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NEGATIVE PARITY COLLECTIVE STATES IN ACTINIDES IN A PHENOMENOLOGICAL APPROACH

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I - INTRODUCTION

The negative parity states in deformed nuclei and in particular in actinides provide an interesting field of studies both for theorists and for experimentalists. In actinides the collective nature of low-lying negative-parity states was predicted in the calculations made within the framework of R.P.A. [1-4]. This prediction is in agreement with the data obtained by Coulomb excitation. Large moments of inertia and small energy intervals between the collective band-heads in actinides accentuate the role of Coriolis mixing of such states. The latter aligns the vibrational angular momentum with the axis of nuclear rotation.

The properties of aligned states differ essentially from those which are known in the region of adiabatically slow rotation (i.e. in the region of small I) [5]. The effects of Coriolis coupling may be conveniently studied in terms of alignment of the angular momentum (j_x) as suggested by Bohr and Mottelson [6]. To estimate j_x one has to know the dependence of the total angular momentum of states (I) on the frequency of rotation (ω). If one knows such a relation for the distorted band ($I_{eff}(\omega)$) and for the rotating core ($I_{core}(\omega)$), one finds j_x from the relation :

$$j_x(\omega) = I_{eff}(\omega) - I_{core}(\omega) \qquad (1)$$

In applying eq.(1) one uses the energies of the states in the distorted band to find $I_{eff}(\omega)$. In the simplest approximation one writes

$$\omega_{eff}(I) = \frac{1}{2}\,(E(I+1) - E(I-1)) \qquad (2)$$

to calculate the function $\omega_{eff}(I)$ for discrete values of I and then to find $I_{eff}(\omega)$ by means of interpolation.

There is no direct way to find such a relation for the $\omega_{core}(I)$ (or $I_{core}(\omega)$) function. Different empirical procedures were suggested to calculate these functions using the energies of the ground-band states [7,8,9]. The possible Coriolis distortions of the ground-state band even at low spins make such an approach rather ambiguous, which is a serious drawback of the theory. Here we present a new procedure for calculating the aligned angular momentum. Our derivation of j_x is model-dependent, but, within the model, definite relations are obtained between the parameters of the intrinsic Hamiltonian and the aligned angular momentum. We shall first present the model, which is a straightforward generalization of that given in ref.[10]. In the present formulation we introduce the attenuation parameters of the Coriolis interaction between the octupole bands which is necessary to describe the spectrum of the negative parity states. Then, we shall describe the procedure used to calculate the inertia parameters of the rotating core and simultaneously the aligned angular momentum. Further, a systematic application of the scheme to some nuclei of the actinide region is presented. The results are summarized in the last section of the paper where some tentative conclusions are drawn concerning the influence of the quasiparticle degrees of freedom and the effects of anharmonicity of octupole vibrations.

II - THE MODEL

The following study of the structure of collective negative-parity states influenced by a strong Coriolis force is based on the Hamiltonian [10] :

$$\hat{H} = \hat{H}_R(\hat{\vec{R}}^2) + \sum_{K=-3}^{3} \omega_{|K|} b_K^+ b_K \qquad (3)$$

where the first term represents the rotational energy of the core and the second term is the intrinsic part of the Hamiltonian including the phonon operators $b_K^+, (b_k)$ which create (destroy) the negative-parity states of predominantly octupole-vibrational nature. The phonon operators are labelled by the quantum number K which is the projection of the vibrational angular momentum on the symmetry axis of the nucleus in a corresponding one-phonon state.

In the following, only the one-phonon states will be considered. The core states are supposed to belong to one rotational band. Then the eigenfunction of the Hamiltonian (3) may be written as a superposition of adiabatic wave-functions [11] :

$$\psi(I,M) = \sqrt{\frac{2I+1}{16\pi^2}} \sum_{K=0}^{3} \psi_K \frac{1}{\sqrt{(1+\delta_{K,0})}} \left(\mathcal{D}_{MK}(\Omega) b_K^+ - (-1)^I \mathcal{D}^I_{M-K}(\Omega) b^+_{-K}\right)|0\rangle \qquad (4)$$

and represented by the column vectors as follows :

$$\Psi(I,M) = \begin{pmatrix} \psi_0^- \\ \psi_1^- \\ \psi_2^- \\ \psi_3^- \end{pmatrix} \qquad I = 1, 3, 5\ldots$$

$$\Psi(I,M) = \begin{pmatrix} \psi_1^+ \\ \psi_2^+ \\ \psi_3^+ \end{pmatrix} \qquad I = 0, 2, 4\ldots \qquad (5)$$

The transformation $\psi \to \hat{R}_x(\pi)\psi$ leaves Ψ invariant when I is even and changes the sign in the other case (notice, that b_K^+ becomes $-b_K^+$ after such a transformation). For this reason the even-I and odd-I states may be called correspondingly as positive ($\sigma = +1$) and negative ($\sigma = -1$) signature states[12].

The first term in the Hamiltonian (3) depends on the rotational angular momentum of the core : $\hat{\vec{R}} = \hat{\vec{I}} - \hat{\vec{j}}$ ($\hat{\vec{I}}$ and $\hat{\vec{j}}$ being respectively the total angular momentum and the angular momentum of phonon excitations). If the properties of the core change slowly with $\vec{R}$ the Coriolis mixing may be taken into account approximately as follows :

$$\hat{H}_R(\hat{\vec{R}}^2) \approx H_R(I(I+1)) - \frac{d\,H_R(I(I+1))}{d(I(I+1))} \cdot (2\,\hat{\vec{I}}.\hat{\vec{j}} - \hat{\vec{j}}^2) \qquad (6)$$

The first term of the right-hand side of (6) is actually an operator $\hat{H}(\hat{\vec{I}}^2)$. Looking for the wave-functions with definite value of the total angular momentum, we may replace it by its eigenvalue $H_R(I(I+1))$. The same goes for the factor before the bracket in the second term. In the following, the $\hat{\vec{j}}^2$ operator will be incorporated in the intrinsic part of the Hamiltonian.

Using the known expressions for the matrix elements of the $\mathcal{D}$-functions, the matrix elements of the operator appearing in eq.(6) may be conveniently written as follows

$$(\hat{\vec{I}}.\hat{\vec{j}})^I_{K,K+1} = \sqrt{I(I+1)} \cdot (\hat{j}_x)_{K,K+1} \cdot \chi(I,K) \qquad (7)$$

where

$$\hat{j}_x = 1/2\,(\hat{j}_+ + \hat{j}_-) \qquad (8)$$

is the x-projection of the vibrational angular momentum and $\chi(I,K)$ is equal to :

$$\chi(I,0) = 1/2\,(1-(-1)^I),$$
$$\chi(I,1) = \left[1 - \frac{2}{I(I+1)}\right]^{1/2},$$
$$\chi(I,2) = \left[1 - \frac{6}{I(I+1)}\right]^{1/2} \qquad (9)$$

In the high-spin approximation, when $I \gg 1$, $\chi(I,K) \approx 1$ and eq.(6) becomes :

$$H_R(\vec{R}^2) = H_R(I(I+1)) - \omega_{rot} \cdot \hat{j}_x \qquad (10)$$

where ω_{rot} is the rotational frequency of the core defined as

$$\omega_{rot} = \frac{d\,E_{core}(\tilde{I})}{d\tilde{I}} \qquad (11)$$

with :

$$\tilde{I} = \sqrt{I(I+1)}, \qquad E_{core}(\tilde{I}) = H_{rot}\,(I(I+1)) \qquad (12)$$

The matrix elements of $(\hat{j}_x)_{K,K'}$ between the one-phonon states are given by the expressions :

$$(\hat{j}_x)_{01} = -\sqrt{6}\;\eta_o \quad (\sigma = -1)$$
$$(\hat{j}_x)_{12} = -\frac{\sqrt{5}}{2}\;\eta_1 \quad (\sigma = \pm 1) \qquad (13)$$
$$(\hat{j}_x)_{23} = -\frac{\sqrt{3}}{2}\;\eta_2 \quad (\sigma = \pm 1)$$

The numerical factors in (13) correspond to the pure octupole states with the intrinsic angular momentum equal to 3 and with the phase convention suggested in [13]. The parameters η_K describe the attenuation of the Coriolis coupling between the one-phonon states.

After the described approximations the Hamiltonian becomes :

$$\hat{H} = E_{core}(\tilde{I}) - \omega_{rot}(\tilde{I}) \cdot \hat{j}_x + \sum_{K \geq 0} \omega_K\, b_K^+ b_K \qquad (14)$$

A Hamiltonian similar to that in eq.(14) was given in ref.(13), in the high-spin approximation of the Coriolis coupling scheme. The operator in (14) has the physical meaning of the Hamiltonian operator transformed to the frame of reference rotating with the angular velocity ω_{rot} and has the same structure as the Routhian operator in the nuclear cranking model [12]. The eigenstates of (14) have definite symmetry properties with respect to the rotation of the intrinsic coordinate axis over the angle π around the x-axis. Thus the signature quantum number $\sigma = \pm 1$ may be attributed to intrinsic states as was already stated before.

Notice that the Coriolis coupling between the states with K;K' = 0;1 in the operator (14) is not affected by the high-spin approximation in which $\chi(I,K) = 1$. This makes this approach applicable to the analysis of the states in $K = 0^-$ band even at low spins, because the admixtures of phonon states with K = 2,3 are small in this case. It is easy to modify the scheme so that the first order effects of the Coriolis coupling of the band $K \neq 0$ with the other bands be treated exactly by a change in the relation between $\tilde{I}$ and I in eq.(12).

III - SPECTRUM OF THE NEGATIVE-PARITY STATES. INERTIA PARAMETERS OF THE CORE

Given the Hamiltonian (14), one calculates the energies of negative-parity states and the corresponding wave-functions, solving the Schrödinger equation :

$$(H^\sigma_{K,K'} - \varepsilon^\sigma_\nu\, \delta_{K,K'})\, \psi^\sigma_{\nu,K'} = 0 \qquad (15)$$

where

$$H_{K,K'} = \omega_K\, \delta_{K,K'} - \omega_{rot}(\tilde{I})(\hat{j}_x)_{K,K'} \qquad (16)$$

Here $\sigma = \pm 1$ is the signature and ν is the label of the distorted band. The formulae for ε^σ_ν and $\psi^\sigma_{\nu K}$ are given in Appendix A. The symmetry properties of the wave-functions (4) lead to the relation :

$$(-1)^I\, \sigma = 1 \qquad (17)$$

The spectrum of negative-parity states splits into 4-families of states with negative signature (odd-I states) and 3-families of states with positive signature (even-I states). The energy of the states is given by :

$$E^\sigma_\nu(I) = E_{core}(\tilde{I}) + \varepsilon^\sigma_\nu(\omega_{rot}(\tilde{I})) =$$
$$= E_{core}(\tilde{I}) - \omega_{rot}(\tilde{I}) \langle \psi^\sigma_\nu | \hat{j}_x | \psi^\sigma_\nu \rangle$$
$$+ \langle \psi^\sigma_\nu | \sum_{K=-3}^{3} \omega_{|K|}\, b_K^+ b_K | \psi^\sigma_\nu \rangle \qquad (18)$$

The right-hand side of eq.(18) contains the function $E_{core}(\tilde{I})$ and its derivative with respect to $dE/d\tilde{I}$ and may be considered as a differential equation which allows one to find $E_{core}(\tilde{I})$ if the left-hand side of (18) is known from the experiment. It is more convenient, however, to use another relation for the definition of the core on the basis of the experimental data. From the expression for the energy E^{σ}_{ν} (I) in eq. (18), it follows :

$$\frac{d\,E^{\sigma}_{\nu}(\tilde{I})}{d\,\tilde{I}} = \omega_{eff}(\tilde{I})' = \omega_{rot}(\tilde{I}) - (j_x)^{\sigma}_{\nu}\ (\omega_{rot}(\tilde{I})).\ \frac{d\,\omega_{rot}(\tilde{I})}{d\tilde{I}} \qquad (19)$$

In writing eq.(19) we use the definition for the aligned angular momentum of octupole vibrations

$$(j_x)^{\sigma}_{\nu} = \langle \Psi^{\sigma}_{\nu} | \hat{j}_x | \Psi^{\sigma}_{\nu} \rangle \qquad (20)$$

Eq. (19) allows us to determine the function $\omega_{rot}(\tilde{I})$ using the experimental data on the energies of states in rotational bands. The explicit relation between j_x and ω_{rot} depends on the parameters of the model which we discuss in more detail further on. Being the first order differential equation for $\omega_{rot}(\tilde{I})$, eq.(19) defines $\omega_{rot}(\tilde{I})$ if an initial condition for this function at a certain value of $\tilde{I}$ (at $\tilde{I}_o$) is formulated. The proper choice of initial condition for $\omega_{rot}(\tilde{I})$ gives simultaneously the solution to eq.(18) :

$$E_{core}(\tilde{I}) = E_o + \int_{\tilde{I}_o}^{\tilde{I}} \omega_{rot}(\tilde{I}')\, d\tilde{I}' \qquad (21)$$

In practice, the calculations were done for several values of ω_{rot} at the lowest value of I and the proper choice was made, taking into account the level spacings of the first states of the 0^- band.

The solution to eq.(19) defines together with $\omega_{rot}(\tilde{I})$ the aligned angular momentum j_x as a function of spin. Graphically j_x is shown in fig. 1, equal to the difference of I values at the points corresponding to the same value of ω_{rot} : one of the points belongs to the line ω_{eff} at a given value of I, while the other is given by the intersection of a horizontal line and the tangent to the curve ω_{core} at the same value of I. This definition of j_x is consistent with eq.(1) if

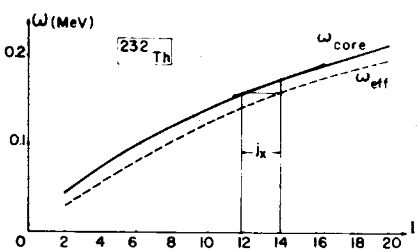

Fig.1 – Frequency of rotation of the core (ω_{core}) from eq.(19) as a function of the spin (solid line) in ^{232}Th. The dashed line represents $\omega_{eff} = 1/2\ [E_{0^-}\ (I+1) - E_{0^-}\ (I-1)]$. The graphical expression of the aligned angular momentum j_x is shown.

$\omega_{core}(I)$ may be approximated by a linear function of I in the interval from I - j_x to I. The examples given below will show that this condition is satisfied.

Finally, one may define the moment of inertia of the core as

$$J_{core} = \tilde{I}/\omega_{rot}(\tilde{I}) \qquad (22)$$

and find its dependence on ω_{rot} or on $\tilde{I}$.

When $\omega_{eff}/\tilde{I} = J^{-1}_{eff}$ and $j_x/\omega_{rot} = \delta J$ do not depend on $\tilde{I}$, the solution to eq.(19) becomes :

$$\omega_{rot} = \tilde{I}/J_{core} \qquad (22)$$

with J_{core} satisfying the relation :

$$1/J_{eff} = 1/J_{core}\ (1 - \delta J/J_{core}) \qquad (23)$$

Thus, the linear dependence of j_x on ω corresponds to the spin-independent renormalization of the moment of inertia.

When ω is small, the Coriolis interaction may be treated in the lowest order of perturbation theory leading to :

$$j_x = \frac{12\,\eta_o^2\,\omega_{rot}}{\omega_1 - \omega_o}\ ;\ (\delta J = 12\,\eta_o^2/(\omega_1 - \omega_o)) \qquad (24)$$

in the case of perturbed $K^{\pi} = 0^-$ band (see eqs. (A,5) - (A,7) in Appendix A). From eq.(23) one obtains an estimate for the moment of inertia of the core at small ω :

$$J_{core} = J_{eff}\ 1/2\ (1 + \sqrt{1 - 4\,\delta J/J_{eff}}) \qquad (25)$$

It must be noticed that eq.(25) gives a solution to the differential equation (19) only when the expression under the square root of eq.(25) is positive.

IV - CALCULATIONS FOR THE ACTINIDE NUCLEI

The procedure outlined in Sect. 3 was used to define the core for the negative-parity states in actinide nuclei ^{232}Th, $^{236,\,238}$U. The experimental data [14-18] which we describe here concern the energies in the distorted 0^- band and also in the K = 1^-, 2^- bands whose collective nature is well established. The moment of inertia of the core J_{core} at small ω_{rot} depends on the attenuation coefficient η_o (see eqs.(24), (25)). The value of η_o is fixed by the condition that the dependence $J_{core}\ (\omega^2_{rot})$ is close to linear in the interval of $\omega_{rot}(I)$ corresponding to the few first points in the angular momentum I. The coefficient η_1 and the energy of the band-heads are obtained using the information on

Table 1

nucleus	ω_K				η_1		η_2	$\omega_{in}(I=2)$
	0^-	1^-	2^-	3^-	η_o	η_1		
^{232}Th	0.708	1.038	1.150	1.500	0.81	0.9	1	0.04545
^{236}U	0.687	0.955	1.117	1.187	0.64	0.8	1	0.03782
^{238}U	0.675	0.920	1.060	1.500	0.64	0.8	1	0.03424

The parameters used in the calculations :
phonon excitation energies , ω_K ;
attenuation coefficents , η_1 ;
initial value of ω_{rot} for the integration of eq.(1'') , $\omega_{in}(I=2)$.

the energy intervals in the bands with $K \neq 0$. The parameters used in the calculations are listed in Table 1. (The left-hand side of eq.(19) is approximated by eq.(2)).

Figure 1 shows the functions $\omega_{eff}(\tilde{I})$ and $\omega_{core}(\tilde{I})$ for the nucleus ^{232}Th. The dependence $\omega_{core}(\tilde{I})$ on $\tilde{I}$ is also smooth in the other considered cases. The calculated and experimental spectra of negative-parity states are compared in figures 2, 3, 4. Notice, that the quality of reproduction of the spectrum of 0^- bands is determined only by the accuracy of solution of eq.(19). The estimations of the energy levels of the states in $K^\pi = 1^-$ and 2^- bands depend strongly on the parameters η_o and η_1. As is seen, the experimental spectra are reproduced quite well within the model.

In the considered nuclei the experimental data on the collective states in the band $K^\pi = 3^-$ are absent. The coupling of this band with the other octupole bands is not important for the estimations of the energies in the 0^- band. Indeed, the states $I = 1^-, 2^-$ are free from admixtures of states with $K = 3^-$. In the region of low spins ω_{core} is small and the admixtures of states with $K^\pi = 3^-$ are small because they come from the high orders of perturbation expansion in ω_{rot}. At large spins the Coriolis force tends to align the vibrational angular momentum perpendicular to its direction in the state $K^\pi = 3^-$, i.e. it modifies the wave function of the aligned state so that it becomes almost orthogonal to the function in the state $K^\pi = 3^-$. Eq.(B.3) in Appendix B determines the asymptotic value of j_x at large ω_{rot} as a function of the parameters η_1. The actual value of η_2 used in calculations is $\eta_2 = 1$. Putting $\eta_2 = 0$ one simulates the situation in which ω_3 is so large that there is no Coriolis coupling with the band $K^\pi = 3^-$. In table 2, the asymptotic values for j_x at large ω_{rot} are shown for different sets of parameters η_1. The table shows that the variations in j_x following the changes in ω_3 are certainly smaller than a few percents. From the above considerations it follows that the function $j_x(\omega)$ depends weakly on ω_3, and the lack of information on the $K^\pi = 3^-$ band should not affect the estimations of the properties of the core in an essential way.

The calculations allow us to compare the moment of inertia of the core J_{core} in eq.(22) with the moment of inertia of the ground-state-band

$$J_{gr} = \tilde{I}/\omega_{gr}, \text{ with } \omega_{gr} = 1/2\,(E_{gr}(I+1) - E_{gr}(I-1)) \tag{26}$$

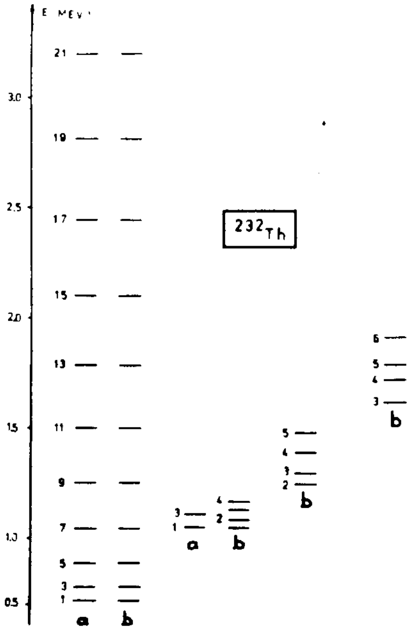

Fig. 2 - Experimental (a) and theoretical (b) spectra of the negative parity collective states in ^{232}Th.

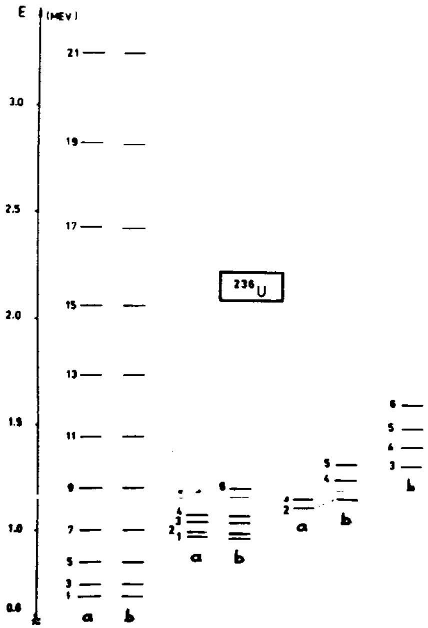

Fig.3. The same as in fig.2 for ^{236}U.

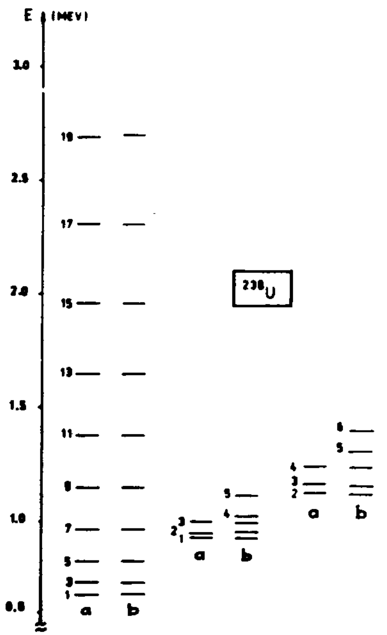

Fig.4. The same as in fig.2 for ^{238}U.

Table 2

	$n_2 = 1$	$n_2 = 0$
^{232}Th	2.562	2.442
$236\text{-}238_U$	2.188	2.014

Asymptotic values of j_x for the case with full Coriolis interaction with $K^\pi = 3^-$ band ($n_2 = 1$) and with no interaction ($n_2 = 0$).

The comparison of different inertia parameters is given in fig. 5. In addition to J_{core} and J_{gr} the effective moment of inertia of the 0^- band is shown here : $J^{0-}_{eff} = I/\omega^{0-}_{eff}$.

The moment of inertia of the core appears to be significantly smaller than $J^{0-}_{eff}(\omega_{rot})$. Having in mind that $\omega_{eff}(I) \approx \omega_{core}(I - j_x)$, we notice that, at large I, where j_x reaches its limiting value, the curves J^{0-}_{eff} and J^{0-}_{core} become almost parallel.

The starting value of j_x at low spins is zero but, as is shown in eq.(23), the Coriolis coupling gives an essential rise to the effective moment of inertia here. Thus $J^{0-}_{eff}(\omega_{rot})$ presents a more or less pronounced minimum at some finite value of ω, which reflects directly the alignment of the vibrational angular momentum.

The difference between J^{0-}_{core} and J_{gr} could be understood as a difference in the polarization effects of the nucleus. For example, rotational coupling effects with other bands, which differ for the ground state band and for the octupole bands, may play a role even at low spin. This difference is rather sensitive to the parameters of the model (we mention that our earlier estimations of it (12), which neglect the attenuation of the Coriolis force, differ from what is obtained here).

The nuclear states may be conveniently characterized by the aligned angular momentum j_x defined formally by eq.(20) and calculated together with $\omega_{rot}(I)$ when solving eq.(19) as was described before. Figs 6a,b,c show the calculated j_x in the nuclei ^{232}Th, ^{236}U, ^{238}U. The attenuation of the Coriolis interaction reduces j_x in comparison with what is expected for pure octupole phonons (see fig.2 in ref. (10)). The dependence of j_x on ω_{rot} at low spin is given by eq.(24), while the high-spin limit follows from eq.(B.3) in Appendix B. As is seen, comparing fig. 6 with the content of Table 2, the high-spin limit is already close for $\omega_{rot} \sim 0.2$ MeV, which corresponds to the highest spins measured in the actinides. The alignment may be clearly seen already in the states with $I \lesssim 9\hbar$.

In fig.6 there are also given the values of j_x^{BF} extracted from the experimental data on the basis of Bengtsson-Frauendorf procedure (7,8). This procedure defines an aligned angular momentum with respect to a ground reference chosen simply as an extrapolation of the linear part of $J^{eff}_{gr}(\omega^2)$. Thus, besides the alignment of the vibrational angular momentum, j_x calculated in this way may contain the alignment of angular momenta associated with other degrees of freedom.

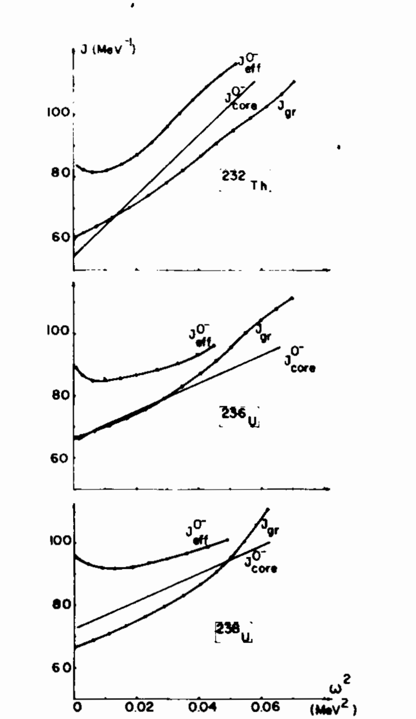

Fig. 5 - Inertia parameters $\mathcal{J}^{0-}_{core}$, $\mathcal{J}^{0-}_{eff}$ and $\mathcal{J}_{gr}$ for ^{232}Th , ^{236}U , ^{238}U .

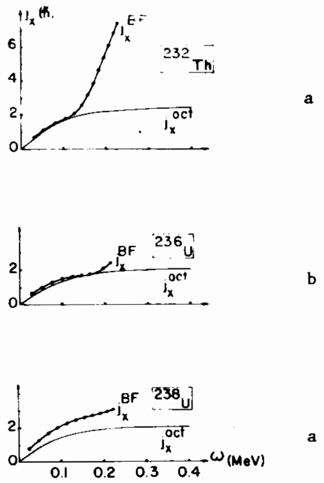

Fig.6 - Vibrational aligned angular momentum in the 0^- bands (j_x^{oct}) for ^{232}Th, ^{236}U and ^{238}U calculated within the present model and compared with the aligned momentum obtained by the Bengtsson-Frauendorf procedure (j_x^{BF}). The parameters used for j_x^{BF} are respectively:

The comparison of j_x obtained by our model and j_x^{BF} allows us to estimate the importance of the degrees of freedom which are not explicity treated within the model.

At high spins, a more or less pronounced rise of j_x^{BF} as compared to j_x^{oct} takes place, indicating an individual alignment of quasi-particle angular momenta. The different behaviour in the three nuclei may be understood if one takes as a guide the microscopic structure of the $K^\pi = 0^-$ states as described in ref.[19], and considers the experimental evidence of alignment effects in even and odd actinides [15,18,20,21].

In this region of nuclei it is well known that both i13/2 protons and j15/2 neutrons are close to the Fermi surface and the irregularities observed in the yrast sequences have been attributed to alignment effects and discussed in relation with the strength of interaction with the crossing s-bands, as predicted by cranked HFB [22] and cranked shell-model calculations [23].

In order to understand the different behaviour of j_x^{BF} at large spins, the following points should be considered :

i) The two-quasiparticle components of the 0^- octupole phonon involve predominantly j15/2 neutrons and much less i13/2 protons.

ii) The importance of the small i13/2 proton component decreases when one goes from ^{238}U to ^{232}Th.

Thus the alignment of the j15/2 neutrons is blocked in the octupole sequence and the more or less important additional alignment which is observed in some cases may be attributed to the transition to the proton s-configuration. One may notice also that ^{232}Th is expected to be a case where the oscillating interaction strength is near its maximum for both neutron and proton crossing s-bands.

At low spins fig.6 shows that in the two nuclei ^{232}Th and ^{236}U j_x^{oct} and j_x^{BF} are close to each other, while in ^{238}U a noticeable difference is seen. This may be directly seen from the experimental level spacings in the $K = 0^-$ band : in ^{238}U the levels are more compressed than in ^{236}U, while the $\Delta_1 = \omega_1 - \omega_0$ energy differences are the same within a few keV. Such differences in the spectra could thus only be associated with the difference of the moment of inertia of the core ($\mathcal{J}^{0-}_{core}$), but not with the choice of the other parameters. We may tentatively associate the larger value of $\mathcal{J}^{0-}_{core}$ in ^{238}U (and the larger j_x^{BF} as compared to j_x^{oct}) with the coupling of the $K = 0^-$ band with an additional $K = 1^-$ band.

V - THE POSSIBILITY OF STABLE OCTUPOLE DEFORMATION INDUCED BY THE ROTATION.

Consider eq.(18) for the energy of negative parity states. The last two terms in it ($\varepsilon_\nu(\omega_{rot})$) determine the excitation energy of one-phonon states. When ω_{rot} is small, ε_ν is close to the corresponding band-head energy. Due to the alignment, the excitation energy of the lowest one-phonon state decreases with ω_{rot}. At a certain value of rotational frequency ω^0_{rot} ε_ν vanishes. Within the model ω^0_{rot} satisfies the equation :

$$(3\eta_0\eta_2)^2(\omega^0_{rot})^4 - 2\{3\eta_0^2\omega_2\omega_3 + 5/4\eta_1^2\omega_3\omega_0 + 3/4\eta_2^2\omega_1\omega_0\}(\omega^0_{rot})^2 + \omega_0\omega_1\omega_2\omega_3 = 0 \qquad (27)$$

At $\omega_{rot} > \omega^0_{rot}$ the excitation energy as defined in this paper becomes negative, which means that the yrast sequence is no more a vacuum state with respect to the phonons of octupole vibrations. Of course the harmonic approximation in the intrinsic hamiltonian for phonon becomes invalid when ε_ν becomes small. But if anharmonicity is small at $\omega_{rot} = 0$, eq.(27) gives the estimations for the frequency of rotation at which the unharmonicity becomes important.

The dependence of ε_ν on ω_{rot} found from the calculations is shown in fig.7. As is seen, the critical frequency ω^0_{rot} appears to be rather large ($\omega^{(0)}_{rot} \sim 0.3$ to 0.4 MeV, $I^{(0)} \sim 30$ to $40\ \hbar$). The model is too schematic to put much importance to the quantitative part of the present discussion. However, it is of interest to follow the consequences of a rotationally induced instability of the vacuum state with respect to the octupole excitation. If the potential energy as a function of octupole deformation parameters has minima corresponding to stable octupole deformation, the yrast sequence at $\omega_{rot} > \omega^0_{rot}$ may represent the rotation in the field with such a deformation. Then the spectrum of high-spin states must contain the pairs of states with opposite parity.

The other possibility is that ω^0_{rot} corresponds to fission. In the considered situation the probability of fission from the aligned state must be larger than from the state of the ground band.

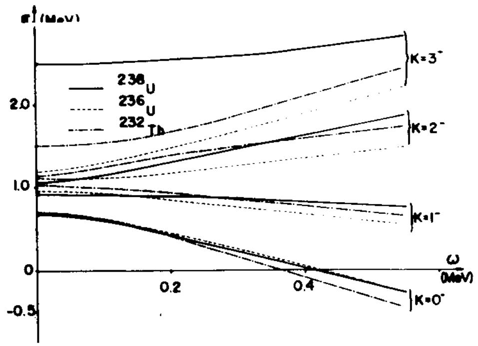

Fig. 7 – Excitation energy of one-phonon negative-parity states as function of ω_{rot} for ^{232}Th, ^{236}U, ^{238}U.

VI – CONCLUSION

In this paper we suggest a method to determine the aligned angular momentum associated with the Coriolis interaction between adiabatic rotational bands. Together with j_x, the inertia properties of the rotating core are determined using the data on the spectrum of distorted bands. Our estimations of j_x depend on the assumptions on the intrinsic Hamiltonian and on the strength of Coriolis interaction. The

calculations which are presented in the paper are made within a very simple phenomenological model where the negative-parity states are treated as one-phonon excitations of a rotating core and the parameters of the intrinsic Hamiltonian are fixed in an empirical way.

The moment of inertia of the core calculated in our way shows sometimes significant differences with the moment of inertia determined from the energies of the ground band states. Such differences are discussed in terms of different polarization effects in the nucleus.

The unharmonicity of phonon excitations may be important at large spins as follows from the discussion in Sect. 5.

Of course, the modifications and the generalization of the present approach with a more fundamental treatment of the nuclear dynamics is possible. In particular, the method may be used to determine the inertia parameters of the core in odd nuclei.

APPENDIX A

From eq.(15), one obtains the following expressions for the eigenvalues $\epsilon^\sigma_\nu = \varepsilon^\sigma_\nu - \omega_o$ and the components of the wave-functions $\tilde{\psi}^\sigma_\nu$ (K) :

$(\Delta_K = \omega_K - \omega_o$ with $K = 1, 2, 3)$

σ = + 1 (even I states)

$$\sum_{n=o}^{3} a_n (\epsilon^+_\nu)^n = 0$$

$$\begin{aligned} a_3 &= 1 \\ a_2 &= -\Delta_1 - \Delta_2 - \Delta_3 \\ a_1 &= \Delta_1 \Delta_2 + \Delta_2 \Delta_3 + \Delta_3 \Delta_1 - \omega^2_{rot} (\tfrac{5}{2}\eta_1^2 + \tfrac{3}{2}\eta_2^2) \\ a_0 &= -\Delta_1 \Delta_2 \Delta_3 + \omega^2_{rot} (\tfrac{5}{2}\eta_1^2 \Delta_3 + \tfrac{3}{2}\eta_2^2 \Delta_1) \end{aligned} \tag{A,1}$$

$$\tilde{\psi}^+_\nu (K) = \frac{\phi^+_\nu(K)}{\sqrt{\sum_{K=1}^{3} (\phi^+_\nu(K))^2}} \tag{A,2}$$

where :

$$\begin{aligned} \phi^+_\nu (1) &= (\Delta_2 - \epsilon^+_\nu)(\Delta_3 - \epsilon^+_\nu) - \tfrac{3}{2}\eta_2^2 \omega^2_{rot} \\ \phi^+_\nu (2) &= -\sqrt{\tfrac{5}{2}}\, \eta_1\, \omega_{rot} (\Delta_3 - \epsilon^+_\nu) \\ \phi^+_\nu (3) &= \sqrt{\tfrac{15}{2}}\, \eta_1\, \eta_2\, \omega^2_{rot} \end{aligned} \tag{A,3}$$

$\sigma = -1$ (odd I states)

$$\sum_{n=0}^{4} a_n (\epsilon_\nu^-)^n = 0$$

$$\begin{aligned}
a_4 &= 1 \\
a_3 &= -\Delta_1 - \Delta_2 - \Delta_3 \\
a_2 &= \Delta_1 \Delta_2 + \Delta_3 (\Delta_1 + \Delta_2) - \omega_{rot}^2 \left[6\eta_o^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right] \\
a_1 &= -\Delta_1 \Delta_2 \Delta_3 + \omega_{rot}^2 \left[6\eta_o^2 (\Delta_2 + \Delta_3) + \frac{5}{2}\eta_1^2 \Delta_3 + \frac{3}{2}\eta_2^2 \Delta_1\right] \\
a_o &= -6\eta_o^2 \Delta_2 \Delta_3 \omega_{rot}^2 + 9\eta_o^2 \eta_2^2 \omega_{rot}^4
\end{aligned} \tag{A,4}$$

$$\tilde{\Psi}_\nu^- (K) = \frac{\phi_\nu^-(K)}{\sqrt{\sum_{K=0}^{3} (\phi_\nu^-(K))^2}} , \tag{A,5}$$

where :

$$\begin{aligned}
\phi_\nu^- (0) &= (\Delta_1 - \epsilon_\nu^-)(\Delta_2 - \epsilon_\nu^-)(\Delta_3 - \epsilon_\nu^-) - \omega_{rot}^2 \left[\frac{3}{2}\eta_2^2 (\Delta_1 - \epsilon_\nu^-) + \frac{5}{2}\eta_1^2 (\Delta_3 - \epsilon_\nu^-)\right] \\
\phi_\nu^- (1) &= \sqrt{6}\, \eta_o \, \omega_{rot} \left[(\Delta_2 - \epsilon_\nu^-)(\Delta_3 - \epsilon_\nu^-) - \frac{3}{2}\eta_2^2 \omega_{rot}^2\right] \\
\phi_\nu^- (2) &= \sqrt{15}\, \eta_o \eta_1 \omega_{rot}^2 \left[\Delta_3 - \epsilon_\nu^-\right] \\
\phi_\nu^- (3) &= -3\sqrt{\frac{5}{2}}\, \eta_o \eta_1 \eta_2 \omega_{rot}^3
\end{aligned} \tag{A,6}$$

The aligned angular momentum is given by :

$$(j_x)_\nu^\sigma = -2 \left[\sqrt{6}\, \eta_o \tilde{\psi}_\nu^\sigma (0) \tilde{\psi}_\nu^\sigma (1) + \sqrt{\frac{5}{2}}\, \eta_1 \tilde{\psi}_\nu^\sigma (1) \tilde{\psi}_\nu^\sigma (2) + \sqrt{\frac{3}{2}}\, \eta_2 \tilde{\psi}_\nu^\sigma (2) \tilde{\psi}_\nu^\sigma (3)\right] \tag{A,7}$$

where, in the case $\sigma = +1$, one should take $\tilde{\psi}_\nu^+(0) = 0$.

APPENDIX B

Asymptotics of large ω_{rot}

Various tendencies in the changes of nuclear properties with accumulation of spin can be understood considering the limit of high ω_{rot} in the above equations, when the differences in ω_K become small compared to ω_{rot}. Here some asymptotic relations for the aligned negative signature band are presented.

The eigenvalue problem (A,4) becomes

$$\epsilon^4 - \left[6\eta_o^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right] \omega_{rot}^2 \epsilon^2 + 9\, \omega_{rot}^4 \eta_0^2 \eta_2^2 = 0 \tag{B,1}$$

From (B,1) it follows in the case of aligned (lowest) band

$$\epsilon = -\omega_{rot} \left[\frac{\left[6\eta_o^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right]}{2} + \sqrt{\frac{\left[6\eta_o^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right]^2}{4} - 9\eta_o^2 \eta_2^2}\right]^{1/2} \tag{B,2}$$

Then the aligned angular momentum is given by :

$$j_x = -\frac{d\epsilon}{d\omega_{rot}} = \left[\frac{\left[6\eta_0^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right]}{2} + \sqrt{\frac{\left[6\eta_0^2 + \frac{5}{2}\eta_1^2 + \frac{3}{2}\eta_2^2\right]^2}{4} - 9\eta_0^2\eta_2^2}\right]^{1/2} \tag{B,3}$$

The wave-functions of the aligned state have the components

$$\begin{aligned}
\tilde{\Psi} (0) &= N j_x \left[j_x - \left(\frac{3}{2}\eta_2^2 + \frac{5}{2}\eta_1^2\right)\right], \\
\tilde{\Psi} (1) &= -N \sqrt{6}\, \eta_o \left(j_x^2 - \frac{3}{2}\eta_2^2\right), \\
\tilde{\Psi} (2) &= N \sqrt{15}\, \eta_o \eta_1 j_x, \\
\tilde{\Psi} (3) &= -3N \sqrt{\frac{5}{2}}\, \eta_o \eta_1 \eta_2 \qquad \left(\sum_K \tilde{\psi}^2_{(K)} = 1\right)
\end{aligned} \tag{B,4}$$

When $\eta_i = 1$, $j_x = 3$ and $\tilde{\psi}(K) = \sqrt{\frac{2}{1+\delta_{K,\nu}}}\, d^3_{3,K}\left(\frac{\pi}{2}\right)$ in accordance with the results of ref.(12) ($d^\lambda_{\mu\tau}(\phi)$ being the middle part of the Wigner $\mathcal{D}$-function).

REFERENCES

1 - Soloviev V.G., Vogel P., Phys. Lett. 1963, 6, p. 126.

2 - Соловьев В.Г., Фогель П., Корнейчук А.А. Изв. АН СССР, сер.физ., 1964, 28, с.1599.

3 - Faessler A., Plastino A., Z. f. Phys. 1967, 203, p. 333.

4 - Neergard K., Vogel P., Nucl. Phys., 1970, A145, p. 33.

5 - Vogel P., Phys. Lett., 1976, 60B, p. 431.

6 - Bohr A., Mottelson B.R., : Int. Conf. Nucl. Structure, Tokyo, 1977. Jour. Ph. Soc. Japan-Suppl. 1978, V.44, p. 157.

7 - Bengtsson R., Frauendorf S., Nucl. Phys. 1979, A314, p. 29.

8 - Bengtsson R., Frauendorf S., Nucl. Phys. 1979, A327, p. 139.

9 - Funke L., Döring J., Dubers F., Kemnitz P., Will E., Winter G., Kiptilij V.G., Kudojarov M.F., Lemberg I.K., Pasternak A.A., Miskin A.S., Hildingsson L., Johnson A., Lindblad T., Nucl. Phys. 1981, A355, p. 228.

10 - Mikhailov I.N., Briançon Ch., Preprint E4-81-402, JINR, 1981,

Briançon Ch., Mikhailov I.N., in : Proceed. Int. Conf. Nucl. Struct. Bormio (Italy), 1982.

11 - Bohr A., Mottelson B.R., Nuclear structure V.2, ch. 4, W.A. Benjamin Inc. Amsterdam, 1974.

12 - Бриансон Ш., Михайлов И.Н. ЭЧАЯ, 1982, т.13, в.2, с.245.

13 - Варшалович Д.А., Москалев А.Н., Херсонский В.К. "Квантовая теория углового момента". "Наука", М., 1975.

14 - Briançon Ch., Libert J., Thibaud J.P., Walen R.J.,

Journ. Phys. Soc. Japan, Suppl. 1978, V. 44, p. 157.

15 - Briançon Ch., Lefebvre A., Libert J., Thibaud J.P., Walen R.J., Proc. Argonne Conf., 1979, ANL/PHY 79-4, p. 477, and Lefebvre A., Thèse de 3$^{\text{ème}}$ cycle, ORSAY (1980).

16 - Ower M., Elze Th., Idzko J., Stelzer K., Emling H., Fuchs P., Grosse E., Schwalm D., Wollersheim M.J., Kaffrel W., Trautmann N., Proc. Int. Conf. on Nuclear Behaviour at High Angular Momentum, Strasbourg (1980).

17 - Grosse E., Balanda A., Emling H., Folkmann F., Fuchs P. and Piercey R.B., Schwalm D., Simon R., Wollersheim H.J., Evers D., Ower H., Physica Scripta, 1981, 24, p. 337.

18 - Simon R.S., De Vito R., Emling H., Kulessa R., Briançon Ch.,

Lefebvre A., Annual Report of GSI and CSNSM (ORSAY), 1980, and Phys. Lett. 1982, 108B, p. 87.

19 - Иванова С.П., Комов А.Л., Малов Л.А., Соловьев В.Г. Изв. АН СССР, сер.физ., 1975, т.39, с.1286.

20 - Briançon Ch., Lefebvre A., Walen R.J., Kulessa R., De Vito R., Emling H., Grosse E., Schwalm D., Simon R.S., Trautmann N., Sletten G., Elze T., Mutterer M., Spreng W., Theobald J., Moore F., Int. Conf. on nucl. and Atomic Phys. with Heavy Ions, (BUCHAREST), 1981.

21 - Kulessa R., De Vito R., Emling H., Grosse E., Schwalm D., Simon R.A Lefebvre A., Briançon Ch., Walen R.J., Elze Th., Mutterer M., Theobald J., Moore F., Trautmann N., Sletten G., G.S.I. Annual Report 1981, C.S.N.S.M. Report (ORSAY), 1981.

22 - Piercey R.B. et al., Phys. Rev. Lett. 46 (1981) 415.
Faessler A., Proc. Int. Meeting Bormio (1982).

23 - Bengtsson R., Suppl. J. de Phys. C10 (1980) 84.
Frauendorf S., Simon R.S., G.S.I. Preprint 80.25 (1980),
Egido J.L., Ring P., J. of Phys. G8 (1982) 43.
Diebel M., Mosel U., Z. f. Phys. 303 (1981) 131.

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Михайлов И.Н. и др. E4-82-489

Изучение коллективных состояний отрицательной четности в актинидах в рамках феноменологической модели

Предложен метод определения момента инерции и выстроенного углового момента, описывающих спектры вращательных полос, искаженных взаимодействием Кориолиса. Метод основан на численном интегрировании уравнения, в котором экспериментальные значения энергетических интервалов между состояниями полосы выражаются как некоторая функция угловой частоты вращения остова и выстроенного углового момента. Такой подход позволяет обнаружить эффекты поляризации остова. Процедура применена для анализа октупольных 0^- полос в актинидах; обсуждены различные поляризационные эффекты, возникающие при возбуждении октупольных колебаний.

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Mikhailov I.N. et al. E4-82-489

Negative Parity Collective States in Actinides in a Phenomenological Approach

A method is suggested to determine the moment of inertia of the rotating core describing the spectrum of rotational bands distorted by the Coriolis force. The method is based on the numerical integration of an equation in which the experimentally found energy intervals between the states of a rotational band are expressed as function of the rotational angular frequency of the core and of the amount of aligned angular momentum. It allows to detect the polarization effects in the core. The procedure is applied to the octupole 0^- bands in actinides, and the polarization effects originating from the excitation of octupole vibrations are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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