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THE NUCLEUS $^{5}_{\Lambda}$He AND Λ-^{4}He SCATTERING

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1. INTRODUCTION

Now we have a reliable theoretical fact for the lack of binding energy of the lightest nuclei ^3H , ^3He and ^4He.

Calculations /1/ of the binding energy of these nuclei given for the majority of modern realistic NN potentials show the lack of the binding energy for 3-body nuclei about 1,5-2 MeV and for ^4He about 4-5 MeV. There exist opinions and estimations/2/ about increasing this discrepancy with increasing atomic number A, if only nucleon degree of freedom is taken into account.

But at the same time in the early seventies some/3/ theoreticians find a considerable overbinding of the $^5_\Lambda He$ nucleus E_{theor} =5.46 MeV, E_{exp}=3.12±0.02 MeV.

There arise questions on reasons for such phenomena, and, maybe, specific properties of the ΛN-interaction can explain the overbinding $^5_\Lambda He$. Two main properties of ΛN-forces make them to differ qualitatively from NN forces. The first property is the transition $\Lambda N \to \Sigma N$ which leads to the two-channel description of a ΛN system and to the appearance of the 3-body ΛNN forces. The second property distinguishing the ΛN-potential consists in the presence of the charge symmetry breaking term in the potential.

The influence of these properties on the binding energy of nucleus $^5_\Lambda He$ is analysed in ref./4/ with a negative result, that means the overbound $^5_\Lambda He$. So, in the framework of the potential model of nucleus $^5_\Lambda He$ the conclusion is necessarily drawn that the overbinding follows from a wrong off-shell behaviour of the ΛN-potentials/5/ or from defects of computing procedures/3,4/. It seems that the common property of methods used in ref./3/ is probably an insufficient consideration of the effects of multiple Λ scattering by the target nucleons and the neglecting of contributions from closed channels, the second approximation as a rule is supported by "kinematical" argument about a comparative remoteness of the nearest threshold in the system Λ- ^4He. But calculations of $\pi\, ^4He$/6/ scattering length including the contribution of closed channels show a considerable amount of this contribution. Bearing in mind a more strong ΛN coupling, as compared with πN-coupling ($a_{\Lambda N} \sim$ 2 fm, $a_{\pi N} \sim 0,1$ fm for corresponding scattering lengths), one may come to the conclusion that the $^5_\Lambda He$ models, which are equivalent to the rigid-core model, may hardly pretend to a quantitative description of this nucleus.

In this paper we present calculations of the binding energy of ${}^{5}_{\Lambda}$He and the cross section of Λ- ^{4}He -scattering at low energies for a few variants of the ΛN potentials and sets of ΛN data. The calculations were performed in the framework of the strong-coupling channels method reformulated on the basis of the Schwinger variational principle/7/.

2. Λ-^{4}He- SCATTERING

In this section we find the S-phase and cross section of the Λ-^{4}He-scattering at low incident energies. The phases of the Λ-^{4}He- scattering δ_ℓ are obtained by solving the equation:

$$V_{\parallel}\,|\phi\rangle = V_{\parallel}\,|\vec{k}_0\rangle + V_{\parallel}[(E - E_1 - h_0)^{-1} - (E - h_0)^{-1}]\,|\phi\rangle + \langle 1|V(E-h_0)^{-1}V|1\rangle|\phi\rangle \qquad (1)$$

$$\mathrm{tg}\,\delta_\ell(k_0) = 16\pi\gamma k_0 \int p^2\,dp\,\phi_\ell(p)\;S_\ell(k_0,p)\;w(k_0,p), \qquad (2)$$

where

$$V = \sum_{i=1}^{A} V_{\Lambda N_i}\;,\quad V_{\Lambda N_i}(k,p) = \frac{1}{4\pi^2 \mu_{\pi N}} \sum_{\nu} v^{\nu}(k,p)\,\mathcal{P}^{\nu},$$

$\mathcal{P}^{\nu}$ is the projector onto a spin state ν, E_1 is the binding energy of the nucleus ^{4}He; $|1\rangle$, wave function of the ground state ^{4}He; h_0, operator of the kinetic energy of the relative motions of Λ-particle and c.m.s. of nucleus; $\vec{k}_0$, relative momentum of Λ ^{4}He, $\gamma = \dfrac{\mu_{\Lambda A}}{\mu_{\Lambda N}}$.

Let us describe the ground state of the nucleus ^{4}He by the shell model with an oscillator potential. The form factor of this state has the form:

$$S(q^2) = \exp(-3\alpha^2 q^2/16) \qquad \alpha^2 = 0.621\,\mathrm{fm}^2. \qquad (3)$$

The value α corresponds to the right value radius of the nucleus ^{4}He. Eq. (1) in momentum space becomes

$$\int p^2\,dp\,S_\ell(k,p)\,w(k,p)\,\phi_\ell(p) = S_\ell(k,p)\,w(k,k_0) + \frac{2\gamma}{\pi}\int_0^\infty p^2\,dp\,[\,4k_1^2 I_1^\ell(k,p) - I_2(k,p)\,S_\ell(k,p) - 3I_3^\ell(k,p)\,]\,\phi_\ell(p), \qquad (4)$$

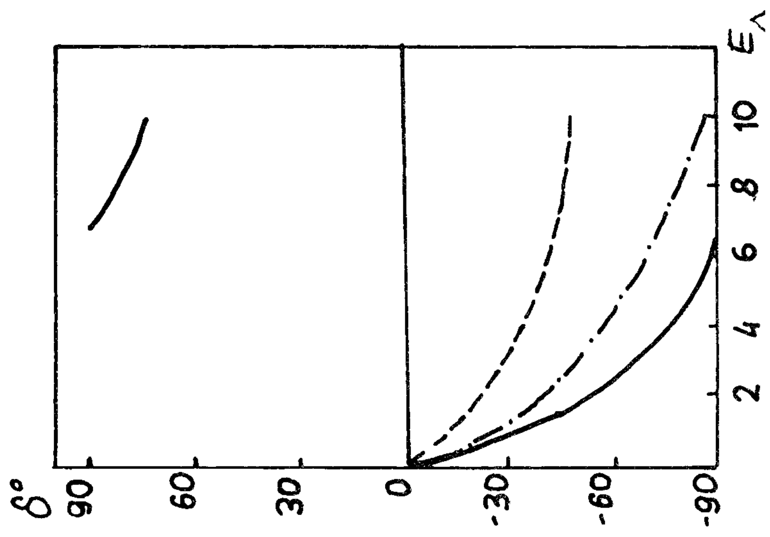

Fig.1

Table 1

Λ-^{4}He cross sections σ (mb) obtained with $V^{(1)}_{\Lambda N}$, $V^{(2)}_{\Lambda N}$ and $V^{(3)}_{\Lambda N}$

E_N (MeV)	$V^{(1)}_{\Lambda N}$	$V^{(2)}_{\Lambda N}$	$V^{(3)}_{\Lambda N}$
2	9.269	9.6	10.1
4	6.482	6.589	6.766
6	4.710	4.729	4.738
8	3.53	3.496	3.426
10	2.704	2.638	2.538
$a_{\Lambda\,^4He}$ (fm)	3.34	3.59	3.46

The last line presents the Λ- ^{4}He scattering lengths.

where $S_\ell(k,p) = e^{-a(k^2+p^2)} j_\ell(-2iakp)\, i^\ell$ is a partial wave harmonic of the form factor (3) $a = \frac{3}{4}\alpha^2$, $k_1^2 = 2\mu_{\Lambda A}|E_1|$, functions I_1, I_2, I_3 are given in the appendix, the function w(k,p) can be expressed in terms of the ΛN potential

$$w(k,p) = \sum_\nu c^\nu v^\nu(k,p),$$

$$c = \frac{1}{4}\sum_i \langle \Lambda 1 | \mathcal{P}_i^\nu | 1 \rangle,$$

where $|\Lambda 1\rangle$ is the spin function of the system ΛA. In the calculations we use s-wave ΛN potentials of three kinds

$$V^{(1)}_{\Lambda N}(p,k) = \frac{\Lambda}{(\beta^2+p^2)(\beta^2+k^2)}. \qquad (5)$$

Parameters of the potential (5) correspond to set B from ref.[8]

$$V_{\Lambda N}(r) = -V_{2\pi}\left(\frac{r}{\lambda_{2\pi}}\right)^{-1} e^{-r/\lambda_{2\pi}} - V_k\left(\frac{r}{\lambda_k}\right)^{-1} e^{-r/\lambda_k} +$$

$$+ V_\omega\left(\frac{r}{\lambda_\omega}\right)^{-1} e^{-r/\lambda_\omega}. \qquad (6)$$

We take potential (6) from ref.[9] with parameters corresponding to the set 1. The range of each term in the potential (6) equals the inverse mass of the corresponding meson (k,ω,2π)

$$V^{(3)}_{\Lambda N}(r) = \begin{cases} -V_0 & r \le R \\ 0 & r > R \end{cases}. \qquad (7)$$

The potential (7) has the same scattering lengths and effective radii as the potentials (5) and (6) in the corresponding states.

Results of the calculations of cross sections, phases and scattering lengths of the Λ-^{4}He scattering are given in table 1 and figs.1-4.

As one can see from fig.1, at the energy of Λ-particle around 6-7 MeV the Λ-^{4}He-scattering phase goes through the π/2, i.e., in the system there arises a narrow resonance. The form of Argand-Plot (fig.4) and behaviour of the amplitude on the second sheet of energy indicate the existence of a pole in K-plane at $k_{res} = k_1 - ik_2$, $k_1 = 0.54\ \mathrm{fm}^{-1}$, $k_2 = 0.014\ \mathrm{f}^{-1}$.

Because there are no barriers in the consideration (we use only s-wave ΛN potentials) the resonance behaviour of Λ-nuclear s-wave phase can be understood only, as a manifestation of multiple scattering of Λ particle on nucleons. The absence of such a resonance in the rigid-core model Λ-He scattering is obvious.

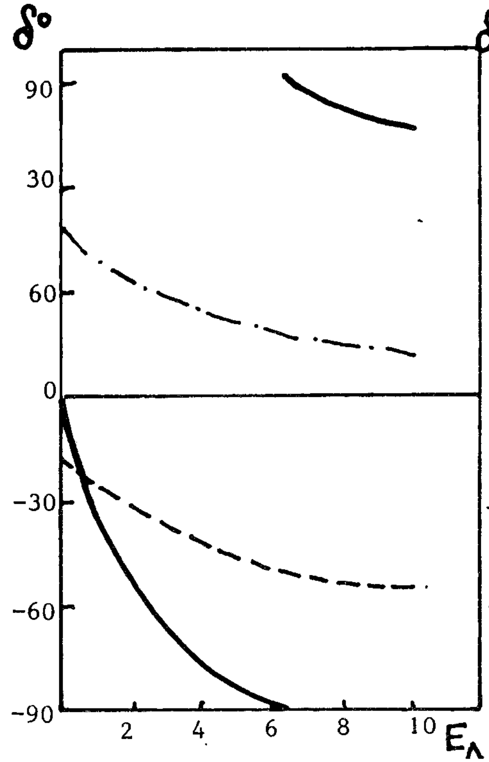

Fig.2

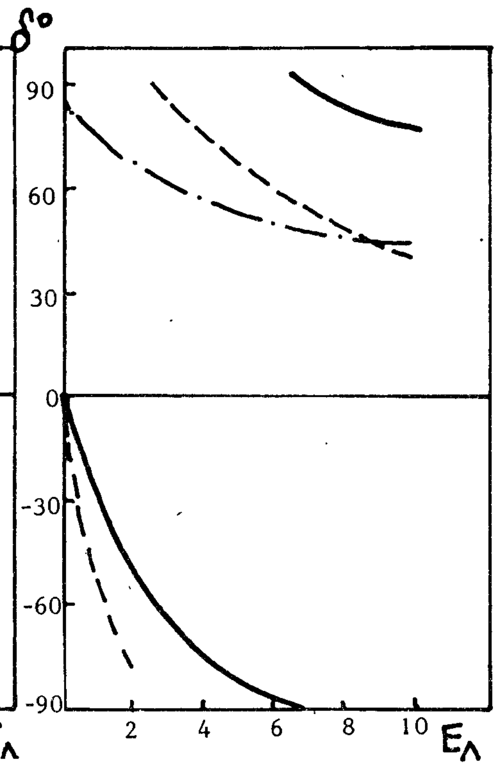

Fig.3

In fig.1 we show the s-phase of Λ-^{4}He scattering: the solid curve is the potential (6); the dash-dotted line corresponds to the change $V_{2\pi} \to V_{2\pi}/2$ in the potential (6); the dotted line corresponds to the change $V_{2\pi} \to 2.\,V_{2\pi}$. As can be seen, variations of the intensity of the 2π-meson term in the potential do not change qualitatively the energy behaviour of the phase before the resonance, but the resonance itself disappears or moves to more high energy region.

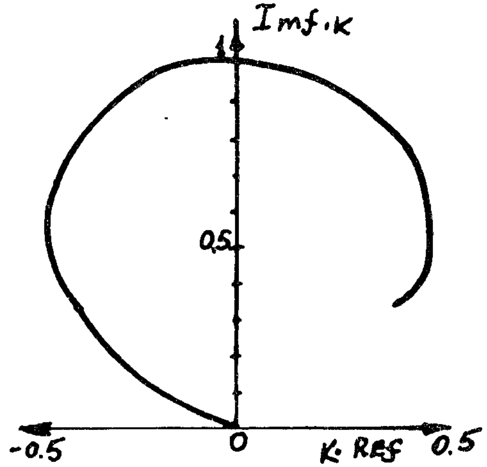

Fig.4

A similar change caused by the K-meson component of the potential (6) is described in fig. 2. In this case the resonance behaviour of the phase-shift seems to disappear at all. The comparison of curves I and II shows that in this energy region the s-phase of $\Lambda - {}^4He$ scattering is more sensitive to variations of that part of the potential which is generated by the K-meson exchange.

Finally, in fig.3 with the notation already adopted, the changes of s-phase of $\Lambda - {}^4He$ scattering, caused by varying the part of the potential, generated by the ω-meson exchange, are shown. In this case, the replacement $V_\omega \to 2V_\omega$ moves the resonance in the $\Lambda - {}^4He$ system towards low energies, approximately to $E \sim 2$ MeV.

The reason for such a behaviour of the resonance is the repulsive character of the ω-meson exchange part of the potential.

As follows from behaviour of the phase shift, as well as from the value of $\mathrm{Im}\, k_{res}$ the width of the resonance found in $\Lambda - {}^4He$ system is very narrow $E \sim 0.1$ MeV.

Apparently, the width may be increased by including contributions from the excited states of ${}^4He$ explicitly. At any rate, it is desirable to seek this resonance experimentally.

3. ${}^5_\Lambda He$

The binding energy of ${}^5He$ is determined by solving the homogeneous eq. (1) at $E < 0$. Results were given in table 2.

The value of the separation energy B_Λ presented in table 2 corresponds to the following approximation in eq. (1).

$$\langle 1|\, V G_0 V\,|1\rangle = \langle 1|\, V\,|1\rangle\, G_0\, \langle 1|\, V\,|1\rangle ,$$

i.e., the neglect of the contribution from closed channel to the kernel of the right-hand side of eq. (1).

As is seen from table 2, in this case the potential gives no bound state for $- {}^5_\Lambda He$.

It is also seen that, one can obtain a better agreement with the experimental value of the ${}^5_\Lambda He$ separation energy using a relatively simple central potential as $V^{(2)}$.

It should be noted that the potential $V^{(2)}$ with set 1 of parameters describes well the separation energy B_Λ for ${}^4_\Lambda He$, ${}^4_\Lambda H$, ${}^3_\Lambda H$ as well.

*Continuation onto the second sheet of energy is performed by a polynomial extrapolation of the function $k \cdot \mathrm{ctg}\, \delta(k)$.

Table 2

		$V^{(2)}_{\Lambda N}$ set 1	set 2	set 3	$V^{(3)}_{\Lambda N}$	Experiment
B_Λ	(MeV)	3.28	4.95	5.74	5.44	3.12 ± 0.02
B_Λ	(MeV)	0		4.75		

CONCLUSION

So, the investigations presented here show that one can reach a better agreement with the experimental value of ${}^5He$ binding energy, remaining in the framework of the potential formulation of the $5(\Lambda 4N)$ —body problem, and using a rather simple ΛN potential.

This potential gives good values for binding energy of 3- and 4-particle hypernuclei. For $\Lambda - {}^4He$ scattering this potential shows a narrow resonance at energy $E_\Lambda \sim 7$ MeV. The appearance of the resonance is caused by effects of multiple scattering of Λ-particle on nucleons of the target and by specific features of elementary ΛN interaction. In fact, no resonance appears in s-wave $\pi - {}^4He$ scattering and in s-wave $n - {}^4He$ scattering at low energy.

APPENDIX

$$I^\ell_1 (k,p) = \int_0^\infty \frac{q^2 dq\, W(k,q)\, W(q,p)\, \delta^\ell (k,q)\, S^\ell(k,q)}{(k_0^2 - q^2)\, (k_0^2 - q^2 - k_1^2)}$$

$$I^\ell_2 (k,p) = \sum_\nu c^\nu \int_0^\infty \frac{q^2 dq\, v^\nu(k,q)\, v^\nu(q,p)}{k_0^2 - q^2 - k_1^2}$$

$$I^\ell_3 (k,p) = \sum_{\mu\nu} c^{\mu\nu} \int_0^\infty \frac{q^2 dq\, T_\ell(p,q,k)\, v^\mu(p,q)\, v^\nu(q,k)}{k_0^2 - q^2 - k_1^2} ,$$

where

$$c^{\mu\nu} = \frac{1}{12} \sum_{i \neq j} \langle 1\Lambda |\, P^\mu_i\; P^\nu_j\, | 1\Lambda\rangle$$

$$T_\ell\ (q,q,k) = \sum_{\ell_1 \ell_2} S_{\ell_1}(p,q)\; S_{\ell_1}(q,k)\; S_{\ell_2}(-p,-k) \times$$

$$\times \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 (2\ell_1 + 1)(2\ell_2 + 1);$$

$\begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}$ – 3j symbol.

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Ядро $^5_\Lambda$He и $\Lambda - ^4$He рассеяние

В работе исследуется проблема пересвязанности ядра и рассеяние Λ-частиц на ^{4}He при низких энергиях. Задача пяти тел $(\Lambda + 4N)$ решается на основе метода сильной связи каналов с использованием вариационного принципа Швингера.

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The Nucleus $^5_\Lambda$He and $\Lambda - ^4$He Scattering

The problem of overbinding of the nucleus $^5_\Lambda$He and low energy Λ scattering by ^{4}He is studied. The five-body problem $(\Lambda + 4N)$ is considered on the basis of the strong coupling channel using the Schwinger variational principle.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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