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MESON-EXCHANGE CORRECTIONS TO THE NUCLEAR WEAK AXIAL CHARGE DENSITY IN THE HARD PION MODEL

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1. INTRODUCTION

The time-like part of the two-body weak axial vector current attracts attention since Kubodera, Delorme and Rho[1] pointed out that it could give rise to large effects even in the one-pion exchange (OPE) limit. This is due to chiral invariance of the strongly interacting nucleon system and can be tested by calculating in detail nuclear axial charge density. For this aim the purely axial weak processes of the beta decay and muon capture between ^{16}N (0^-; 120 keV) and ^{16}O (0^+; groundstate) are well suited because they are known to be very sensitive to the time component of the current and the one- and two-body parts of the transition operator are of the same magnitude O(1/M). The idea is to calculate the ratio $\Lambda_\mu/\Lambda_\beta$ of the partial muon-capture rate $\Lambda_\mu(0^+-0^-)$ to the partial beta-decay rate $\Lambda_\beta(0^--0^+)$ in order to learn how the induced pseudoscalar coupling constant g_p is related to the axial nucleon form factor g_A[2]. The answer to the question how significant is the mesonic exchange correction (MEC) to this first forbidden transition depends on a variety of conditions (see refs.[3-8] and the discussion therein) and particularly on the form of the two-body current operator,which might not be as simple as usually applied. It is our purpose to investigate the heavy-meson exchange contributions to the time component of the weak axial vector current. For this aim the two-nucleon MEC-operator is constructed using the phenomenological Lagrangian (PL) version of the hard pion model[9,10]. The meson exchanges, which are taken into account, are those due to pions, rho- and A_1-mesons.

2. THE TWO-BODY AXIAL MEC-OPERATOR

The two-nucleon axial MEC-operator is constructed in the one-boson approximation starting with the hard-pion PL-model as proposed by Ogievetsky and Zupnik[11].The explicit momentum dependence of various form factors is specified up to ≈ 1 GeV and the current algebra (CA) and the PCAC-hypothesis together with the important concept of the rho-dominance of the isospin current are consistently combined. The PL-technique enables one to apply the standard Feynman rules in order to write down all graphs of interest for a given process. The hard-pion PL is chosen to be invariant under the local $SU_2 x SU_2$-transformation. This chiral gauge invariance is assumed to be broken only due to the non-zero masses m_ρ and m_{A_1} of the rho- and A_1-mesons respectively. The presence of graphs with A_1-meson exchange within the hard pion method guarantees the consistency of the chiral approach with vector dominance /12,13/. In the Ogievetsky-Zupnik version of the hard-pion model the effective PL for the $A_1\rho\pi$-system is completely determined by four phenomenological parameters, m_ρ, m_{A_1} and the rho- and A_1-mesons coupling constants g_ρ and g_{A_1} respectively. So it is well defined how many ρ-and A_1-exchange diagrams must be taken into account, in other words, in contrast to the CA- and PCAC-method, an unambiguous counting of the pion and heavy-meson exchange graphs is ensured. The standard ideology accepted in the elementary-particle physics is that the hard pion method works up to the energy scale ~1 GeV (m_ρ, m_{A_1}) and in this domain the corresponding PL's provide a good description already in the tree-approximation of hadron amplitudes. That is the reason why the two-nucleon MEC-operator is well defined as a set of all possible tree-graphs in the hard-pion PL-model /9,10/. Because of the basic assumption such an operator possesses the correct chiral $SU_2 \times SU_2$-transformation properties and reproduces all standard PCAC-results in the soft-pion limit. The non-Born MEC-operators, which contribute significantly to the transition rates $\Lambda_\mu(\Lambda_\beta)$ are presented in the figure. In what follows the space part of the one-meson exchange current will be completely neglected, and we shall treat the space part of the nuclear current only in the I.A. in the standard way /14/. After non-relativistic reduction and transformation to the coordinate space the time part of the axial vector MEC-operator is obtained as:

$$J^4_{exch} = \sum_{i \neq j} e^{-i\vec{k}_\nu \cdot \vec{r}_i}\, \vec{\sigma}_j \cdot \hat{r}_{ij} \{ C_1 Y_1(A m_\pi r) + C_2 F_0(r) \}\, i\, (\vec{\tau}_i \times \vec{\tau}_j)_-$$

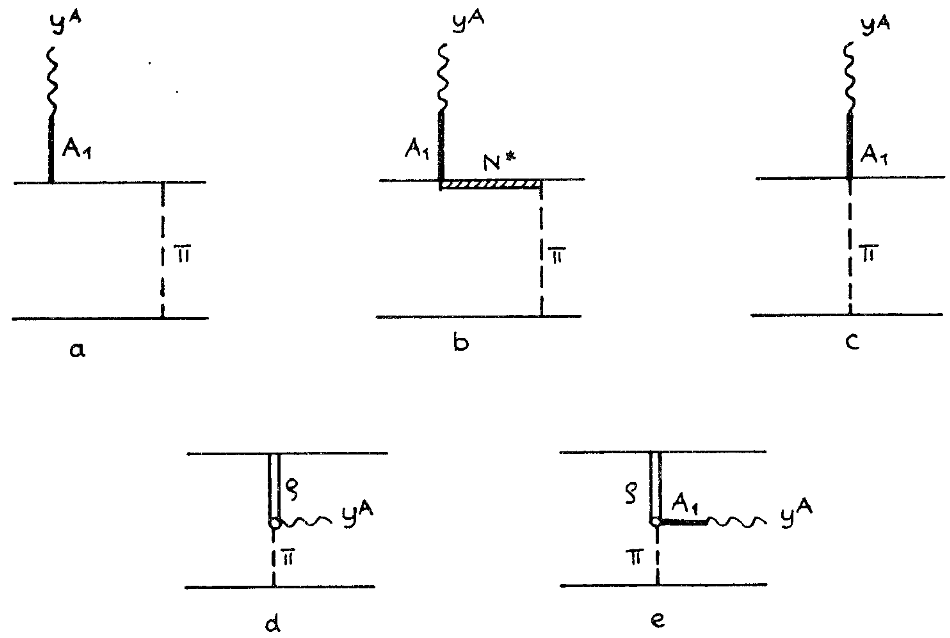
$$Y_1(x) = \frac{e^{-x}}{x}\left(1 + \frac{1}{x}\right)$$

$$\vec{r} = \frac{1}{b\sqrt{2}}(\vec{r}_i - \vec{r}_j) \qquad \vec{R} = \frac{1}{b\sqrt{2}}(\vec{r}_i + \vec{r}_j) \qquad \hat{r}_{ij} = \frac{\vec{r}}{|\vec{r}|} \tag{2.1}$$

$$b = \sqrt{\hbar / M\omega}$$

$$C_1 = -\frac{g_A g_\rho^2 m_\pi^2}{8\pi m_\rho^2}\left[1 + \frac{2 f^2_{\pi N N^*} m_\rho^2}{9 g_\rho^2 M (M + M^*)}\right] \qquad C_2 = -\frac{g_A g_\rho^2}{16\pi}.$$

Here $\vec{r}_i$, $\vec{\sigma}_i$, $\vec{\tau}_i$ refer to the position, spin, and isospin component of the i-th nucleon undergoing the weak transition, k is



Feynman graph representation of the two-nucleon axial MEC-operator in the tree-approximation: a) pair term, b) isobar excitation current, c) contact term, d) $\rho\pi$-weak decay current, e) $A_1\rho\pi$-current. J^A stands for the weak leptonic axial-vector current. The S-matrix is sandwiched between the initial and final nuclear states.

the linear momentum associated with the axial current, f_π $(f_{\pi N N^*})$ stands for the πNN-(πNN*) coupling constants; M*, for the mass of the $\Delta(1236)$ isobar state, b is the oscillator length parameter. The radial dependence $F_0(r)$

$$F_0(r) = \int_0^1 dt\, e^{-Br} j_0(Akrt) \left\{ 1 + \left(\frac{1}{m_\rho^2} + \frac{\kappa_V}{4M^2}\right) a^2 \left[1 - \frac{2}{Br}\left(1 + \frac{1}{Br}\right)\right] + \frac{\kappa_V}{4 m_\rho^2 M^2} a^4 \left[1 - \frac{4}{Br}\left(1 + \frac{1}{Br}\right)\right]\right\} \tag{2.2}$$

$$a = \sqrt{t(1-t)k^2 + t(m_\rho^2 - m_\pi^2) + m_\pi^2}$$

$$\kappa_V = \mu_p - \mu_n = 3.7 \qquad B = ab\sqrt{2} \qquad A = b\sqrt{2}$$

determines the contribution from both the rho-weak decay current and the $A_1\rho\pi$-current (graphs d,e) and arises when the functional form of the amplitude

$$J(\vec{r}) = \int \frac{\ell^{i\vec{q}\cdot\vec{r}}\, d\vec{q}}{[(\vec{k}+\vec{q})^2 + m_\rho^2][\vec{q}^{\,2} + m_\pi^2]} \tag{2.3}$$

is transformed into the convenient integral representation

$$J(\vec{r}) = \pi^2 \int \frac{dt}{a}\, \ell^{-ar + it\vec{k}\cdot\vec{r}}. \tag{2.4}$$

The exact treatment of this quantity is necessary in the muon-capture process because of the large transferred four-momentum $k^2 = 0.8 m_\pi^2$. For zero-transferred momentum the contribution from the rho-meson exchange graphs can be simplified using the obvious relation

$$\frac{1}{(m_\pi^2 + q^2)(m_\rho^2 + q^2)} = \frac{1}{m_\rho^2 - m_\pi^2}\left(\frac{1}{m_\pi^2 + q^2} - \frac{1}{m_\rho^2 + q^2}\right).$$

It leads to

$$\int \frac{\ell^{i\vec{q}\cdot\vec{r}}\, d\vec{q}}{[\vec{q}^{\,2} + m_\pi^2][\vec{q}^{\,2} + m_\rho^2]} = \frac{2\pi^2}{m_\rho^2 - m_\pi^2}\,\frac{1}{r}\,[\ell^{-m_\pi r} - \ell^{-m_\rho r}]. \tag{2.5}$$

As a result, the radial dependence becomes

$$F_0(r) = \frac{2}{m_\rho^2 - m_\pi^2}\Big\{ m_\pi^2 \Big(1 + \frac{\kappa_V m_\pi^2}{4M^2}\Big)\Big(1 + \frac{m_\pi^2}{m_\rho^2}\Big) Y_1(A m_\pi r) - m_\rho^2 \Big(1 + \frac{\kappa_V m_\rho^2}{4M^2}\Big)\Big(1 + \frac{m_\rho^2}{m_\rho^2}\Big) Y_1(A m_\rho r)\Big\} \tag{2.6}$$

According to the KSFR relation

$$2 f_\pi^2 g_\rho^2 = m_\rho^2$$

and the Goldberger-Treiman relation

$$M g_A = g_r f_\pi$$

we rewrite (2.1) in the more convenient form

$$J^4_{exch} = \sum_{i\neq j} \ell^{-i\vec{k}_\nu\cdot\vec{r}_i}\, \vec{\sigma}_j \cdot \hat{r}_{ij}\, \{C_1' Y_1(A m_\pi r) + C_2' Y_1(A m_\rho r)\}\, i(\vec{\tau}_i \times \vec{\tau}_j)_-$$

$$C_1' = -\frac{g_r^2 m_\pi^2}{8\pi g_A M^2}\Big\{\Big[\frac{1}{2} + \frac{1}{2}\,\frac{m_\rho^2}{(m_\rho^2 - m_\pi^2)}\Big(1 + \frac{\kappa_V m_\pi^2}{4M^2}\Big)\Big(1 + \frac{m_\pi^2}{m_\rho^2}\Big)\Big] + \frac{4 f_{\pi NN^*}^2 f_\pi^2}{9M(M + M^*)}\Big\}$$

$$C_2' = -\frac{g_r^2 m_\pi^2}{8\pi g_A M^2}\Big\{-\frac{m_\rho^2}{m_\rho^2 - m_\pi^2}\,\frac{m_\rho^2}{m_\pi^2}\Big(1 + \frac{\kappa_V m_\rho^2}{4M^2}\Big)\Big\}. \tag{2.7}$$

The term in the brackets is exactly identical with the expression usually exploited in calculating MEC to the transition operator of the nuclear axial charge density not only in the beta-decay process, where the approximation $k \sim 0$ is nearly valid and therefore the formulae (2.6) and (2.7) are justified, but also in the muon-capture [3,7,8]. This holds when $m_\rho^2/(m_\rho^2 - m_\pi^2) \sim 1$, $m_\pi^2/4M^2 \ll 1$ and $m_\pi^2/m_\rho^2 \ll 1$ is assumed. From now on the notation MEC_4^{rst} will be used for this restricted form of the operator. Cheng, Lorazo and Goulard calculated correctly the rho-meson propagator in the $\rho\pi$-exchange graph and as an effect, the term $\frac{g_r^2 m_\pi^2}{8\pi g_A M^2}\,\frac{m_\rho^2}{m_\rho^2 - m_\pi^2}\,\frac{m_\rho^2}{m_\pi^2}\, Y_1(A m_\rho r)$ is added to MEC_4^{rst} [7]. We found by our method the same term (C_2' from (2.7)) to be larger by a factor of $(1 + \kappa_V m_\rho^2/4M^2)$. This is due to the contribution from the $A_1\rho\pi$-diagram and shows the real importance of taking into account the A_1-meson exchange. The last term in C_1', the coefficient in front of which is $4/9\, f_{\pi NN^*}^2 f_\pi^2/M(M + M^*)$, represents the contribution coming from the Δ-isobar excited nucleon state and is generally omitted because of its small magnitude. A correct treatment of the muon-capture rates, however, requires the employment of the complete expression (2.1).

3. The MESON EXCHANGE CORRECTIONS TO THE TRANSITION RATES

The partial transition rates of the muon-capture reaction

$$\mu^- + {}^{16}O(0^+; gst) \to {}^{16}N(0^-; 120\ keV) + \nu_\mu$$

and the inverse beta-decay process

$$ {}^{16}N(0^-; 120\ keV) \to {}^{16}O(0^+; gst) + e^- + \bar{\nu}_e$$

are usually calculated starting with the non-relativistic Hamiltonian [15] and are determined as

$$\Lambda_\mu(0^+ \to 0^-) = 16.75\, g_A^2(q^2) 10^3\, |\langle 0^- | J_{IA}^{(\mu)} | 0^+ \rangle|^2$$

$$\Lambda_\beta(0^- \to 0^+) = 21.3\, g_A^2(0)\, |\langle 0^- | J_{IA}^{(\beta)} | 0^+ \rangle|^2 \tag{3.1}$$

$$J_{IA}^{(\mu)} = \frac{1}{M}(000;p) + \frac{1}{\sqrt{3}}\,\frac{G_A - G_P}{g_A(q^2)}(110)$$

$$J_{IA}^{(\beta)} = \frac{1}{M}(000;p) - \frac{1}{\sqrt{3}}\, g_\beta(110).$$

The conventional values of the weak coupling constants are used

$$g_V = 0.97 \qquad g_A(q^2)\big|_{q^2 = 0.8 m_\pi^2} = 1.24 \qquad g_p = 7.5 g_A$$

$$g_M = 3.7\, g_V \quad G_A - G_p = g_A\left(1 - (g_p/g_A - 1)\frac{E_\nu}{2M}\right) \quad g_A(0) = 1.26$$

$$g_\beta = 0.932\left(1 + \frac{3\alpha Z}{2RE_\nu^\circ}\right) /8/.$$

The energy of the outgoing neutrino in the muon-capture is $E_\nu = 95.1$ MeV and the maximal electron energy in the beta-decay is $E_\nu^\circ = 11.05$ MeV. The operators (110) and (000;p) are defined as

$$(110) = -\sqrt{3}\, j_1(E_\nu r)\,\vec{\sigma}\cdot\hat{r}\,\tau_- ,$$
$$(000;p) = j_0(E_\nu r)\,\vec{\sigma}\cdot\vec{\nabla}\,\tau_- . \tag{3.2}$$

The meson-exchange corrections to the partial transition rates are included via

$$\Lambda_\mu(0^+ \to 0^-) = 16.75\, g_A^2(q^2) 10^3 \left| \langle 0^- | J_{1A}^{(\mu)} + \frac{1}{\sqrt{2}\, g_A(q^2)} J_{exch}^{4(\mu)} | 0^+ \rangle \right|^2$$

$$\Lambda_\beta(0^- - 0^+) = 21.3\, g_A^2(0) \left| \langle 0^- | J_{1A}^{(\beta)} + \frac{1}{\sqrt{2}\, g_A(0)} J_{exch}^{4(\beta)} | 0^+ \rangle \right|^2 \tag{3.3}$$

$$J^h = J_{1A} + \frac{1}{\sqrt{2}\, g_A} J_{exch}^4 .$$

In the simple picture of a closed core hypothesis for the 0^+ state and particle-hole configuration for the 0^--excited state

$$|0^-; 120\ \text{keV}\rangle = |(2s_{1/2} 1p_{1/2}^{-1})^{J=M=0}_{T=1\ T_3=-1}\rangle$$

the nuclear matrix element in the impulse approximation is determined as

$$\langle 0^- | J_{1A} | 0^+ \rangle = \frac{1}{\sqrt{3}} (2s_{1/2} \| J_{1A} \| 1p_{1/2}). \tag{3.4}$$

We found easily the MEC-part in this simplified situation to give

$$\langle 0^- | J_{exch}^4 | 0^+ \rangle =$$
$$= \frac{1}{2} \frac{1}{\sqrt{3}} \sum_{\substack{n\ell j = 1s, 1p \\ J = 0,1,2 \\ T, T'}} n_{1p_{1/2}, n\ell j}^{-1} \hat{J} (2s_{1/2}; n\ell j; JT \| J_{exch}^4 \| 1p_{1/2}; n\ell j; JT'; 1)(-)^T \tag{3.5}$$

We defined the renormalization coefficient δ as

$$\delta = \frac{\langle 0^- | J^h | 0^+ \rangle}{\langle 0^- | J_{1A} | 0^+ \rangle} \tag{3.6}$$

and included also the parameter

$$z = \frac{\delta_\mu^2}{\delta_\beta^2}. \tag{3.7}$$

Its value determines the reduction of the ratio $\Lambda_\mu(0^+ - 0^-)/\Lambda_\beta(0^- - 0^+)$ after taking into account MEC-effects. In order to check our formulae we calculated the renormalization of the $(2s_{1/2} - 1p_{1/2})$ single-particle matrix element for the muon capture (δ_μ) and the beta decay (δ_β) using both MEC_4^{rst} and the representations (2.1) and (2.7). For each operator the parameter z is also presented (see the Table).

4. DISCUSSION

The expressions (2.7) show that the correct treatment of the heavy-meson graphs leads to an additional term (C_2'), which works in the opposite direction of the dominating one-pion exchange contributions. As an effect the partial rates derived from the hard-pion operator are significantly smaller as compared to the MEC_4^{rst} -results (the table). In the muon capture, however, the contributions coming from the usually omitted $(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\hat{r}_{ij}$ -part, which contains spherical Bessel-functions of odd power and arises when the operator (2.1) is transformed from the absolute to relative and centre-of-mass coordinates, compensate to some extent the heavy-meson exchange corrections. In the beta-decay process the term $(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\hat{r}_{ij}$ is suppressed because of the small transferred momentum. So the influence of the ρ- and A_1 - diagrams in this case is not strongly attenuated and the enhancement of the single-particle matrix element is essentially smaller (by 28%) as the result obtained after the standard version MEC_4^{rst}, was used. The parameter z, however, increases only by about 10% and this is in our opinion not so crucial for the ratio g_p/g_A as it has been feared /7/ because of the experimental uncertainties. So we conclude that in the simplified nuc-

Table

The reduced matrix elements of the two-body exchange operator is labelled by $(nr1\ ,nr2;\ ;\ JT\|\ J^4_{exch}\ \|\ nr3,\ nr4;\ JT')$. The notation $1\equiv 1s_{1/2}$, $2\equiv 1p_{3/2}$, $3\equiv 1p_{1/2}$ and $5\equiv 2s_{1/2}$ is used. For other symbols see the text

quantum numbers of the r.m.e.					muon capture r.m.e.		beta decay r.m.e.	
nr1	nr2	nr3	nr4	JTT'	MEC^{rst}_4	MEC^{h-p}_4	MEC^{rst}_4	MEC^{h-p}_4
5	1	3	1	001	-	-.000823	-	-.000125
5	1	3	1	101	-.372979	-.283696	-.372123	-.311493
5	3	3	3	001	-.002289	.006014	.027297	.035498
5	2	3	2	101	-.001402	.003744	.016716	.021743
5	2	3	2	201	-.001809	-.001152	.021580	.027958
$\langle 0^- \mid J^4_{exch} \mid 0^+ \rangle$					-.189293	-.138502	-.152629	-.112372
$\langle 0^- \mid J_{IA} \mid 0^+ \rangle$					-.324667		-.109847	
δ					1.33	1.24	1.78	1.57
Λ (sec^{-1})					$4.82 \cdot 10^3$	$4.2 \cdot 10^3$	1.29	1.01

type of the operator	MEC^{rst}_4	MEC^{h-p}_4	experiment
z	0.56	0.62	(0.57)

lear-structure picture still much room remains for mesonic exchange corrections even if heavy-meson exchanges are taken into account.

The reason why MEC are expected to play an essential role in the 0^+-0^--transition is of a principal nature and reflects the approximate global chiral symmetry properties of the strongly interacting nucleon system. The chiral invariance of a system consisting of non-zero mass particles can be realized only via the mechanism of spontaneous symmetry breaking, which leads, as is well known, to the appearance of massless Goldstone particles which could be, with a good accuracy, identified with pions. If the philosophy of the chiral invariance of strong interaction is true and the mechanism of spontaneous breaking takes place, one should not expect, that the generator of axial chiral rotation (the axial charge) can be obtained as a simple sum of the purely nucleonic contributions with the pionic mode completely neglected, as impulse approximation suggests. In order to expose the interplay between the heavy-meson exchanges and the nuclear-structure correlation effects explicit use must be made of the correct 0^+ and 0^--wave functions. The latter arise from the diagonalization of the nuclear residual interaction within the complete subspace of all non-spurious $0\hbar\omega-2\hbar\omega$ $(1\hbar\omega-3\hbar\omega)$ configurations (see for example ref. /16/). For this aim a self-contained package of computer programs CURRME for calculating the reduced matrix elements (r.m.e.) of the one- and two-body parts of the operator of the nuclear weak axial charge density has been written in FORTRAN IV. We apply the standard shell-model technique to transform the antisymmetrized two-particle wave-function from the absolute to the relative and centre-of-mass coordinates and use the Brody-Moshinsky coefficients (see the Appendix). We generated numerically the r.m.e. both for the muon-capture and the beta-decay processes. In the model subspace spanned over the $1s_{1/2}$ up to the $2p_{1/2}$ oscillator shells the J^4_{exch}-operator is determined by 946 various non-vanishing (coupled) reduced matrix elements of the type

$$(n_1 \ell_1 j_1;\ n_2 \ell_2 j_2;\ JT \|\ J^4_{exch} \|\ n_3 \ell_3 j_3;\ n_4 \ell_4 j_4;\ J'T')\ \delta_{JJ'}$$

$$n_1 \ell_1 j_1 \le n_2 \ell_2 j_2 \qquad n_3 \ell_3 j_4 < n_4 \ell_4 j_4$$

$$\ell_1 + \ell_2 - \ell_3 - \ell_4 = \text{odd}$$

$$\rho_1 = 2n_1 + \ell_1 + 2n_2 + \ell_2 \qquad \rho_2 = 2n_3 + \ell_3 + 2n_4 + \ell_4$$

$$|\rho_1 - \rho_2| = 1 \text{ or } 3 \qquad \rho_1,\ \rho_2 < 6$$

and the 1s-shell being completely occupied.

The result of the extended calculation of the nuclear matrix element will be presented in a forthcoming publication.

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APPENDIX

The Reduced Matrix Elements of the Weak Axial Charge Density MEC-Operator

Using the standard general method of second quantization for an arbitrary spherical tensorial operator $F^{k\kappa}_{t\tau}$ of rank r, projection κ in the coordinate-spin space and rank t projection τ in the isospin space the following representation (in obvious notations) in the jj-coupling scheme can be written

$$F^{k\kappa}_{t\tau} = \frac{1}{4} \sum_{\substack{1'2',1,2\\ J,M,\theta,\theta_3\\ J',M',\theta',\theta'_3}} n^{-1}_{12} n^{-1}_{1'2'} \hat{J}\hat{J}'\hat{\theta}\hat{\theta}' (-)^{J-M+\theta-q} \begin{pmatrix} J & j_2 & j_1 \\ -M & m_2 & m_1 \end{pmatrix} \begin{pmatrix} J' & j'_2 & j'_1 \\ -M' & m'_2 & m'_1 \end{pmatrix} \begin{pmatrix} J' & k & J \\ -M' & \kappa & M \end{pmatrix}$$
$$\times \begin{pmatrix} \theta & 1/2 & 1/2 \\ -\theta_3 & \tau_2 & \tau_1 \end{pmatrix} \begin{pmatrix} \theta' & 1/2 & 1/2 \\ -\theta'_3 & \tau'_2 & \tau'_1 \end{pmatrix} \begin{pmatrix} \theta' & t & \theta \\ -\theta'_3 & \tau & \theta_3 \end{pmatrix}$$
$$\times (n_{1'}\ell_{1'} j_{1'};\, n_{2'}\ell_{2'} j_{2'};\, J'\theta' \| F^k_t \| n_1\ell_1 j_1;\, n_2\ell_2 j_2;\, J\theta) \times a^+_{1'} a^+_{2'} a_2 a_1 \qquad \text{(A.1)}$$

$$n_{12} = (1 - \delta_{n_1\ell_1 j_1;\, n_2\ell_2 j_2} (-1)^{J+T-1/2}. \qquad \text{(A.2)}$$

The antisymmetrized two-particle states are kept in mind as

$$|n_1\ell_1 j_1; n_2\ell_2 j_2; JM; TT_3\rangle = n_{12} \sum_{\substack{m_1 m_2 m_{\ell_1} m_{\ell_2}\\ m_{s_1} m_{s_2} \tau_1 \tau_2}} (j_1 m_1 j_2 m_2 | JM)(1/2\tau_1 1/2\tau_2 / TT_3) \qquad \text{(A.3)}$$
$$\times (\ell_1 m_{\ell_1} 1/2 m_{s_1} | j_1 m_1)(\ell_2 m_{\ell_2} 1/2 m_{s_2} / j_2 m_2) \frac{1}{\sqrt{2}} [\psi_1(1)\psi_2(2) - \psi_1(2)\psi_2(1)].$$

The single particle state $|n\ell m_\ell m_s; 1/2\tau\rangle$ refers to the harmonic oscillator potential and is defined by

$$\psi_{n\ell m_\ell m_s \tau} = R_{n\ell}(r)\, Y_{\ell m_\ell}(\theta,\phi)\, \chi_{m_s} \chi_\tau\,, \qquad \text{(A.4)}$$

where $Y_{\ell m_\ell}(\theta,\phi)$ are the spherical harmonics normalized over the unit sphere,

$$R_{n\ell}(r) = \sqrt{\frac{2n!}{\Gamma(n+\ell+3/2)}}\, r^\ell\, e^{-\frac{r^2}{2}} L_n^{\ell+1/2}(r^2) \qquad \text{(A.5)}$$

$L_n^{\ell+1/2}(r^2)$ are the Laguerre polynomial as defined in /17/, the radial distance is expressed in units $b = \sqrt{\hbar/M\omega}$ and the phase of the radial functions is chosen so that they are positive at the coordinate origin.

Using the standard technique of spherical tensor decomposition and the Racah-algebra technique, the following expression for the reduced matrix elements of the operator (2.1) is obtained in the framework of the classical shell-model

$$(n\ell s(j); NL; JT \| J^{k=0}_{t=1} \| n'\ell' s'(j'); N'L'; JT') =$$

$$= \sqrt{6}\,\sqrt{3}\,(\sqrt{6})^2 \sum_{\substack{i=1,2\\ g=1,2}} (-1)^{c_i} \begin{pmatrix} c_i & 1 & A_i \\ 0 & 0 & 0 \end{pmatrix} (-1)^{J+j'+\ell+s'} \xi_i [1+\xi_i(-1)^{s+s'}]\, \hat{J} \qquad \text{(A.6)}$$

$$\hat{A}_i \hat{A}_i \hat{c}_i \hat{c}_i \hat{s}\hat{s}' \hat{T}\hat{T}' \hat{j}\hat{j}' \hat{\ell}\hat{\ell}' \hat{L}\hat{L}' \begin{pmatrix} \ell & A_i & \ell' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & c_i & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} j & j' & c_i \\ L' & L & J \end{Bmatrix}$$

$$\begin{Bmatrix} 1 & s & s' \\ 1/2 & 1/2 & 1/2 \end{Bmatrix} \begin{Bmatrix} T & T' & 1 \\ 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \end{Bmatrix} \begin{Bmatrix} A_i & 1 & c_i \\ \ell & s & j \\ \ell' & s' & j' \end{Bmatrix} c_g \int_0^\infty R_{n\ell}(r)\, j_{c_i}(Qr)\, \Phi_g(r)\, R_{n'\ell'}(r) r^2\, dr$$

$$\int_0^\infty R_{NL}(R)\, j_{c_i}(QR)\, R_{N'L'}(R)\, R^2\, dR$$

$$Q = \frac{kb}{\sqrt{2}} \qquad \begin{array}{llll} i=1 & A_i=1 & c_i=0 & \xi_i=1 \\ i=2 & A_i=0 & c_i=1 & \xi_i=-1 \end{array} \qquad \Phi_g(r) = \begin{cases} g=1 & Y_1(Am_\pi r) \\ g=2 & F_0(r) \end{cases}$$

The transformation to the jj-coupling scheme can then be performed according to the relation

$$(n_1\ell_1 j_1; n_2\ell_2 j_2; JT \| F^k_t \| n_3\ell_3 j_3; n_4\ell_4 j_4; J'T') = n_{12} n_{34} \times$$
$$\times \sum_{\substack{\lambda S \lambda' S'\\ jj'\\ n\ell NL\\ n'\ell' N'L'}} \tilde{n}_{12}\tilde{n}_{34}\, \hat{\lambda}'\hat{S}'\, \hat{j}_3 \hat{j}_4\, \hat{\lambda}\hat{S}\, \hat{j}_1 \hat{j}_2 \begin{Bmatrix} \ell_1 & 1/2 & j_1 \\ \ell_2 & 1/2 & j_2 \\ \lambda & S & J \end{Bmatrix} \begin{Bmatrix} \ell_3 & 1/2 & j_3 \\ \ell_4 & 1/2 & j_4 \\ \lambda' & S' & J' \end{Bmatrix} \qquad \text{(A.7)}$$

$$U(L\ell JS: \lambda j)\, U(L'\ell' J'S'; \lambda' j') \langle n\ell NL\lambda | n_1\ell_1\, n_2\ell_2 \rangle$$

$$\langle n'\ell' N'L'\lambda' | n_3\ell_3\, n_4\ell_4 \rangle (-1)^{\lambda+\lambda'+\ell+\ell'+j+j'+J+J'} [1+(-1)^{\ell_3+\ell_4+L+S+T}]$$

$$\times (n\ell S(j); NL; JT \| F^k_t \| n'\ell' S'(j'); N'L'; J'T').$$

$$\tilde{n}_{12} = (1 - \delta_{(n_1\ell_1)(n_2\ell_2)} (-)^{L+S+T})^{1/2}$$

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Обменные поправки к оператору плотности ядерного слабого аксиального заряда в модели жестких пионов

Рассматриваются обменные поправки к временной компоненте оператора слабого аксиального тока. Выясняется влияние обмена тяжелыми мезонами на отношение парциальных скоростей перехода в реакциях захвата мюонов в основное состояние $^{16}O\ (0^+)$ с переходом в возбужденное состояние $^{16}N\ (0^-;\ 120$ кэВ$)$ и обратной реакции бета-распада. Графики с обменом тяжелых ρ и A_1 мезонов рассматриваются в сформулированной на языке феноменологических лагранжианов модели жестких пионов. В простом представлении о структуре состояния 0^+ как замкнутой $1p_{1/2}$ -оболочки и о состоянии 0^- как частично-дырочной конфигурации $(2s_{1/2}1p_{1/2}^{-1})^{J=0}_{T=1}$ вычисляется ренормализация одночастичного приведенного матричного элемента $(2s_{1/2}-1p_{1/2})$ за счет обменных эффектов. Она несколько меньше по сравнению с получаемой в отсутствие этих графиков. Отношение скоростей перехода изменяется в сторону эксперимента. Учет обмена тяжелыми мезонами не приводит к уменьшению роли обменных токов в рассматриваемой задаче.

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Kirchbach M., Truhlik E. E4-82-586

Meson-Exchange Corrections to the Nuclear Weak Axial Charge Density in the Hard Pion Model

The heavy-meson exchange contributions to the time component of the weak axial-vector current are treated using the phenomenological Lagrangian version of the hard-pion model. The meson exchanges, which are taken into account, are those due to pions, rho- and A_1 -mesons. This operator is applied to describe the purely axial weak transition $0^+ \xrightarrow[\text{desintegration beta}]{\text{capture muon}} 0^-$ in the nuclear $A=16$ system. In the simple picture of a closed-core hypothesis for the 0^+-state and particle-hole configuration $(2s_{1/2}1p_{1/2}^{-1})^{J=0}_{T=0}$ for the 0^--state the renormalization of the single-particle matrix element $(2s_{1/2}-1p_{1/2})$ for both processes is calculated. It is shown, that in this case still much room remains for mesonic exchange corrections even if heavy meson exchanges are taken into account.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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