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V.L.Lyuboshitz

THE PROBABILITY DISTRIBUTION OF THE DELAY TIME OF A WAVE PACKET IN STRONG OVERLAP OF RESONANCE LEVELS

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In our previous papers[1-5] the time development nuclear reactions at a large density of resonance levels of the compound nucleus corresponding to the same angular momentum has been discussed. It is shown that in the model of equivalent channels under the condition $\Gamma >> \frac{nD}{2\pi}$ (Γ is the width of an individual quasistationary level, n is the number of channels, and D is the average distance between the neighbouring levels) the average duration of collision is much larger than the lifetime $\hbar/\Gamma$ for an individual level and is equal to

$$t_0 = \frac{2\pi\hbar}{nD}. \tag{1}$$

In so doing the time development of processes is distinctly non-exponential in character with the root-mean-square deviation $\Delta t \le 2\sqrt{\frac{\pi}{n\Gamma D}} << t_0$. Later on similar results have been obtained in papers[6-9].

In the present paper the probability distribution of the time delay of a wave packet at the stage of compound nucleus formation is studied at arbitrary values of the parameter Γ/nD*. The initial relation, which connects the function of the time delay distribution for a given reaction with the corresponding S-matrix element, takes the form (the angular momentum is supposed to be fixed)

$$P_{ij}(t) = \frac{1}{2\pi\hbar}\int \phi_{ij}(\epsilon) e^{-i\epsilon/\hbar} d\epsilon. \tag{2}$$

Here the indices i and j correspond to the final and initial state, respectively,

$$\phi_{ij}(\epsilon) = \frac{\langle S_{ij}(E) S^*_{ij}(E-\epsilon)\rangle - |\langle S_{ij}(E)\rangle|^2}{\langle |S_{ij}(E)|^2\rangle - |\langle S_{ij}(E)\rangle|^2} \tag{3}$$

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*The formation of compound nucleus is accompanied by instantaneous elastic scattering, which we do not consider here; for $\Gamma >> nD/2\pi$ the process of instantaneous scattering corresponds to the average values of S-matrix elements which are equal to zero (see[2-3]).

is the function of amplitude correlation, the symbol $\langle\ \rangle$ denotes the averaging over the energy spectrum of packets. The average time delays at the stage of compound nucleus are determined by the formula

$$\bar{t}_{ij} = \int t P_{ij}(t) dt = -i\hbar \frac{d\phi'_{ij}(\epsilon)}{d\epsilon}\Big|_{\epsilon=0}. \tag{4}$$

They satisfy the sum rule

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \bar{t}_{ij}\,(\langle|S_{ij}(E)|^2\rangle - |\langle S_{ij}(E)\rangle|^2) = i\hbar\, Sp \langle \hat{S}(E) \frac{d\hat{S}^+(E)}{dE}\rangle. \tag{5}$$

A further consideration is based on general results of the theory of overlapping resonances. It has been shown formerly that the unitary many-channel S-matrix corresponding to a definite angular momentum satisfies the relations[3,4]

$$i\,Sp(\hat{S}(E)\frac{d\hat{S}^+(E)}{dE}) = \sum_k \frac{\Gamma_k}{(\mathcal{E}_k - E)^2 + \Gamma_k^2/4}, \tag{6}$$

$$\det(\hat{S}(E)\hat{S}^+(E-\epsilon)) = \prod_k (1 + i\epsilon\Gamma_k/((\mathcal{E}_k - E + i\frac{\Gamma_k}{2})(\mathcal{E}_k - E + \epsilon - \frac{i}{2}\Gamma_k))), \tag{6'}$$

which are independent of constant background. Besides, in accordance with [10],

$$\det \hat{S}(E) = \det \hat{U} \prod_k (1 + i\frac{\Gamma_k}{\mathcal{E}_k - E - \frac{i}{2}\Gamma_k}), \tag{7}$$

where $\hat{U}$ is the background matrix. The averaging of the sum rule (6) over the energy spectrum of a packet with the effective width $\Delta E \gg \Gamma_k, D$ yields

$$i Sp\langle \hat{S}(E)\frac{d\hat{S}^+(E)}{dE}\rangle = \frac{2\pi}{D} \tag{8}$$

and an analogous procedure for expression (7) leads to the Simonius formula[11]

$$|\det\langle\hat{S}(E)\rangle| = |\langle\det\hat{S}(E)\rangle| = e^{-\pi\Gamma/D}, \tag{9}$$

where Γ is the mean width of resonance levels. In the one-channel case from (5), (8) and (9) there follows the result

$$\bar{t} = \frac{2\pi\hbar}{D}(1 - e^{-\frac{2\pi\Gamma}{D}})^{-1} \tag{10}$$

obtained previously in paper[5] and independently in[6]. In the framework of the model of n equivalent channels (see[3-5]) the average time delays at the stage of compound nucleus are identical. Then

$$|\langle S_{ij}(E)\rangle| = \exp(-\frac{\pi\Gamma}{nD})\delta_{ij} \tag{9'}$$

and, in accordance with (5),

$$\bar{t}_{ij} = \bar{t} = \frac{\hbar}{\Gamma}\frac{x}{1 - e^{-x}}, \quad x = \frac{2\pi\Gamma}{nD}. \tag{11}$$

Thus, when passing to multichannel reactions, the average distance between the neighbouring levels D is replaced by nD. Let us emphasize that the above relations are not connected with concrete hypotheses about the statistical distribution of resonance parameters, and they are the consequences of S-matrix unitarity. In distinction to the average time delay, the behaviour of correlation function $\phi(\epsilon)$ characterizing the time development of reaction is already sensitive to statistical hypotheses. Let us assume that all levels have the same widths Γ and their energies are distributed according to the Poisson law with density $\rho = 1/D$. It is easy to show that when points y_k are distributed in an interval $(y_1 y_2)$ according to the Poisson law, for any continuous function the formula[1]

$$\langle \prod_k a(y_k)\rangle = \exp(\frac{1}{D}\int_{y_1}^{y_2}(a(y) - 1)dy) \tag{12}$$

is valid, where D is the average distance between the neighbouring points. Averaging equality (6') over energy interval $\Delta E \gg \Gamma, D$ and using (12), we obtain (see[1,2,4])

$$\langle\det\hat{S}(E)\hat{S}^+(E-\epsilon)\rangle = \exp(i\frac{2\pi\epsilon}{D(1 - i\epsilon/\Gamma)}). \tag{13}$$

In the one-channel case from (3), (9) and (13) one gets the simple expression for correlation function:

$$\phi(\epsilon) = (e^{-\frac{x\epsilon}{\epsilon + i\Gamma}} - e^{-x})(1 - e^{-x})^{-1}, \quad x = \frac{2\pi\Gamma}{D}. \tag{14}$$

In the model of n equivalent channels[4,5]

$$\langle\hat{S}(E)\hat{S}^+(E-\epsilon)\rangle_{ij} = \exp(i\frac{2\pi\epsilon}{nD(1 - i\epsilon/\Gamma)})\delta_{ij}. \tag{15}$$

It is easy to see that if all processes at the stage of compound nucleus have the same time development, the correlation function is described by one-channel formula (14) with $x = 2\pi\Gamma/nD$ according to (3), (9') and (15). The probability distribution of time delay conforming to the correlation function (14) can be calculated in the explicit form by formula (2). Expanding the numerator of the function $\phi(\epsilon)$ in a series and using the residue theorem, we find

$$P(x,\tau) = e^{-\tau}(e^{x}-1)^{-1}\sum_{m=0}^{\infty}\frac{x^{m+1}\tau^{m}}{m!(m+1)!}, \qquad \tau > 0; \tag{16}$$

$$P(x,\tau) = 0, \qquad \tau < 0;$$

Here

$$\tau = \frac{\Gamma t}{\hbar}, \qquad \int_0^{\infty} P(x,\tau)\,d\tau = 1. \tag{17}$$

From (16) one gets automatically the formula (11) for the average time delay. A relative fluctuation of the time delay can be determined in a similar manner. Indeed,

$$\overline{\tau^2} = \int_0^{\infty} P(x,\tau)\,\tau^2 d\tau = (x^2+2x)(1-e^{-x})^{-1}. \tag{18}$$

Hence

$$\eta_\tau(x) = \left(\frac{\overline{\tau^2}-(\bar\tau)^2}{(\bar\tau)^2}\right)^{1/2} = \left[\frac{2}{x}(1-e^{-x})-e^{-x}\right]^{1/2} \tag{19}$$

and $\frac{d\eta_\tau(x)}{dx} < 0$, $\eta_\tau(x) \le \eta_\tau(0) = 1$ (see [5] as well). Equation (16) can be rewritten in the form

$$P(x,\tau) = e^{-\tau}(e^{x}-1)^{-1}\sqrt{\frac{x}{\tau}}\,I_1(2\sqrt{x\tau}), \tag{20}$$

where $iI_1(z) = J_1(iz)$ is a first order Bessel function of imaginary argument. When $\sqrt{x\tau} \ll 1$, then

$$P(x,\tau) = e^{-\tau}\frac{x}{e^{x}-1} \tag{21}$$

and for $\sqrt{x\tau} \gg 1$, taking into account the asymptotic behaviour of the Bessel functions, we have

$$P(x,\tau) = \frac{1}{\sqrt{4\pi}(x\tau)^{1/2}}\sqrt{\frac{x}{\tau}}\,e^{-(\sqrt{x}-\sqrt{\tau})^2}. \tag{22}$$

In accordance with eq. (21), for $x \ll 1$ and $t \ll \frac{\hbar}{\Gamma}\frac{1}{x}$ there takes place the exponential distribution of time delay. In this case, according to eqs. (11) and (19), $\bar t = \frac{\hbar}{\Gamma}$, $\eta_\tau = 1$. Note that

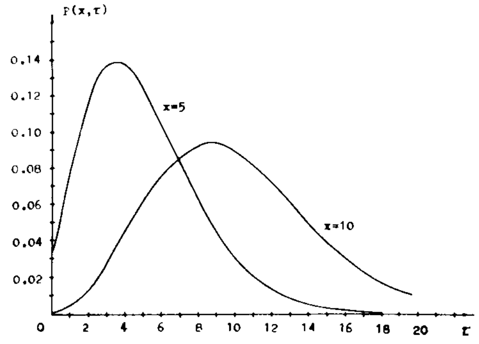

Fig.1. Probability distribution of time delay at the stage of compound nucleus calculated by formula (20) ($x = \frac{2\pi\Gamma}{nD}$, $\tau = \frac{\Gamma t}{\hbar}$).

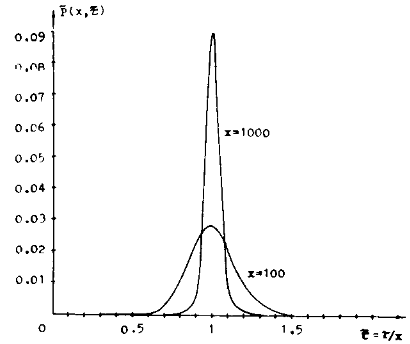

Fig.2. Plot of the probability distribution of time delay at a very large density of resonances levels ($x = \frac{2\pi\Gamma}{nD}$, $\tilde\tau = \frac{\tau}{x}$).

in the discussed case the delay probability for times $t \lesssim \hbar/\Gamma x$ is extremely small and this interval can be generally excluded from the consideration. Quite another situation appears under the condition $x = \frac{2\pi\Gamma}{nD} \gg 1$. In this case the probability of delay fo for times $t \lesssim \hbar/\Gamma x$ is very small ($\sim e^{-x}$), and for $t \gg \hbar/\Gamma x$ the asymptotic formula (22) is valid. It is easy to see that in this case the average time delay $\bar t = \frac{2\pi\hbar}{nD}$ ($\bar\tau = x$) and the relative fluctuatuin $\eta_\tau = \sqrt{\frac{2}{x}} \ll 1$. It is evident that, in accordance with the results [1-4], the function of "the relative time delay" distribution $\tilde\tau = \frac{\tau}{x}$ ($\tilde P(x,\tau) = xP(x,\tau)$) takes the form of a very narrow (δ-shaped) peak at $x \to \infty$. It should be emphasized that in this limited situation the S-matrix elements tend to zero. The plots of the function of the time delay distribution are presented in figs.1 and 2.

As was noted above, the formulae (10), (11) for average time delay are independent of the character of statistical distribution, and they are also valid at the deviation from the Poisson law (in particular, at the equidistant location of levels). From the analysis performed in paper [12] one gets the relation

$$\eta_\tau = \gamma\sqrt{\frac{2}{x}} \quad \text{as values } x \gg 1,$$

where $\gamma = \left(\frac{\overline{\Delta^2 D^2}}{\bar{D}^2}\right)^{1/2}$ is a relative fluctuation of the distance between levels (the value $\gamma = 1$ corresponds to the Poisson distribution). For most relatistic models $\gamma \lesssim 1$. In particular, in the case of the Wigner distribution of levels $\gamma = 0.52$.* Thus, the general conclusion of papers /2-5/ is confirmed that the relative fluctuation of time delay at the stage of compound nucleus is small at a very large density of overlapping levels.

It should be emphasized that our consideration, as in papers /3-5/, is valid under the condition that the number of independent superpositions of compound nucleus levels with amplitudes different from zero (the number of "incoming states") coincides with the number of channels. This regime corresponds to a minimal correlation between levels which appears as a consequence of unitarity and leads to geometric effective cross sections (after summation over partial waves). However, the situations are in principle possible when the number of "incoming states" (or "doorways"), which we denote by m, is smaller than the number of channels n. This question has been already discussed in paper /5/. According to ref. /5/ in the general case the formula (1) for the life-time of compound nucleus is replaced by the relation at $\Gamma \gg mD/2\pi$

$$t_0 = \frac{2\pi\hbar}{mD} \geq \frac{2\pi\hbar}{nD} \gg \frac{\hbar}{\Gamma}, \quad 1 \leq m \leq n \, . \qquad (23)$$

In particular, at $m = 1$ (Newtonian case /13/) the time development of nuclear reactions is the same as in the one-channel case ($t_0 = \frac{2\pi\hbar}{D}$) independently of the number of channels. First Baz' has noticed this in paper /14/.

An interesting conclusion can be draw from (23): in spite of ordinary treatments the duration of nuclear reactions is able not only to decrease with rising the excitation energy of compound nucleus but also it is able to increase. Indeed, it is well known that the distance between levels decreases fastly

* At the equidistant location of levels the time delay distribution for a wave packet with a wide energy spectrum, which has been found in paper /6/ for the one-channel case, has the form in our denotations:

$$P(x,\tau) = (1 - e^{-x}) \sum_{k=1}^{\infty} \delta(\tau - kx)\, e^{-x(k-1)} .$$

It is easy to show, that then

$$\bar{\tau} = x(1 - e^{-x})^{-1}, \quad \eta_\tau = e^{-x/2} .$$

with increasing excitation energy. In principle it is possible that the quantity mD decreases also. Although in other conditions the time of reaction evolution can decrease with increasing excitation energy but much slower than it is customary to assume. This behaviour is confirmed by the results of recent experiments on the determination of the duration of fission reactions at the interaction of ^3He and ^{238}U nuclei /15/. Apparently, a further experimental investigation of the duration of nuclear reactions at excitation energies about several tens MeV will be of importance for understanding the structure of high-excited nuclei.

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Любошиц В.Л. Вероятностное распределение времени задержки волнового пакета при сильном перекрывании резонансных уровней — E4-82-494

На основе теории перекрывающихся резонансов исследуется временной ход ядерных реакций при большой плотности уровней. В рамках модели n эквивалентных каналов получено аналитическое выражение для функции распределения вероятности времени задержки пакета при образовании составного ядра /формула /20//. Показано, что при сильном перекрывании резонансных уровней относительная флуктуация времени задержки на стадии составного ядра мала. Обсуждается возможность увеличения длительности ядерных реакций с ростом энергии возбуждения.

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Lyuboshitz V.L. The Probability Distribution of the Delay Time of a Wave Packet in Strong Overlap of Resonance Levels — E4-82-494

The time development of nuclear reactions at a large density of levels is investigated using the theory of overlapping resonances. The analytical expression for the function describing the time delay probability distribution of a wave packet is obtained in the framework of the model of n equivalent channels (formula (20)). It is shown that a relative fluctuation of the time delay at the stage of the compound nucleus is small. The possibility is discussed of increasing the duration of nuclear reactions with rising excitation energy.

The investigationh has been performed at the Laboratory of High Energies, JINR.

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