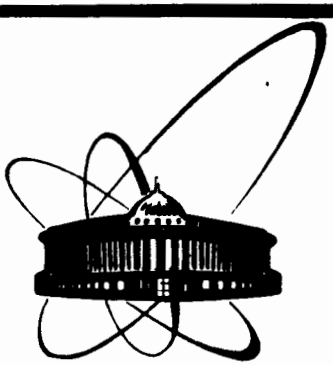

объединенный
институт
ядерных
исследований
дубна

E4-82-468

M.Kh.Khankhasayev

POTENTIAL DESCRIPTION OF THE LOW-ENERGY PION ELASTIC SCATTERING ON ^{4}He

Submitted to "ЯФ"

1982

1. In this paper we consider the $(\pi, {}^4He)$ elastic scattering in the framework of the unitary potential approach recently developed in refs.[1-4]. This approach is based on the so-called method of evolution with respect to coupling constant (CCE) (see review [5]). In the description of the pion-nucleus interaction one must very carefully treat the unitarity condition because the pions can be absorbed in nuclear matter (unlike, e.g., the nucleons).

In ref.[1] an iteration procedure for the calculation of π-nucleus phase shifts in the low-energy limit (quasi-two-particle case) was developed. The basic element of this expansion is the two-body u-matrix of pion interaction with a separate nuclear nucleon. In refs.[2-4] we have generalized this approach to arbitrary energies of the projectile and have carefully studied the first-order approximation in the two-body u-matrix. It was shown that this approximation gave an acceptable description of the low-energy pion-light-nucleus scattering data. The goal of the present paper is the calculation of the second-order corrections. This enables us to establish the convergence region of the considered series. The comparison of the calculated phase shifts, cross sections. etc., with the low-energy data can provide us with some information about the role of the true pion absorption channel.

The paper is organized as follows. In Sec.2 we derive the general expressions for the first two iterations for π-nucleus phase shifts. In Sec.3 the first-order approximation is considered. Unlike to refs.[2-4], where nucleons were considered to be "frozen", here their Fermi motion in the target nucleus was taken into account. Sec. 4 is devoted to the calculation of the real and imaginary parts of the π-nucleus phase shifts in the second-order approximation. In Sec.5 we discuss the off-shell behaviour of the two-body u-matrix. The application of the present approach to the description of the $(\pi, {}^4He)$-elastic scattering is given in Secs.6 and 7. In Sec.8 we have discussed the main results of this paper.

2. In the framework of the CCE-method the Hamiltonian for the system is

$$H = K_\pi + H_A + \lambda U, \quad U = \sum_{i=1}^{A} U^i , \qquad (2.1)$$

where K_π is the pion kinetic-energy operator, H_A is the nuclear Hamiltonian, U^i labels the pion interaction with an i-th nucleon and λ plays the role of πN-coupling constant. The pure nuclear problem with the Hamiltonian

$$h = K_\pi + H_A \tag{2.2}$$

is assumed to be known and the system evolution in the coupling constant λ from $\lambda = 0$ to the realistic value $\lambda = 1$ is considered.

The matrix elements $U_{\mu\nu} \equiv \langle\mu|U|\nu\rangle$ of the potential over the eigenfunctions $|\mu\rangle, |\nu\rangle$, etc., of the Hamiltonian H are the basic quantities in the CCE-method [5]. In the absence of the π-nucleus bound states there is one-to-one correspondence between the eigenfunctions of H and h. Thus, one can represent $U_{\mu\nu}$ as:

$$U_{\mu\nu}(\lambda) \equiv \langle \vec{k}_\mu, \alpha_\mu | V(\lambda) | \vec{k}_\nu, \alpha_\nu \rangle, \tag{2.3}$$

where $V(\lambda)$ is some Hermitian operator and $|\vec{k}_\mu, \alpha_\mu\rangle$, etc., are eigenfunctions of the channel Hamiltonian (2.2). Here $\vec{k}_{\mu,\nu}$ denotes the pion momentum in the π-nucleus c.m.s. (Acm-system) and $\alpha_{\mu,\nu}$ the nuclear states ($\alpha_{\mu,\nu} = 0,1,2,\ldots; \alpha_{\mu,\nu} = 0$ labels the ground state). There is a system [1,5] of exact integral equations for $U_{\mu\nu}$ (or V). In ref. [1] an iteration procedure for solving these equations was developed. The series obtained for $V(\lambda)$ is the expansion in powers of the so-called two-body u-matrix. Each term of this expansion is Hermitian. This provides the unitarity of the scattering matrix at each step of successive approximations. Two first terms of this expansion are

$$V(\lambda) = V^{(I)}(\lambda) + V^{(II)}(\lambda), \quad V^{(II)}(\lambda) = V_1^{(II)}(\lambda) + V_2^{(II)}(\lambda), \tag{2.4}$$

where

$$V^{(I)}(\lambda) = \sum_{i=1}^{A} u^i(\lambda), \tag{2.5}$$

$$V_1^{(II)}(\lambda) = -\sum_{i\neq j=1}^{A} \sum_s \int_0^\lambda d\lambda_1 [G_0^{(+)}(E_s^\circ) u^i(\lambda_1) |s\rangle_0 {}_0\langle s| u^j(\lambda_1) + \text{h.c.}], \tag{2.6}$$

$$V_2^{(II)}(\lambda) = \sum_{i,j=1}^{A} \sum_s \int_0^\lambda d\lambda_1 \{[G^{(+)}(E_s) u^i(\lambda_1) |\vec{k}_s, \alpha_s\rangle \langle \vec{k}_s, \alpha_s | u^j(\lambda) - G_0^{(+)}(E_s^\circ) u^i(\lambda_1) |s\rangle_0 {}_0\langle s| u^j(\lambda)| + \text{h.c.}\}. \tag{2.7}$$

The two-body u-matrix is defined as follows:

$$u^i(\lambda) \equiv \sum_{n,m} |n\rangle_0 u^i_{nm}(\lambda) {}_0\langle m|,$$

where u^i_{nm} is the exact two-body matrix element of the pion-i-th nucleon interaction (see Sec.5), $|n\rangle_0$, E_n° etc., are eigenfunctions and eigenvalues of the free Hamiltonian $H_0 = K_\pi + K_A$ (K_A denotes the nucleus kinetic-energy operator), $G_0^{(\pm)}(E) = (E - H_0 \pm i\delta)^{-1}$, $G^{(\pm)}(E) = (E - h \pm i\delta)^{-1}$ are the Green functions.

In the low-energy limit (the two-body case) the π-nucleus phase shifts are determined by the matrix element $\langle \vec{k}, 0 | V(\lambda) | \vec{k}', 0 \rangle$ ($\vec{k}$ and $\vec{k}'$ are pion momenta before and after collision in Acm) as

$$\delta(k) = -\pi \epsilon_A(k) \{ \int_0^1 d\lambda \langle \vec{k}, 0 | V(\lambda) | \vec{k}', 0 \rangle \}, \tag{2.8}$$

where $\epsilon_A(k)$ is the level density: $\epsilon_A = k^2 / |2\pi^2 dE_0/dk|$, $E_0 = \omega_\pi(k) + \omega_A(k)$ is the collision energy counted from the nuclear ground state, $\omega_\pi(k) = (k^2 + m^2)^{1/2}$, $\omega_A(k) = (k^2 + (AM)^2)^{1/2}$, m and M denote the pion and nucleon masses. By the braces in the general relations like (2.8) we denote an appropriate partial harmonics with respect to the angular momentum, spin, and isospin. For the zeroth spin-isospin nuclei, like ^{4}He, the symbol is:

$$\{f\}_L \equiv \frac{1}{2} \int_{-1}^{1} dx P_L(x) f(x),$$

where P_L are the Legendre polynomials and $x = \hat{\vec{k}}\,\hat{\vec{k}}'$.

The generalization of (2.8) to the case of an arbitrary energy of the projectile is as follows [2]:

$$\delta(k) = -\pi \epsilon_A(k) \{ \int_0^1 d\lambda \langle \vec{k}, 0 | U_0(E, \lambda) | \vec{k}', 0 \rangle \}, \tag{2.9}$$

where $U_0(E, \lambda)$ is some effective energy-dependent operator. In general $U_0(E, \lambda)$ is non-Hermitian. Its non-Hermitian part is due to the contribution of inelastic channels to the elastic one. By an exact integral equation (see ref. [2]) $U_0(E, \lambda)$ is expressed through the $V(\lambda)$ (2.4). Two first iterations of this equation give:

$$U_0(E, \lambda) = V(\lambda) - 2\pi i \int_0^\lambda d\lambda_1 V(\lambda_1) \delta(E - h) \hat{Q} V(\lambda), \tag{2.10}$$

where $\hat{Q} = \sum_{n>0} |n\rangle\langle n|$ is the projection operator for the excited nuclear states. The contribution of the second term in the r.h.s. disappears when the projectile cannot excite any nuclear state energy and $U_0(E, \lambda) = V(\lambda)$. Hence, in the low-energy limit the two-body unitarity condition is justified in our approach.

Substituting (2.4) and (2.10) into (2.9) gives us the following expression for two first iterations of the π-nucleus phase shifts:

$$\delta(k) = \delta^{(I)}(k) + \delta^{(II)}(k), \quad \delta^{(II)}(k) \equiv \delta_R^{(II)}(k) + i\delta_{Im}^{(II)}(k), \tag{2.11}$$

where $\delta^{(I)}$ is the first-order approximation:

$$\delta^{(I)}(k) = -\pi\epsilon_A(k)\{\int_0^1 d\lambda \langle \vec{k},0|V^{(I)}(\lambda)|\vec{k}',0\rangle\}, \quad (2.12)$$

$\delta_R^{(II)}$ and $\delta_{Im}^{(II)}$ are real and imaginary parts of the second-order correction:

$$\delta_R^{(II)}(k) = -\pi\epsilon_A(k)\{\int_0^1 d\lambda \langle \vec{k},0|V^{(II)}(\lambda)|\vec{k}',0\rangle, \quad (2.13)$$

$$\delta_{Im}^{(II)}(k) = 2\pi^2\epsilon_A(k)\{\int_0^1 d\lambda \int_0^\lambda d\lambda_1 \langle \vec{k},0|V^{(I)}(\lambda_1)\delta(E-h)\hat{Q}V^{(I)}(\lambda)|\vec{k}',0\rangle. \quad (2.14)$$

Operators $V^{(I)}$ and $V^{(II)}$ are defined in (2.5)-(2.7). Note the matrix elements in (2.12)-(2.14) are similar in structure to the first and second-order optical potential (see refs./6-9/). Thus, for its calculation we shall use below methods elaborated in the optical model.

3. The first-order approximation. Using the overlap function

$$F_{00}^{(1)}(\vec{k}_1,\vec{k}_1') = \int \prod_{i=2}^{A} \frac{d\vec{k}_i}{(2\pi)^3} \delta(\vec{k} + \sum_{j=1}^{A} \vec{k}_j) \bar{\psi}_0(\vec{k}_1,\vec{k}_2,\ldots,\vec{k}_A)\psi_0(\vec{k}_1',\vec{k}_2,\ldots,\vec{k}_A),$$

where ψ_0 is the ground state w.f., the quantity (2.12), can be represented /2,6/ in the form:

$$\delta^{(I)}(k) = -A\pi\epsilon_A(k)\{\int_0^1 d\lambda \int \frac{d\vec{p}}{(2\pi)^3} F_{00}^{(1)}(\vec{p},\vec{p}-\vec{q})\langle \vec{k},\vec{p}|u^1(\lambda)|\vec{k}',\vec{p}-\vec{q}\rangle\}. \quad (3.1)$$

Here $\vec{q} = \vec{k} - \vec{k}'$ is the transfer momentum. In ref./2/ we have calculated (3.1) in the static limit, i.e., when $m/M \to 0$. In refs./3,4/ for its calculation the so-called factorization approximation was used. In addition there was admitted the picture, in which the nucleons are "frozen" in the target nucleus. Here we take into account the nucleon Fermi motion. Following ref./6/ we make use of the fact that the ^{4}He -ground state w.f. is well described by the 1s -harmonic oscillator function $\Phi_0(p) \sim \exp(-a_0^2p^2/2)$, where a_0 is the oscillator parameter. Then using the Jacoby coordinates in (3.1) one can easily obtain

$$\delta^{(I)}(k) = -A\pi\epsilon_A(k)\{\rho_{00}(\vec{q}) \int_0^1 d\lambda \bar{u}^1(\vec{k},\vec{k}',\lambda)\}, \quad (3.2)$$

where

$$\rho_{00}(\vec{q}) = \int \frac{d\vec{p}}{(2\pi)^3} F_{00}^{(1)}(\vec{p},\vec{p}-\vec{q}) = \exp(-a^2\vec{q}^2/4) \quad (3.3)$$

is the nuclear form factor, $a^2 = [(A-1)/A]a_0^2$, and $\bar{u}^1$ is the two-body u -matrix averaged over the nuclear density

$$\bar{u}^1(\vec{k},\vec{k}';\lambda) = \int \frac{d\vec{p}}{(2\pi)^3} F_{00}^{(1)}(\vec{p},\vec{p}) \langle k,\vec{p}+\vec{p}_0|u^1(\lambda)|\vec{k}',\vec{p}+\vec{p}_0-\vec{q}\rangle. \quad (3.4)$$

Here $\vec{p}_0 = -\vec{k}/A + [(A-1)/2A]\vec{q}$, the index "1" of the u -matrix stands for its spin-isospin part (see Sec.5). For the zeroth spin-isospin nuclei, like ^{4}He, the result of the averaging of u^1 over the nuclear w.f. is obtained from (3.4) by the substitution of the isoscalar part of u -matrix- u_0 for u^1. The nuclear density of ^{4}He has the form /6/:

$$F_{00}^{(1)}(\vec{p},\vec{p}) = (2\sqrt{\pi}b)^3 \exp(-\vec{p}^2b^2), \quad b^2 = [A/(A-1)]a_0^2. \quad (3.5)$$

In the calculation the form factor ρ_{00} is assumed to be charge form factor determined by the electron scattering /10/. The parameter "a" is expressed through the nuclear charge radius, $R_{ch} = 1.71$ fm for ^{4}He and the proton radius, $r_p = 0.76$ fm; $a^2 = 2(R_{ch}^2 - r_p^2)/3$, i.e., $a = 1.25$ fm. Thus, for a_0 and b we get: $a_0 = 1.44$ fm and $b = 1.67$ fm.

The dependence of $\vec{p}_0$ in (3.4) on $\vec{q}$ makes this integral to be very time-consuming. Thus, taking into account that the nuclear form factor has a pronounced maximum at $q = 0$ we shall take $\vec{p}_0 = -\vec{k}/A$ that reduces the integration over $\vec{p}$ to be two-dimensional. This "Fermi-folding approximation" is usually used in the optical model calculations /6,8/.

4. The second-order approximation for the π -nucleus phase shifts. A. The real part of this correction, $\delta_R^{(II)}$ (2.13), is the sum (see (2.4)) of two terms: $\delta_R^{(II)} = \delta_{R,1}^{(II)} + \delta_{R,2}^{(II)}$, where

$$\delta_{R,i}^{(II)}(k) = -\pi\epsilon_A(k)\{\int_0^1 d\lambda \langle \vec{k},0|V_i^{(II)}(\lambda)|\vec{k}',0\rangle\}, \quad i = 1,2. \quad (4.1)$$

Here $V_i^{(II)}$ are defined by (2.6) and (2.7). The first term describes the pion rescattering on the noninteracting nucleons and the second one - through intermediate nucleon interaction.

Consider the correction $\delta_{R,1}^{(II)}$. To evaluate the matrix element $\langle \vec{k},0|V_1^{(II)}|\vec{k}',0\rangle$ it is advantageous to introduce the two-particle overlap function:

$$F_{00}^{(2)}(\vec{k}_1,\vec{k}_2;\vec{k}_1',\vec{k}_2') = \quad (4.2)$$

$$= \int \prod_{i=3}^{A} \frac{d\vec{k}_i}{(2\pi)^3} \delta(\vec{k} + \sum_{j=1}^{A} \vec{k}_j) \bar{\psi}_0(\vec{k}_1,\vec{k}_2,\vec{k}_3,\ldots,\vec{k}_A)\psi_0(\vec{k}_1',\vec{k}_2',\vec{k}_3,\ldots,\vec{k}_A).$$

Using it one can represent the matrix element of $V_1^{(II)}$ as:

$$\langle \vec{k},0|V_1^{(II)}(\lambda)|\vec{k}',0\rangle = A(A-1) \int \frac{d\vec{k}'}{(2\pi)^3} \int \frac{d\vec{p}_1 d\vec{p}_2}{(2\pi)^6} \int_0^\lambda d\lambda_1 \times$$

$$F_{00}^{(2)}(\vec{p}_1,\vec{p}_2-\vec{q}'';\vec{p}_1+\vec{q}',\vec{p}_2)<\vec{k}',\vec{p}_1|u^1(\lambda_1)|\vec{k}'',\vec{p}_1-\vec{q}'><\vec{k}'',\vec{p}_2+\vec{q}''|u^2(\lambda_1)|\vec{k}',\vec{p}_2> \quad (4.3)$$

$$\left[\frac{1}{E^\circ(\vec{k},\vec{p}_1)-E^\circ(\vec{k}'',\vec{p}_1')-i\gamma}+\frac{1}{E^\circ(\vec{k}',\vec{p}_2)-E^\circ(\vec{k}'',\vec{p}_2')+i\gamma}\right],$$

where $\vec{q}'=\vec{k}-\vec{k}''$, $\vec{q}''=\vec{k}''-\vec{k}'$, $\vec{p}_1'=\vec{p}_1-[(A-1)/A]\vec{q}'$, $\vec{p}_2'=\vec{p}_2+[(A-1)/A]\vec{q}''$, the energy $E^\circ(\vec{k},\vec{p})\equiv E_0(\vec{k})+[A/2(A-1)M]\vec{p}^{\,2}$, $E_0(\vec{k})=\omega_\pi(\vec{k})+\omega_A(\vec{k})$. In the calculation of the energy denominators the nonrelativistic kinematics for nucleons was used.

Now to simplify (4.3) we employ the factorization approximation, i.e., in the integral of (4.3) we put:

$$F_{00}^{(2)}(\vec{p}_1,\vec{p}_2-\vec{q}'';\vec{p}_1+\vec{q}_1',\vec{p}_2)\simeq C_{00}(\vec{q}',\vec{q}'')F_{00}^{(1)}(\vec{p}_1,\vec{p}_1)F_{00}^{(1)}(\vec{p}_2,\vec{p}_2), \quad (4.4)$$

where

$$C_{00}(\vec{q}',\vec{q}'')=\int\frac{d\vec{p}_1\,d\vec{p}_2}{(2\pi)^6}F_{00}^{(2)}(\vec{p}_1,\vec{p}_2-\vec{q}'';\vec{p}_1+\vec{q}',\vec{p}_2)= \quad (4.5)$$

$$=<0|\exp[i(\vec{q}'\vec{r}_1+\vec{q}''\vec{r}_2)]|0>$$

is the correlation function. Next, noting that $C_{00}(\vec{q}',\vec{q}'')$ has a pronounced maximum at $\vec{q}'=\vec{q}''=0$ we suppress the q-dependence of the energy denominators in (4.3). With these approximations we get:

$$\delta_{R,1}^{(II)}(k)=-\pi\epsilon_A(k)A(A-1)\{\int\frac{d\vec{k}''}{(2\pi)^3}2P\left(\frac{1}{E_0(k)-E_0(k'')}\right)C_{00}(\vec{q}',\vec{q}'')\times \quad (4.6)$$

$$\times\sum_{\beta=0}^{3}\mathcal{F}_\beta\int_0^1 d\lambda\int_0^\lambda d\lambda_1\bar{u}_\beta(\vec{k},\vec{k}'';\lambda_1)\bar{u}_\beta(\vec{k}'',\vec{k}';\lambda_1)\},$$

where the sign P means the principal value integration. In the derivation of (4.6) the spin-isospin structure of the $(\pi,{}^4He)$-system was taken into account: $\mathcal{F}_\beta: 1,-\frac{2}{3},-\frac{1}{3}[(\hat{\vec{k}}\hat{\vec{k}}'')-(\hat{\vec{k}}\hat{\vec{k}}'')(\hat{\vec{k}}''\hat{\vec{k}}'')]$, $-\frac{2}{3}[(\hat{\vec{k}}\hat{\vec{k}}')-(\hat{\vec{k}}\hat{\vec{k}}'')(\hat{\vec{k}}''\hat{\vec{k}}'')]$. Integrals over λ of the two-body matrix elements, u_β, in terms of the πN-phase shifts are presented in Sec.5.

Let us consider now the $\delta_{R,2}^{(II)}$-correction in (4.1). It is expressed through the matrix element of $V_2^{(II)}$ (2.7). In ref.[1] we have shown that this correction is strictly zero in the static limit, i.e., when $m/M\to 0$, due to the cancellation of G and G_0 in (2.7). Indeed, in that limit the energy denominators in G and G_0 do not depend on the nucleon variables. Thus, due to the completeness of eigenfunctions of h and H_0 we get the desired result. Note that the similar approximation we have just used in obtaining (4.6) when we have neglected the $\vec{q}$-dependence of the energy denominators in (4.3). Hence, below we shall put that $\delta_R^{(II)}=\delta_{R,1}^{(II)}$. One can show that this implies the ~20%-level of accuracy of our calculations of the second-order corrections.

B. The imaginary part of the second-order correction, $\delta_{Im}^{(II)}$, (2.14) has the form:

$$\delta_{Im}^{(II)}(k)=-2\pi^2\epsilon_A(k)\{\int\frac{d\vec{k}''}{(2\pi)^3}\int_0^1 d\lambda\int_0^\lambda d\lambda_1\sum_{\alpha''>0}<\vec{k},0|V^{(I)}(\lambda_1)|\vec{k}'',\alpha''>\times \quad (4.7)$$

$$\times\delta(E_0(k)-E_{\alpha''}(k''))<\vec{k}'',\alpha''|V^{(I)}(\lambda)|\vec{k}',0>\},$$

where $V^{(I)}=\sum_i u^i$ (see (2.5)), $E_0(k)=\omega_\pi(k)+\omega_A(k)$, $E_\alpha(k)=E_0(k)+\Delta_\alpha$, Δ_α is the nuclear excitation energy counted from the ground state.

The matrix elements in (4.7) are similar in structure to that one in the first-order approximation (2.12). Thus we generalize (3.2) to any excited states:

$$<\vec{k},0|\sum_i u^i(\lambda)|\vec{k}'',\alpha''>\simeq A\,\rho_{0\alpha''}(\vec{q}')\bar{u}^1(\vec{k},\vec{k}'';\lambda). \quad (4.8)$$

Here $\rho_{0\alpha''}$ is the transition form factor, $\vec{q}'=\vec{k}-\vec{k}''$. In the calculation of (4.7) we have also used the closure approximation by the set: $E_\alpha(k)=E_0(k)+\Delta$, where Δ is the nuclear mean excitation energy. For ${}^4He$ its value is about 20 MeV. In ref.[8] it was shown that the variation of Δ from zero to 20 MeV in integrals like (4.7) gives the effect of about 10%. Therefore, in the subsequent calculation we set (as in ref.[8]) $\Delta=0$ to avoid the unnecessary complexity. Then by using the identity:

$$\sum_{\alpha>0}\rho_{0\alpha}(\vec{q}')\rho_{\alpha 0}(\vec{q}'')=\frac{1}{A}\rho_{00}(\vec{q}'+\vec{q}'')+(1-\frac{1}{A})C_{00}(\vec{q}',\vec{q}'')-\rho_{00}(\vec{q}')\rho_{00}(\vec{q}''),$$

and averaging over spin-isospin variables in (4.7) we get:

$$\delta_{Im}^{(II)}(k)=-2[A\cdot\pi\epsilon_A(k)]^2\{\int\frac{d\hat{\vec{k}}''}{4\pi}\int_0^1 d\lambda\int_0^\lambda d\lambda_1\sum_{\beta=0}^{3}\bar{u}_\beta(\vec{k},\vec{k}'';\lambda_1)\times \quad (4.9)$$

$$\times\bar{u}_\beta(\vec{k}'',\vec{k}';\lambda)[\frac{1}{A}\rho_{00}(\vec{q})A_\beta-\rho_{00}(\vec{q}')\rho_{00}(\vec{q}'')\delta_{\beta 0}+(1-\frac{1}{A})\mathcal{F}_\beta C_{00}(\vec{q}',\vec{q}'')]\}.$$

Here $k=|\vec{k}|=|\vec{k}'|=|\vec{k}''|$, $A_\beta: 1,2,(\hat{\vec{k}}\hat{\vec{k}}')-(\hat{\vec{k}}\hat{\vec{k}}'')(\hat{\vec{k}}'\hat{\vec{k}}''),-2[(\hat{\vec{k}}\hat{\vec{k}}')-(\hat{\vec{k}}\hat{\vec{k}}'')(\hat{\vec{k}}'\hat{\vec{k}}'')]$, the coefficients $\mathcal{F}_\beta$ are defined in (4.6). As will be shown in Sec.5, the integrals of products of the two-body u-matrices can be expressed in terms of πN-phase shifts.

5. Our goal is to express the π-nucleus phase shifts in terms of the πN-ones and nuclear bound state characteristics. To do this, we must transform the πN-u-matrix in (3.2), (4.6), and (4.9) from the Acm-system into the πN c.m. frame (2cm-system). Considering that the u-matrix has the same transformation properties as the scattering matrix, t, we have

$$<\vec{k},\vec{p}\,|\,u(\lambda)\,|\,\vec{k}',\vec{p}\,'> = \gamma <\vec{\kappa}\,|\,\tilde{u}(\lambda)\,|\,\vec{\kappa}'>, \qquad (5.1)$$

$$\gamma = \left[\frac{\omega_\pi(\vec{\kappa})\,\omega_N(\vec{\kappa})\,\omega_\pi(\vec{\kappa}')\,\omega_N(\vec{\kappa}')}{\omega_\pi(\vec{k})\,\omega_N(\vec{p})\,\omega_\pi(\vec{k}')\,\omega_N(\vec{p}\,')}\right]^{1/2}. \qquad (5.2)$$

Here $\tilde{u}(\lambda)$ and the pion initial and final momenta $\vec{\kappa}$ and $\vec{\kappa}'$ refer to the 2cm, $\omega_\pi(\vec{k}) \equiv (\vec{k}^2+m^2)^{1/2}$, $\omega_N(\vec{k}) \equiv (\vec{k}^2+M^2)^{1/2}$. The quantities $\vec{\kappa}$, $\vec{\kappa}'$ can be expressed in terms of $\vec{k}$, $\vec{p}$, $\vec{k}'$, $\vec{p}\,'$ (for details see refs.[2,3,6]).

The spin-isospin dependence of the u-matrix can be expressed as follows:

$$<\vec{\kappa}\,|\,\tilde{u}(\lambda)\,|\,\vec{\kappa}'> = \sum_{\beta=0}^{3} <\vec{\kappa}\,|\,u_\beta(\lambda)\,|\,\vec{\kappa}'>\, O_\beta, \qquad (5.3)$$

where $O_\beta = 1, (\vec{t}\vec{\tau})$, $i(\vec{\sigma}\vec{n})$, $i(\vec{\sigma}\vec{n})(\vec{t}\vec{\tau})$. Here $\vec{\sigma}/2$ is the nucleon spin operator, and $\vec{\tau}/2$ and $\vec{t}$ are, respectively, the nucleon and pion isospin operators. Neglecting the spin-dependent terms (β=2,3), the matrix elements of $u_{0,1}$ can be written as

$$<\vec{\kappa}\,|\,u_\beta(\lambda)\,|\,\vec{\kappa}'> = \sum_I c_I^\beta \sum_{\ell_j} (j+\tfrac{1}{2})\, u_{\ell_j}^I(\kappa,\kappa';\lambda)\, P_\ell(\hat{\kappa}\hat{\kappa}'), \qquad (5.4)$$

where I=1/2,3/2 labels a given isotopic πN-state, $c^0_{1/2}=1/3$, $c^0_{3/2}=3/2$, $c^1_{3/2}=-c^1_{1/2}=1/3$, $j=\ell\pm 1/2$.

In each eigenchannel $\nu(I,\ell,j)$ the on-energy-shell ($\kappa=\kappa'$) u_ν-matrix determines the πN-phase shift[5]

$$\delta_\nu(\kappa) = -\pi\epsilon_2(\kappa)\int_0^1 d\lambda\, u_\nu(\kappa,\kappa;\lambda), \qquad (5.5)$$

where $\epsilon_2(\kappa)$ is the level density. Now substituting (5.4) and (5.5) into (3.4) and (3.2) gives us the desired expression of the first-order approximation, $\delta^{(I)}$, in terms of πN-phase shifts and the nuclear form factor (see refs.[2-4]).

The off-energy-shell dependence of $u_\nu(\kappa,\kappa',\lambda)$ in (4.6) (following ref.[7]) is assumed to have a separable form:

$$u_\nu(\kappa,\kappa';\lambda) = u_\nu(\kappa,\kappa';\lambda)\, g_\nu(\kappa')/g_\nu(\kappa) \qquad (5.6)$$

with $g_\nu(\kappa) = \kappa^\ell \exp(-\beta_\nu\kappa^2)$. The parameter β_ν is related to the range of πN-interaction. Acceptable values of β_ν probably lie[7] in the region $0.1\ \mathrm{fm}^2 < \beta_\nu < 0.4\ \mathrm{fm}^2$.

Consider now the integrals over λ of products of u-matrices in (4.6) and (4.9):

$$A_{\nu\nu'} \equiv \int_0^1 d\lambda \int_0^\lambda d\lambda_1 [u_\nu(\kappa,\kappa;\lambda_1)\, u_{\nu'}(\kappa',\kappa';\lambda) + (\nu \rightleftarrows \nu')], \qquad (5.7)$$

$$B_{\nu\nu'} \equiv \int_0^1 d\lambda \int_0^\lambda d\lambda_1 [u_\nu(\kappa,\kappa;\lambda_1)\, u_{\nu'}(\kappa',\kappa';\lambda_1) + (\nu \rightleftarrows \nu')]. \qquad (5.8)$$

Integrating by parts with taking into account (5.5) gives an expansion the leading term of which is:

$$A_{\nu\nu'} = B_{\nu\nu'} = [1/(\pi^2\epsilon_2(\kappa)\epsilon_2(\kappa'))]\,\delta_\nu(\kappa)\,\delta_{\nu'}(\kappa'). \qquad (5.9)$$

The estimation of higher-order terms (see for example Eq. (31) in ref.[1]) gives a contribution of about 20%. Therefore we may safely neglect it within the accuracy of our calculations (see Sec.4).

The formulae we have obtained in this section complete the definition of the two first iterations of the $(\pi, {}^4\mathrm{He})$-phase shifts. Substitution of (5.4), (5.6) into (4.6) and (4.9) gives us the desired expressions of the second-order corrections in terms of the πN-phase shifts, the nuclear form factors, and the correlation functions. In the numerical calculations we have neglected the spin-dependence of πN-interaction. In ref.[9] it is shown that it contributes only at the pion energy above 100 MeV.

6. We present now the calculational results of the two first iterations (3.2), (4.6) and (4.9) for $(\pi, {}^4\mathrm{He})$-phase shifts. The parameters used in the calculations have been defined before (see eqs. (3.3), (3.5) and (5.6)). The πN-phase shifts we have used here are from ref.[11]. Only s and p-πN-waves were taken into account. The correlation function (4.5) due to the Gaussian form of the ${}^4\mathrm{He}$-ground state wave function is

$$C_{00}(q,q') = \exp\left[-\frac{1}{4}a^2(q^2+q'^2) + \frac{1}{6}a^2(q\cdot q')\right], \qquad (6.1)$$

where $a \simeq 1.25$ fm (see Sec.3). In (6.1) the effect of the short-range NN correlations have not been included. These correlations give a rather small contribution in the low-energy region. Their contribution is determined by the parameter $(\ell_c/a)^2 \approx 0.02$, where $\ell_c = 0.4$ fm is the correlation length (see ref. [7]). The NN short-range correlation gives[7] an appreciable effect only at energies higher than, say, 180 MeV in a large momentum transfer region.

Figures 1,2 show the energy dependence of the s and p-$(\pi, {}^4\mathrm{He})$-phase shifts and inelasticity parameters up to 140 MeV. The dashed lines are the first-order approximation ($\delta=\delta^{(I)}, \eta^{(I)}=1$) and the

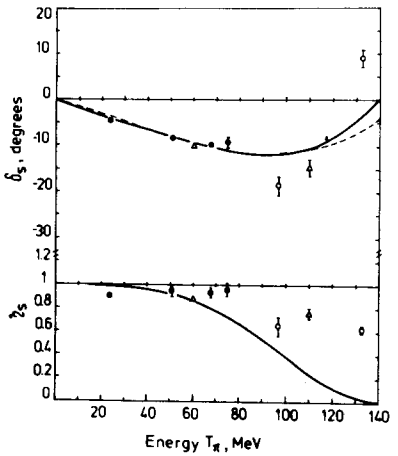

Fig.1. The $(\pi, {}^4He)$ -phase shift, δ_S, and the inelasticity parameter, η_S, calculated from $\delta^{(I)} + \delta^{(II)}$ by using $\beta_\nu = 0.2$ fm^2 (solid curves) and $\delta^{(I)}$ (dashed curve, $\eta^{(I)} = 1$). The results of PSA at 24, 51, 68 and 75 MeV (refs./13,14/) are shown by black points, at 98 and 135 (ref./15/) by open circles, and at 60 and 110 MeV (ref./16/) by triangles.

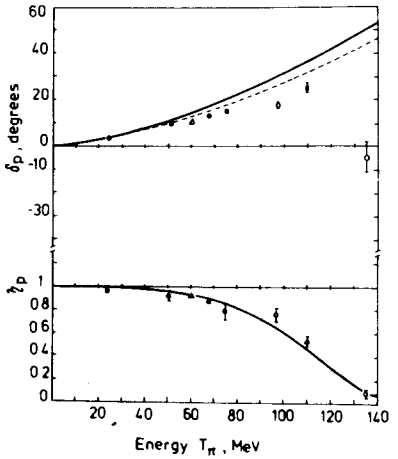

Fig.2. The same as in Fig.1, but for the δ_P -$(\pi, {}^4He)$ -phase shift and for the inelasticity parameter η_P.

full ones the sum of the two first iterations $(\delta = \delta^{(I)} + \delta_R^{(II)}$, $\eta = \exp(-2\delta_{Im}^{(II)})$. We also present in Figs.1,2 the results of some phase shift analyses (PSA).

From Figs.1,2 we see that for the real parts of phase shifts, $\delta_{s,p}$, the second-order correction $\delta_R^{(II)}$ (4.10) gives an appreciable contribution ($\gtrsim$10%) at energies T_π above 80 MeV*. One can admit this value as defining the convergence range of the considered series. Note that at lower energies the PSA data are in good agreement with the calculational results.

It must be noted that the correction $\delta_R^{(II)}$ becomes essential at very low energies: below ~40 MeV. At 24 MeV (see Table 1) its contribution is about 30%. The contribution of $\delta_R^{(II)}$ increases with decreasing energy. If the $(\pi, {}^4He)$ -scattering length, $a_{\pi\,{}^4He}$, calculated in the first-order approximation (3.2)with the Solomon/11/ πN -phase shifts is equal to -0.033 fm, then the sum of two first iterations gives $a_{\pi^4He} = -0.126$ fm. This

*Here T_π is the pion kinetic energy in the lab.system.

value is in a fairly good agreement with the experimental result: $Re\,a_{\pi\,{}^4He} = -(0.143 \pm 0.04)$ fm/17/. Nevertheless, the numerical convergence of the iteration scheme is not accidental (see ref./1/). Smallness of $\delta^{(I)}$ as compared to $\delta_R^{(II)}$ at very low energies is due to the specific isotopic structure of the problem resulting in the cancellation of two "large" quantities in the first-order approximation.

Now let us consider the potential description of the PSA data for inelasticity parameters. We see from Figs.1,2 that especially in the s-wave the potential predictions diverge from the data. For that reason we have carried out some additional investigation of our iteration scheme. We have estimated the third and fourth-order corrections and have found that they give more than 10%-contribution at energies above 70 MeV. Thus, the observed discrepancy indicates the importance of the inclusion into consideration of the pion absorption channel. A more detailed discussion of it will be given below.

7. In the preceding section we have established the convergence range of the iteration scheme to be lower than 70 MeV. Here we shall analyse the $(\pi, {}^4He)$ scattering data at 24 and 51 MeV from refs./12,14/. At these energies there are data both for π^+ and π^--mesons. Hence it is possible to define the so-called mean differential and total cross sections as: $d\bar{\sigma}/d\Omega = (d\sigma^+/d\Omega + d\sigma^-/d\Omega)/2$, etc., which are of an accuracy of $O(1/\alpha^2)$, $\alpha = 1/137$, determined only by the strong interaction. It is convenient to use these quantities for comparing the theoretical and experimental results. It should be noted that the present available precision of experimental data in the low-energy region is not high (see, for example, Fig.4). Also there are no differential cross section data at small angles $\theta_{cm} < 30° - 40°$). That makes the PSA data be not free from some ambiguities. Especially this concerns the PSA predictions of values for the total cross section. This quantity is obtained by the extrapolation of the scattering data to the zero angle. Thus, we consider the comparison of the calculations with the data presented below only as preliminary.

The $(\pi, {}^4He)$-phase shifts at 24 and 51 MeV are shown in Tables 1,2. The first two columns contain the results obtained in the first-order approximation (3.2) with ($\delta^{(I)}$) and without ($\delta_{Ft}^{(I)}$) inclusion of the Fermi motion. We see that the Fermi averaging reduces the s-wave at 24 MeV by more than 20% and at 51 MeV by 15%. The effect of this reduction on the differential cross section at 24 MeV is shown in Fig.3. The Fermi averaging shifts the differential cross section minimum to the right. Columns 3 and 4 show the second-order correction effect (with the Fermi averaging): $Re\,\delta = \delta^{(I)}(3.2) + \delta^{(II)}(4.6)$, $Im\,\delta = \delta_{Im}^{(II)}(4.9)$, the range parameter of $\delta_R^{(II)}$ is $\beta_\nu = 0.4$ fm^2 and 0.2 fm^2, respec-

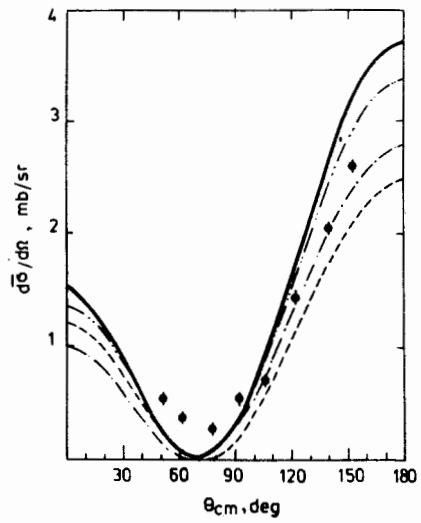

Fig.3. Pion- ^{4}He differential cross section at $T_\pi = 24$ MeV. Black points label mean differential cross section data from ref.[12]. Dash and the dash-dotted lines are obtained by using $\delta^{(I)}$ and $\delta^{(I)}_{Fr}$, respectively. Solid and the dash-double-dotted lines are calculated from $\delta^{(I)}+\delta^{(II)}$ by using $\beta_\nu = 0.2$ fm^2 and $\beta_\nu = 0.4$ fm^2 respectively.

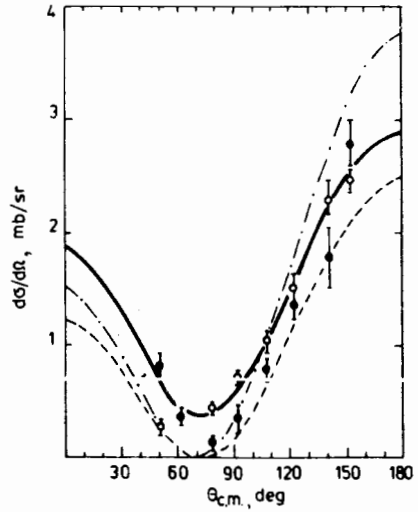

Fig.4. The $(\pi, ^4\mathrm{He})$-differntial cross section at 24 MeV calculated from $\delta^{(I)}+\delta^{(II)}$ by using $\beta_\nu = 0.2$ fm^2 (dash-dotted line) and $\delta^{(I)}$ (dashed line). The data from ref.[12] for $(\pi^+, ^4\mathrm{He})$ are denoted by open circles and for $(\pi^-, ^4\mathrm{He})$ by black points. Solid lines are obtained by using the PSA data from ref.[13].

tively. We see that the influence of the β_ν-variation is rather small (about 10%). The dependence of $d\sigma/d\Omega$ on β_ν at 24 MeV is shown in Fig.3. We see that a larger β_ν (corresponding to a longer πN interaction range) yields a smaller differential cross section both at small and large scattering angles. Hence in accordance with ref.[7] we can conclude that the difficulties encountered in fitting the low energy data cannot be resolved by simple adjustments in β_ν.

Finally, in column 5 in Tables 1,2 we have presented the PSA-data from refs.[13,14]. The corresponding differential cross sections are shown in Figs.4 and 5. We see that the two first iterations well reproduce the PSA data for Reδ. We obtain the most distinction in the p-wave at 51 MeV (about 30%). The description of the inelasticity parameters is worse. At 24 MeV Im$\delta_{S,P}$ (4.9) is by an order smaller than PSA predictions and at 51 MeV the calculated value of Imδ_P is twice smaller than the data.

Table 1

The $(\pi, ^4\mathrm{He})$-phase shifts at 24 MeV

Phase shifts, in degrees	$\delta^{(I)}_{Fr}$	$\delta^{(I)}$	$\delta^{(I)}+\delta^{(II)}$, $\beta_\nu = 0.4$ fm^2	$\delta^{(I)}+\delta^{(II)}$, $\beta_\nu = 0.2$ fm^2	PSA data [13]
Re δ_S	-3.47	-2.69	-3.59	-3.81	-4.28
Im δ_S	0.00	0.00	0.34	0.34	2.53
Re δ_P	3.34	3.34	3.75	3.91	3.49
Im δ_P	0.00	0.00	0.12	0.12	1.19
Re δ_D	0.18	0.19	0.19	0.19	0.25
Im δ_D	0.00	0.00	0.06	0.06	0.07
σ_{EL}, mb	10.2	9.1	12.3	13.5	13.8
σ_{TOT}, mb	10.2	9.1	21.6	22.8	87.2
Re$f(0)$, fm	0.32	0.35	0.36	0.37	0.31

Table 2

The $(\pi, ^4\mathrm{He})$-phase shifts at 51 MeV

Phase shifts, in degrees	$\delta^{(I)}_{Fr}$	$\delta^{(I)}$	$\delta^{(I)}+\delta^{(II)}$, $\beta_\nu = 0.4$ fm^2	$\delta^{(I)}+\delta^{(II)}$, $\beta_\nu = 0.2$ fm^2	PSA data [14]
Re δ_S	-7.91	-6.71	-7.18	-7.47	-8.40
Im δ_S	0.00	0.00	2.56	2.56	1.75
Re δ_P	9.86	10.53	12.04	12.68	9.05
Im δ_P	0.00	0.00	1.07	1.07	2.30
Re δ_D	1.26	1.34	1.39	1.43	1.00
Im δ_D	0.00	0.00	0.15	0.15	0.15
σ_{EL}, mb	36.0	37.4	46.7	51.5	30.6
σ_{TOT}, mb	36.0	37.4	81.7	86.4	79.6
Re$f(0)$, fm	0.79	0.88	0.97	1.01	0.61

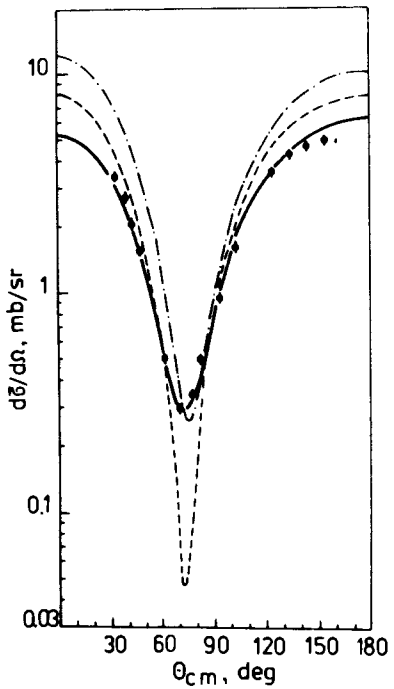

Fig.5. The same as in Fig.4 but for $(\pi, {}^4He)$ -scattering at 51 MeV. Black points label mean differential cross sections data from ref./14/.

Note that this discrepancy goes beyond the 20%-accuracy of our calculations. Thus, we conclude that the pure potential calculations cannot fit the PSA data for inelasticity parameters. The discrepancy between theory and data for $d\sigma/d\Omega$ in Figs.4 and 5 reflects this result.

Tables 1,2 also show the total elastic, σ_{EL}, the total, σ_{TOT}, cross sections and the real part of the forward scattering amplitude, Ref(0). We see that the potential calculations well reproduce the PSA predictions for Ref(0). It is consistent with that this quantity is mainly determined by the real parts of the phase shifts which are well reproduced by our calculations, too. In principle the difference between the PSA results for σ_{TOT} and the calculated ones can give an estimation of σ_{ABS}. Here as was mentioned above one must remember that the PSA data for σ_{TOT} can be not free from some ambiguity at present. Thus, now we can conclude only that at very low energies (~20 MeV) σ_{INEL} is mainly contributed by the pion absorption channel. At higher energies (~50 MeV) we observe an interesting situation when the rate of σ_{ABS} in σ_{INEL} is relatively small (see Table 2), but the absorption channel strongly affects the elastic one.

8. In this paper by calculating the second-order terms in the unitary pion-nucleus phase shifts expansion we have established the convergence range of the series to be $T_\pi \leq 70$ MeV for $(\pi, {}^4He)$-scattering. We have observed that the second-order corrections somewhat destroy the obtained earlier /2-4/ agreement with the data in the first-order approximation especially in 50-MeV region. This disagreement comes mainly from the difference between the calculated inelasticity parameters, η_L, and those of PSA. Our potential calculations show a simple regularity of the behaviour of η_L at a given energy with increasing orbital momentum L: $\eta_S < \eta_P < \eta_D < \ldots$, etc. Thus, the most part of the inelasticity is concentrated in the s-wave. This potential picture may be

simply explained by the increase of the centrifugal barrier with increasing L. But the PSA data show (see Table 2 and Figs.1,2) the deviation from this regularity in s and p-waves: $\eta_S \simeq \eta_P$. Hence, one may consider this as a direct indication of the strong influence of the pion absorption channel on the elastic one. The importance of a careful treatment of the absorption channel to get a satisfactory description of pion-nucleus scattering data in the low-energy region was pointed out in refs./18,19/ based on the first-order optical model calculations. Here the role of the pure pion-absorption channel due to the unitarity of the considered approach is reflected more impartially.

The author is grateful to V.B.Belyaev, D.A.Kirzhnitz, T.I.Kopaleishvili and R.A.Eramzhyan for helpful discussions. He is also indebted to V.Ju.Ponomarev for assistance in numerical calculations.

REFERENCES

1. Беляев В.Б. и др. ЯФ, 1980, 31, с.1466.
2. Khankhasayev M.Kh. JINR, E4-80-691, Dubna, 1980.
3. Khankhasayev M.Kh. JINR, E4-80-873, Dubna, 1980.
4. Khankhasayev M.Kh. Czech.Journ.of Phys., 1982, B32, p.207.
5. Киржниц Д.А., Крючков Г.Ю., Такибаев Н.Ж. ЭЧАЯ, 1979, 10, с.741.
6. Landau R.H., Phatak S.C., Tabakin F. Ann.of Phys., 1973, 78, p.299; Landau R.H. Phys.Rev., 1977, C15, p.2127.
7. Lee T.-S.H., Chakravarti S. Phys.Rev., 1977, C16, p.273.
8. Wakamatsu M. Nucl.Phys., 1978, A312, p.427.
9. Mach R., Sapozhnikov M.G. JINR, E4-82-189, Dubna, 1982.
10. Frosch R.F. et al. Phys.Rev., 1967, 160, p.874.
11. Solomon M. TRIUMF Report TRI-74-2, 1974.
12. Nordberg M.E., Kinsey K.F. Phys.Lett., 1966, 20, p.692.
13. West G.B. Phys.Rev., 1967, 162, p.1677.
14. Crowe K.M. et al. Nucl.Phys., 1969, 180, p.1349.
15. Balestra F. et al. Nuovo Cim., 1978, A43, p.604.
16. Binon F. et al. Nucl.Phys., 1978, A298, p.499.
17. Backenstoss G. et al. Nucl.Phys., 1974, A232, p.519.
18. Landau R.H., Thomas A.W. Nucl.Phys., 1978, A302, p.461; Phys.Lett., 1976, B61, p.361; Johnson R.R. et al. Nucl.Phys., 1978, A296, p.444.
19. Liu L.C., Shakin C.M. Phys.Rev., 1977, C16, p.333.

Received by Publishing Department
on June 18 1982.

Ханхасаев М.Х.

E4-82-468

Потенциальное описание низкоэнергетического упругого рассеяния пионов на ^{4}He

Упругое (π, ^{4}He) -рассеяние рассматривается в рамках унитарного потенциального подхода, в основе которого лежит закон эволюции системы с изменением константы связи. В данном подходе итерационная процедура строится непосредственно для вычисления фаз пион-ядерного рассеяния. Вычисляются поправки второго порядка к изученному ранее низшему приближению теории. Показано, что достаточно первых двух итераций для описания потенциального упругого (π, ^{4}He) -рассеяния при энергиях пиона ниже 70 МэВ. Сравнение теории и эксперимента при 24 и 51 МэВ указывает на существенную роль канала с истинным поглощением пиона в описании низкоэнергетического пион-ядерного рассеяния.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Khankhasayev M.Kh.

E4-82-468

Potential Description of the Low-Energy Pion Elastic Scattering on ^{4}He

The (π, ^{4}He)-elastic scattering is considered in the framework of the unitary potential approach based on the method of evolution in coupling constant. In this approach an iteration procedure for the direct calculation of π-nucleus phase shifts is developed. The second-order corrections to the considered earlier first-order approximation for pion-nucleus phase shifts are calculated. It is shown that the two first iterations are quite enough for the description of the potential (π, ^{4}He)-scattering below 70 MeV pion energy. The comparison of the calculated differential cross sections with the scattering data at 24 and 51 MeV indicates the importance of the inclusion of the true pion absorption channel to obtain a good fit of the data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982