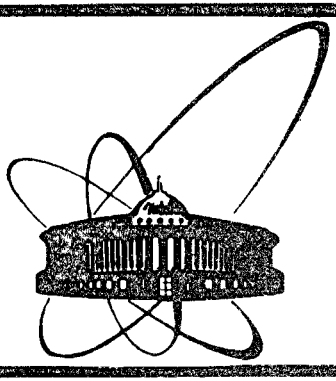

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THE OSCILLATIONS OF DOUBLE-CENTRE NUCLEAR DENSITY

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1. INTRODUCTION

Deep inelastic heavy ion collisions with nuclei are accompanied by the excitation of many states of a complicated structure. Among them the nuclear density vibrations (giant resonances) in colliding nuclei or in a double nuclear system many be the most important for understanding the dissipative phenomena [1-3] and the mechanism of preequilibrium emission of light particles [2,4-6].

In ref. [7] we began to analyse collective excitations of the double nuclear system. A hydrodynamic approach has been used to derive equations for vibrational excitations of a nucleus with the equilibrium density $\rho_0(\vec{x})$ which, in principle, can considerably differ from the spherical one. We have analysed the transition to the known case of the density oscillations inside the hard sphere [8] and demonstrated an approximate way of taking into account the nuclear surface diffuseness.

In this report we have derived the simplest analytic solution for vibrations of the two-centre density, modelling a double nuclear system.

2. EQUATIONS FOR DESCRIPTION OF NUCLEAR DENSITY OSCILLATIONS

It is convenient to describe the density oscillations in terms of operators of the density $\rho_0(\vec{x}) \equiv \hat{\Psi}^{+}(\vec{x}) \cdot \Psi(\vec{x})$ and the flux density $\vec{j}(\vec{x}) \equiv \frac{h}{2mi}(\hat{\Psi}^{+}(\vec{x}) \cdot \vec{\nabla} \Psi(\vec{x}) - \vec{\nabla} \hat{\Psi}^{+}(x) \cdot \Psi(\vec{x}))$, where $\hat{\Psi}^{+}(\vec{x})$, $\Psi(\vec{x})$ are operators of the nucleon field, and with the use of the hydrodynamical representation for the Hamiltonian of a Fermi system with the effective two-body interaction $V(\vec{x})$ [7,9,10]:

$$H = \frac{m}{2} \int d^3x \vec{j}(\vec{x}) \rho^{-1}(\vec{x}) \vec{j}(\vec{x}) + U[\rho],$$

$$U[\rho] \equiv \frac{h^2}{8m} \int d^3x \frac{|\vec{\nabla}\rho(\vec{x})|^2}{\rho(\vec{x})} + \mathcal{E}[\rho], \qquad (1)$$

$$\mathcal{E}[\rho] \equiv \int d^3x d^3y \, \rho(\vec{x}) V(\vec{x} - \vec{y}) \rho(\vec{y}),$$

which is equivalent in view of the equation of motion for the operators $\rho(\vec{x})$ and $\vec{j}(\vec{x})$ to the usual nuclear Hamiltonian [4-7].

If we restrict our analysis to the only vibrational motion (supposing the nuclear motion to be irrotational), it is suitable to use the velocity potential operator $\vec{j}(\vec{x}) \equiv \frac{1}{2}\{\rho(\vec{x}), \vec{\nabla}\phi(\vec{x})\}_+$.

The operators ρ and ϕ are connected with canonical commutation relations:

$$[\phi(\vec{x}),\ \phi(\vec{y})] = [\rho(\vec{x}),\ \rho(\vec{y})] = 0,$$
$$[\rho(\vec{x}),\ \phi(\vec{y})] = i\frac{h}{m}\delta(\vec{x}-\vec{y}), \qquad (2)$$

following from the definition of these operators and the commutation relations for the operators of the nuclear field:

$$\{\Psi(\vec{x}),\ \Psi^{+}(\vec{y})\}_+ = \delta(\vec{x}-\vec{y}).$$

Extracting the mean density $\rho_0(\vec{x})$ and the operator of its deviation $\delta\rho(\vec{x})$ from the density operator: $\rho(\vec{x}) \equiv \rho_0(\vec{x}) + \delta\rho(\vec{x})$ and demanding $\rho_0(\vec{x})$ to be the equilibrium one we get the equation of the Thomas-Fermi type for $\rho_0(\vec{x})$:

$$\frac{\delta U[\rho]}{\delta\rho_0(\vec{x})} = \frac{\delta \mathcal{E}[\rho]}{\delta\rho_0(\vec{x})} - \frac{h^2}{4m}\left(\frac{\Delta\rho_0(\vec{x})}{\rho_0(\vec{x})} - \frac{|\vec{\nabla}\rho_0(\vec{x})|^2}{2\rho_0^2(\vec{x})}\right) = 0, \qquad (3)$$
$$\int d^3x \rho_0(\vec{x}) = A.$$

A is the number of nucleons of a nuclear systems.

The equations of motion for $\delta\rho$ and ϕ operators in the harmonic approximation are the following:

$$\frac{\partial}{\partial t}\delta\rho(\vec{x},t) + \vec{\nabla}(\rho_0(\vec{x})\cdot\vec{\nabla}\phi(\vec{x},t)) = 0,$$
$$\frac{\partial}{\partial t}\phi(\vec{x},t) - \frac{h^2}{4m^2}\vec{\nabla}(\rho_0^{-1}(\vec{x})\cdot\vec{\nabla}\delta\rho(\vec{x},t)) + \qquad (4)$$
$$+\left(\frac{\delta^2 \mathcal{E}[\rho]}{m\delta^2\rho_0(\vec{x})} + \frac{h^2}{4m^2}\left(\frac{\Delta\rho_0(\vec{x})}{\rho_0^2(\vec{x})} - \frac{|\vec{\nabla}\rho_0(\vec{x})|^2}{\rho_0^3(\vec{x})}\right)\right)\delta\rho(\vec{x},t) = 0.$$

This sytem of equations has the first integral of motion (an energy $H_{[2]}$):

$$H_{[2]} \equiv \frac{m}{2}\int d^3x \rho_0(\vec{x})|\vec{\nabla}\phi(\vec{x},t)|^2 + \frac{1}{2}\int d^3x \frac{\delta^2\mathcal{E}[\rho]}{\delta\rho_0^2(\vec{x})}\cdot\delta\rho(\vec{x},t)^2 +$$
$$+\frac{h^2}{8m}\int d^3x\left(\frac{\Delta\rho_0(\vec{x})}{\rho_0^2(\vec{x})} - \frac{|\vec{\nabla}\rho_0(\vec{x})|^2}{\rho_0^3(\vec{x})}\right)\delta\rho(\vec{x},t)^2 +$$
$$+\frac{h^2}{8m}\int d^3x \frac{|\vec{\nabla}\delta\rho(\vec{x},t)|^2}{\rho_0(\vec{x})}. \qquad (5)$$

Equations (4) can be derive from (5) by the usual rules ($m\phi$ and $\delta\rho$ are canonically conjugated quantities). $H_{[2]}$ is a part of the Hamiltonian of the second order in powers of operators $\delta\rho$ and ϕ (the harmonic approximation means neglecting all the terms of a higher order).

In order to analyse the stationary oscillations operators $\delta\rho$ and ϕ can be expanded, as usual, in terms of creation and annihilation operators (b_s^+ and b_s) of the boson ($[b_s, b_{s'}^+] = \delta_{ss'}$, "$s$" is its quantum number):

$$\delta\rho(\vec{x}) = \frac{-h}{m}\sum_s g_s(\vec{x})(b_s^+ + b_s),$$
$$\phi(\vec{x}) = i\frac{h}{m}\sum_s f_s(\vec{x})(b_s^+ - b_s). \qquad (6)$$

The amplitudes $f_s(\vec{x})$, $g_s(\vec{x})$ satisfy the following relation of orthonormalization:

$$\delta_{ss'} = -\frac{2h}{m}\int d^3x f_s(\vec{x}) g_{s'}(\vec{x}),$$
$$\delta(\vec{x}-\vec{y}) = -\frac{2h}{m}\sum_s f_s(\vec{x}) g_s(\vec{y}) \qquad (7)$$

resulting from the definition (6) and commutation relations between $\delta\rho$, ϕ; b_s^+, b_s operators.

With the relations (6), (7) the equations of motion (4) can be rewritten in terms of amplitudes $f_s(\vec{x})$, $g_s(\vec{x})$ and frequencies ω_s ($H_{[2]} = \sum_s h\omega_s b_s^+ b_s + \text{const}$):

$$\vec{\nabla}(\rho_0(\vec{x})\vec{\nabla}f_s(\vec{x})) = \omega_s g_s(\vec{x}),$$
$$\frac{h^2}{4m}\left(\vec{\nabla}\left(\frac{1}{\rho_0(\vec{x})}\vec{\nabla}g_s(\vec{x})\right) - \left(\frac{\Delta\rho_0(\vec{x})}{\rho_0^2(\vec{x})} - \frac{|\vec{\nabla}\rho_0(\vec{x})|^2}{\rho_0^3(\vec{x})}\right)g_s(\vec{x})\right) - \qquad (8)$$
$$-\frac{1}{m}\frac{\delta^2\mathcal{E}[\rho]}{\delta\rho_0^2(\vec{x})}\cdot g_s(\vec{x}) = \omega_s f_s(\vec{x}).$$

To solve these equations together with equation (3) for the equilibrium density and the relation of orthonormalization (7) is a very difficult task. In paper [7] we suggested an approximate variant for the equations (8). One equation of the Schrödinger type for $f_s(\vec{x})$ and ω_s (with an approximate functional connection between $f_s(\vec{x})$ and $g_s(\vec{x})$) instead of the system of two coupled equations (8) has been written, that way has been a success in qualitative investigating the influence of the nuclear surface diffuseness on the properties of oscillations of spherical nuclei. In the next section an analytic solution

of equations (7), (8), suitable for the description of density vibrations in the double nuclear system will be derived.

3. TWO-CENTRE NUCLEAR DENSITY VIBRATIONS

A successive scheme of solving the equations (3), (7), (8) seems to be as follows: One chooses a two-body interaction $V(\vec{x})$ (or a functional $\mathcal{E}[\rho]$) and derives a continuous normalized solution for the equilibrium density $\rho_0(\vec{x})$. Substituting it into (8) one solves this problem for eigenvalues ω_s and functions f_s, g_s with the orthonormalization conditions (7). But in practice this way meets with considerable difficulties.

It is extremely difficult to get a continuous, normalized in the whole space solution of the Thomas-Fermi type for equations (3) while choosing realistic interactions (for instance, those used in Nuclear Matter Theory [11]), especially with the dependence of the interaction on the density and exchange effects (see ref.[12]). If even we were a success in obtaining such a solution, we would get a very complicated system of integro-differential equations (8). That is why we shall act as follows: We won't select an interaction, we'll take the form of the density in its ground state, and calculate the second variation $\delta^2\mathcal{E}/\delta\rho^2$ in equations (8) with the help of equations (3). By this we'll effectively coordinate all the quantities in equations (8) for ω_s, $f_s(\vec{x})$, , $g_s(\vec{x})$. This is the simpliest task for a spherical or one-dimensional case. It is more difficult for a nonspherical case. Further we'll consider the simplest analytically solvable variant, modelling the double nuclear system.

Let us select the equilibrium density of the form:

$$\rho_0(r,\ z,\ \Phi) = \bar{\rho}\ \exp(-\pi(r,\ z)), \tag{9a}$$

where (r, z, Φ) are cylindrical coordinates of a point. $\bar{\rho}$ can be found from the normalization density conditions (3).

The equation $\pi(r, z) = 0$ is an equation of the boundary of the nucleus (we suppose an axial symmetry), i.e., the surface, where the density has the maximum value. Let us select the simplest form for π, having two distinguished centers at points $z = \pm z_0$ [13,14]:

$$\pi(r,\ z) = \lambda(r^2 + (|z| - z_0)^2) - \tilde{R}^2\ . \tag{9b}$$

where parameters λ, $\tilde{R}$ are connected with the geometric sizes of the nuclear system.

For this case:

$$\frac{4m}{h^2}\frac{\delta\mathcal{E}[\rho]}{\delta\rho_0(\vec{x})} = \frac{\Delta\rho_0(\vec{x})}{\rho_0(\vec{x})} - \frac{1}{2}\frac{|\vec{\nabla}\rho_0(\vec{x})|^2}{\rho_0^2(\vec{x})} = 4\lambda(\pi(r,z) + \tilde{R}^2) - 6\lambda\ ,$$

$$\frac{4m}{h^2}\rho_0(\vec{x})\frac{\delta^2\mathcal{E}[\rho]}{\delta\rho_0^2(\vec{x})} = -2\lambda\ . \tag{9c}$$

It is necessary to mention that one has been a success in deriving these expressions due to a special form of the density (9). Substituting (9) into (7) and (8) we get:

$$\begin{aligned}
&\Delta\tilde{f}_s(\vec{x}) - \lambda^2(r^2 + (|z| - z_0)^2 - 3/\lambda)\tilde{f}_s(\vec{x}) = \omega_s\,\tilde{g}_s(\vec{x}),\\
&\Delta\tilde{g}_s(\vec{x}) - \lambda^2(r^2 + (|z| - z_0)^2 - 5/\lambda)\,\tilde{g}_s(\vec{x}) = \frac{4m^2}{h^2}\omega_s\,\tilde{f}_s(\vec{x}),\\
&\tilde{f}_s(\vec{x}) \equiv \rho_0(\vec{x})^{1/2}\,f_s(\vec{x});\quad \tilde{g}_s(\vec{x}) \equiv \rho_0(\vec{x})^{-1/2}\,g_s(\vec{x}).
\end{aligned} \tag{10}$$

The functions $\tilde{f}_s(\vec{x})$, $\tilde{g}_s(\vec{x})$ are diagonal in the basis of the two-centre oscillator [13,14]:

$$\begin{aligned}
&\tilde{f}_s(\vec{x}) \equiv \tilde{f}_{n_r\Lambda\nu}(\vec{x}) = F_{n_r\Lambda\nu}\cdot\Psi_{n_r\Lambda\nu}(\vec{x}),\\
&\tilde{g}_s(\vec{x}) \equiv \tilde{g}_{n_r\Lambda\nu}(\vec{x}) = G_{n_r\Lambda\nu}\cdot\Psi_{n_r\Lambda\nu}(\vec{x}),\\
&\Psi_{n_r\Lambda\nu}(\vec{x}) \sim (\lambda^{1/2}r)^{\Lambda}e^{-\lambda r^2/2}\,\mathcal{L}^{\Lambda}_{n_r}(\lambda r^2)e^{i\Lambda\Phi}\,D_\nu(\sqrt{2\lambda}\,(|z| - z_0)),
\end{aligned} \tag{11}$$

where $\mathcal{L}^{\Lambda}_{n_r}$ are Laguerre polynimials, D_ν are Weber parabolic-cylinder functions [15]. The frequencies $\omega_{n_r\Lambda\nu}$ and constants $F_{n_r\Lambda\nu}$, $G_{n_r\Lambda\nu}$ can be found from the equations (7), (10), (11):

$$\begin{aligned}
&\omega_{n_r\Lambda\nu} = \frac{h\lambda}{m}\sqrt{\tilde{N}(\tilde{N} - 1)}, \qquad \tilde{N} \equiv 2n_r + \Lambda + \nu,\\
&G_{n_r\Lambda\nu} = -\frac{m}{2hF_{n_r\Lambda\nu}}\omega_{n_r\Lambda\nu},\\
&F_{n_r\Lambda\nu} = 0.5\left(\frac{\tilde{N} - 1}{\tilde{N}}\right)^{1/4}.
\end{aligned} \tag{12}$$

The eigenfunctions $\Psi_{n_r\Lambda\nu}(\vec{x})$ and eigenvalues $\tilde{N}$ depend on the distance between centers $2z_0$ and can be found analogously to the Two-Centre Shell Models [13,14], with the inclusion of the conditions of continuity of the wave functions and their first derivatives at the origin of the coordinate system.

Substituting (9), (11), (12) into (6) we get explicit expressions for the operators $\delta\rho$ and ϕ, i.e., a complete hydrodynamic description of the symmetric two-centre nuclear density vibrations.

Selecting other forms for $\rho_0(\vec{x})$ (analogously to the nuclear fission [16]) one may try to generalize this method to more realistic cases of asymmetric double nuclear systems.

It is necessary to note that the formulas above can be applied only for the analysis of the isoscalar density vibrations.

In conclusion we should mark that it is very important to have a consistent solution to equations (3) for the density in the ground state and to equations (4), (8) for $\delta\rho$ and ϕ operators. Moreover, the presence of surface terms * gives us a possibility to obtain a solution correct in the whole space.

Let us illustrate this statement by the following examples. We will select the simplest parametrization of the effective interaction in the Finite Fermi System Theory [17,18]:

$$V(\vec{x}) = \frac{\pi^2 h^3}{2m^* p_F} \cdot f_0[\rho_0(\vec{x})]\delta(\vec{x}), \qquad (13)$$

where m^* is an effective nucleon mass, p_F a Fermi momentum, f_0 the scalar scattering amplitude.

Having neglected the surface terms of the pressure operator in equations (8) and taking into acccount that $\delta^2\mathcal{E}/\delta\rho_0^2 \simeq \simeq \pi^2 h^3 f_0/m^* p_F$ we'll obtain a more habitual form of these equations:

$$\vec{\nabla}\left(\frac{\rho_0(\vec{x})}{\rho_0(0)}\vec{\nabla} f_s(\vec{x})\right) + \frac{\omega_s^2}{u^2} f_s(\vec{x}) = 0,$$
$$g_s(\vec{x}) \simeq -\frac{\omega_s}{u^2}\rho_0(0) f_s(\vec{x}), \qquad (8a)$$

where for convenience the "sound" velocity u in the nucleus is introduced as usual $\delta^2\mathcal{E}/\delta\rho_0^2 \simeq m^* u^2/\rho_0(0)$.

If we'll also omit the surface terms in the equation of continuity (the first eq. (8a)), we'll obtain the usual Helmholtz equation for ω_s and $f_s(\vec{x})$ [8]. It is necessary to keep in mind that in that case the Helmholtz equation will be correct only inside the nucleus (where $\rho_0(\vec{x}) \sim \rho_0(0) = 0.17$ fm^{-3}, $f_0 \simeq f_{in} = 0.2 \div 0.5$) and needs corresponding conditions at the boundary of the nucleus. In order to get the solution in the whole space, all the surface terms should be taken into account. Particularly, B.Rumjantzev in his papers [2] pointed out that solving equations of the type (8a) with natural boundary conditions ($f_s(|\vec{x}| \to \infty) \to 0$) leads to a physically unsatisfactory result (to a continuous spectrum ω_s) if one takes the interaction (13) and the density in the ground state in the Wood-Saxon

*Such terms are indispensable for the right description of commutation properties of the kinetic energy tensor in terms of density and flux operators.

form: $\rho_0^{WS}(\vec{x}) \sim (1 + \exp((|\vec{x}| - R)/a))^{-1}$. To our mind, the reasons are the following. First, the density $\rho_0^{WS}(\vec{x})$ is not consistent with the choice of the interaction (13) in the whole space (but only near the centre of a nucleus) and, second, the surface terms of the pressure operator were not taken into account (they were omitted while the transition from the equations (8) to (8a)). The solution of equations (7), (8) for the density $\rho_0(\vec{x})$ of the Gauss type, derived in this section, is free from these drawbacks. However, if we solved the approximate system (8a) we would meet the difficulties above. To our regret we cannot vividly illustrate these assertions now, because we have failed to obtain an analytical solution of the equations (3), (7), (8) to describe the Wood-Saxon type density vibrations. And the first attempts to solve, this task numerically have shown that further hard efforts are needed. We are going to investigate this problem in a subsequent paper.

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Картавенко В.Г. Колебания двухцентровой ядерной плотности E4-82-554

В гидродинамическом приближении исследуются колебания ядерной плотности, имеющей два выделенных центра. Показано, что если равновесная плотность может быть представлена в виде двухцентрового гауссоида $\rho_0(\vec{x}) \sim \exp(-\lambda(r^2+(|z|-z_0)^2))$, то спектр коллективных возбуждений, отклонение плотности от равновесной и потенциал поля скоростей простым образом связаны со спектром и волновыми функциями двухцентрового осциллятора.

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Kartavenko V.G. The Oscillations of Double-Centre Nuclear Density E4-82-554

The density oscillations in a composite nuclear system are analysed. A hydrodynamical approach is used to investigate the vibrations of the two-centre nuclear density. It is shown that for an equilibrium density of the form $\rho_0(\vec{x}) \sim \exp(-\lambda(r^2+(|z|-z_0)^2))$ the spectrum of excitations, the deviation of the density from the equilibrium one and the velocity potential are connected in a simple way with the spectrum and wave functions of the two-centre oscillator.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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