expansion of the wave function of the pair over the complete set of internal states Φ_α (see [1]):

$$\Psi(x_1, x_2) = \sum_\alpha F_\alpha(X)\Phi_\alpha(x), \qquad (2)$$

where $X=(x_1+x_2)/2$; $x=x_2-x_1$ are the coordinates of the centre of mass and the relative distance of the particles, we can retain a single term with $\alpha = 0$.

For the wave function F_0 describing the motion of the quasideuteron centre of mass, we get the Schrödinger equation [1]*:

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*We shall not take into account the relativistic effect of the mass defect due to the binding energy and the corresponding increase in the centre of mass energy as in the case of united particle we have considered before.

$$-\frac{h^2}{2M}F_0''(X) + V_{00}(X)F_0(X) = 2EF_0(X); \qquad M=2m, \tag{3}$$

where

$$V_{00}(X) = \int \Phi_0^2(x)\,[V(x_1 = X - \frac{x}{2}) + V(x_2 = X + \frac{x}{2})]\,dx. \tag{4}$$

The averaging in (4) of the fields $V(x_1)$ and $V(x_2)$ acting on separated particles over the ground state of internal motion of the pair leads to their smoothing and decreasing of the height of the potential barrier V_{00} in comparison with the case of united particles (2V). The effect of increasing penetrability due to this fact has been discovered in [2], where the model numerical calculations have been performed*. The corresponding increase (about ten times) of the probability of α-decay has been shown in papers [3].

It may seem that the effect can be intesified by increasing the size of the complex particle. But the enlargement of the dimension is accompanied by the lowering of the levels of excited states so that their neglect in the sum in (2) becomes unjustified. We get then a system of coupled Schrödinger equations (see [1]), taking into account the energy losses of the centre of mass, which hinder the motion. The occurrence of the competitive channels sets up the limits for the increasing of the barrier penetrability. It turns out to be possible to remove these limits for the many-body systems with long range correlations. For instance, according to the theory of superfluidity [4,5], the pairing forces lead to the occurrence of an energy gap Δ between the ground state of a system and the spectrum of the excited states. This allows one to use a single level approximation in (2), where now Φ_0 is to be taken as a wave function of superfluid state of N particles:

$$\Phi_0 = \prod_s (u_s + v_s a_s^+ a_{-s}^+)|0\rangle, \tag{5}$$

where a_s^+ is the creation operator of a particle in the s state of the average field of N particles, $|0\rangle$ is the vacuum state $(a_s|0\rangle = 0)$, the coefficients v_s are the probability amplitudes to find the pairs of particles on the level "s" with the energy ϵ_s; $u_s^2 = 1 - v_s^2 = \frac{1}{2}(1 + \frac{\epsilon_s}{\sqrt{\epsilon_s + \Delta^2}})$. The function (5) corresponds to the conservation of average number of particles only: $\langle\Phi_0|\sum_{\pm s} a_s^+ a_s|\Phi_0\rangle = \sum_s v_s^2 = N$. The signs of s may signify the spin (1/2) direction of particles in the pair.

The interaction with the barrier can be presented by the expression:

$$V_{00}(X) = \langle\Phi_0|\sum_{\pm s}\langle\phi_s(x_i)V(X-x_i)\phi_s(x_i)\rangle_{x_i} a_s^+ a_s|\Phi_0\rangle = \sum_{\pm s} v_s^2 \langle\phi_s(x_i)V(X-x_i)\phi_s(x_i)\rangle_{x_i}, \tag{6}$$

where ϕ_s is the wave function of the independent motion of a single particle in s-state of the average field.

In the expression (6) for simplicity we have not taken into account the translation invariance [6] of states of the internal motion of the complex particle. But the corresponding corrections are small for large N. The potential V_{00} in (6) is calculated by averaging the single particle barriers over the states of the system which has a large dimension. By choosing suitable N, Δ and a dimension of a complex particle, we can get the penetration coefficient D for it even bigger than for a single particle.

A classical analogue of the discussed phenomenon is the motion of a train over a small hill, which cannot be passed by a single truck, but the trucks coupled together with the same initial kinetic energies can easily overcome the hill if the length of the train is big enough.

The effect of supertransitivity must reveal itself in many quantum phenomena, particularly, in nuclear physics by heavy ion collisions, by fission, in the interaction of closed shells and so on.

In the theory of Josephson's effect the process of motion of particles inside the potential barrier between two superconductors is not considered [7]. The scheme proposed here can be useful for a simple explanation of this process.

*As was pointed out to the author by G.N.Afanasiev, the length of the interval [a,b] in (1) increases for small values of energy E due to averaging in (4), which can make the transition through the barrier more difficult there.

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Захарьев Б.Н. E4-82-668

Туннельная сверхпроницаемость потенциальных барьеров

Дальнодействующие коллективные корреляции /сверхтекучего типа и др./ сильно облегчают частицам прохождение через потенциальные барьеры. Это происходит благодаря передаче дополнительной энергии частицам, двигающимся в области барьера, от находящихся вне его.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Tunnel Superpenetrability of Potential Barriers

Long range collective correlations of particles (of the superfluidity type and others) simplify very much for them passing through high potential barriers. This happens due to the transfer of the supplementary energy from the particles outside the barrier to those inside it.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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