объединенный
институт
ядерных
исследований
дубна

E4-82-355

D.V.Kreopalov, M.K.Volkov

THE DECAY $\eta \to \pi^{\circ}\gamma\gamma$ IN THE MODEL OF MESONS WITH QUARK LOOPS

Submitted to "Ядерная физика"

1982

1. INTRODUCTION

In paper [1] a model which can describe the behaviour of all known mesons in the low-energy sphere was proposed* . The model is based on the a study of four-quark interactions. It automatically reproduces models, which have been well-known earlier for various sorts of mesons. So, for instance, the σ-model is emerged for the scalar and pseudo-scalar meson sector, the model of Yang-Mills type for the vector meson sector, the model with vector meson dominance for electromagnetic interactions. The pseudo-vector and tensor mesons are described in the same way.

All constants of the strong interaction are defined by quark loops. These loops are defined by finite integrals or logarithmically divergent expressions. The square-divergent integrals can also emerge, for instance, when one describes masses of mesons or the interaction of tensor mesons. Since higher divergences are absent in this model, it is renormalizable. All the strong-interaction constants of various sorts of mesons are connected with each other and are expressed via the only constant g_ρ defining the decay process $\rho\to 2\pi$**.

Five various types of couplings between quarks such as scalar, pseudoscalar, vector, pseudovector, and tensor couplings are possible in the initial effective four-quark interaction. Our further statement is that all these types of couplings describe the interaction only between mesons really existing in Nature. In particular, instead of fictitions σ-particles emerging in the old version of the σ-model, we propose to regard experimentally detected particles (resonances), ϵ and δ. Masses of these resonances ($m_\epsilon \approx 700$ MeV, $m_\delta = 981$ MeV) appreciably deviate from the prediction of the σ-model, in which the formula $m_\sigma^2 = m_\pi^2 + 4m_q^2$ [1] ($m_q \approx 240$ MeV) takes place*. However, this fact should be regarded as the breakdown of the chiral symmetry, really existing in Nature.

* The model [1] continues the model [2] proposed earlier.

** The mass of quark m_q emerging in the model is expressed through one more parameter, the constant of the decay $\pi^+\to\bar{\mu}\nu$: $m_q = gF$ (The Coldberger-Treiman identity). g is a constant of the strong interaction between quarks and pions, $g = g_\rho/\sqrt{6}$.

It will be shown in this paper how our statement allows us to describe the decay $\eta\to\pi^0\gamma\gamma$ in a good agreement with experimental data obtained recently [3]. Models with the more exact chiral symmetry could not provide a good agreement with experiment [4,5].

In the next section all the processes corresponding to the decay $\eta\to\pi^0\gamma\gamma$ and also the decays close to them, $\eta'\to\rho\gamma$ and $\eta'\to\omega\gamma$, will be described.

In section 3 the decay $\eta\to\pi^0\gamma\gamma$ is discussed in the realistic case and in the limit of the chiral symmetry.

2. THE DECAYS $\rho^0\to\eta\gamma$, $\omega\to\eta\gamma$, $\eta'\to\rho\gamma$, $\eta'\to\omega\gamma$, $\delta\to\eta\pi$, $\delta^0\to\gamma\gamma$

The Lagrangian, which describes the interaction between quarks and mesons we are interested in has the form [1]

$$\mathcal{L} = \bar{q}\{g[\epsilon + \vec{\tau}\vec{\delta} + i\gamma_5(\frac{\eta_8+\sqrt{2}\,\eta_0}{\sqrt{3}} + \vec{\tau}\vec{\pi})] + \frac{g_\rho}{2}\gamma^\mu[\omega_\mu + \vec{\tau}\vec{\rho}_\mu]\}q - \frac{e}{g_\rho}(m_\rho^2\rho_\mu^0 + \frac{m_\omega^2}{3}\omega_\mu)A^\mu . \qquad (1)$$

Here $q = \binom{u}{d}$ are quark fields, having three colours (we suppose the summation over colour indices in (1)) ϵ, δ, π, η, ω, ρ are fields of scalar pseudoscalar, and vertor mesons, respectively, A^μ is photon field, m_ρ and m_ω are masses of ρ- and ω-mesons, e is the electromagnetic charge, g and g_ρ are strong coupling constants, $g = \sqrt{2\pi}$, $g_\rho = \sqrt{6}\,g$, $\vec{\tau}$ are Pauli matrices, γ^μ are Dirac matrices.

The decay processes $\rho^0\to\eta\gamma$, $\omega\to\eta\gamma$, $\eta'\to\rho\gamma$, $\eta'\to\omega\gamma$ are described by triangle quark diagrams of the anomalous type, shown in fig.1**. Their amplitudes are

* This note applies more to the δ-resonance since the ϵ-resonance has a larger width (~400 MeV) due to which the divergence of its mass from the predicted mass of the σ-particle is not too big. ($m_\sigma \approx 500$ MeV).

** These processes were calculated by the same way in paper [6]. In contrast to [6] we use one independent constant of the strong interaction.

$$T_{\rho^\circ\to\eta\gamma} = -\frac{\sqrt{3}\,e g_\rho}{8\pi^2 F}(\cos\theta - \sqrt{2}\sin\theta)\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^\mu_\rho\,\epsilon^\nu_\gamma\,p^\rho q^\sigma\,;\; T_{\omega\to\eta\gamma} = \frac{1}{3}T_{\rho_0\to\eta\gamma}$$

$$T_{\eta'\to\rho\gamma} = \frac{\sqrt{3}\,e g_\rho}{8\pi^2 F}(\sqrt{2}\cos\theta + \sin\theta)\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^\mu_\rho\,\epsilon^\mu_\gamma\,p^\rho q^\sigma;\; T_{\eta'\to\omega\gamma} = \frac{1}{3}T_{\eta'\to\rho\gamma}. \quad (2)$$

Here F=95 MeV is the pion decay constant, θ is the angle of singlet-octet mixing of the η-meson. We shall choose the old value θ=-11° for it, though the other experimental value is known θ =-18°[7]. q is the momentum of photon, p is the momentum of the ρ or ω mesons, ϵ^μ_ρ and ϵ^ν_γ are the ρ-meson and photon polarization, respectively, and $\epsilon_{\mu\nu\rho\sigma}$ is the fully antisymmetric tensor. The decay widths corresponding to the amplitudes (2) are

$$\Gamma_{\rho^\circ\to\eta\gamma} = 54\text{ keV},\quad \Gamma_{\omega\to\eta\gamma} = 6{,}4\text{ keV},$$
$$\Gamma_{\eta'\to\rho\gamma} = 97\text{ keV},\quad \Gamma_{\eta'\to\omega\gamma} = 9{,}8\text{ keV} \quad (3)$$

The experimental values are consistent with these estimates

$$\Gamma^{exp.}_{\rho^\circ\to\eta\gamma} = (55\pm14)\text{ keV}^{[8]},\quad \Gamma^{exp.}_{\omega\to\eta\gamma} = (3^{+2,5}_{-1,6})\text{ keV}^{[8]},$$
$$\Gamma^{exp.}_{\eta'\to\rho\gamma} = (83{,}4\pm4{,}5)\text{ keV}^{[8]},\quad \Gamma^{exp.}_{\eta'\to\omega\gamma} = (7{,}56\pm1{,}4)\text{ keV}^{[9]}. \quad (4)$$

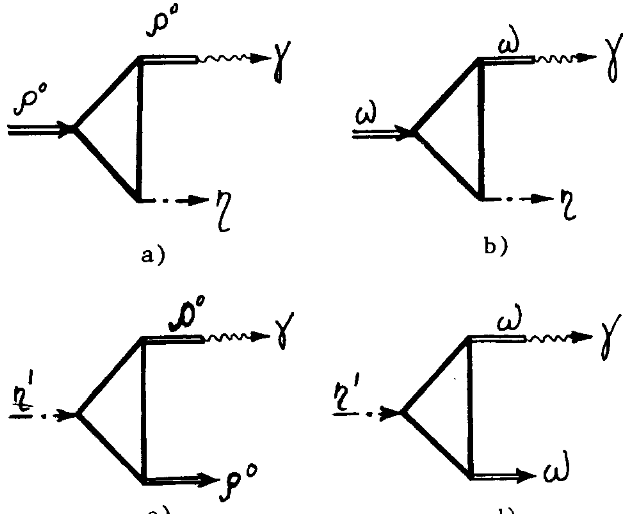

Fig.1

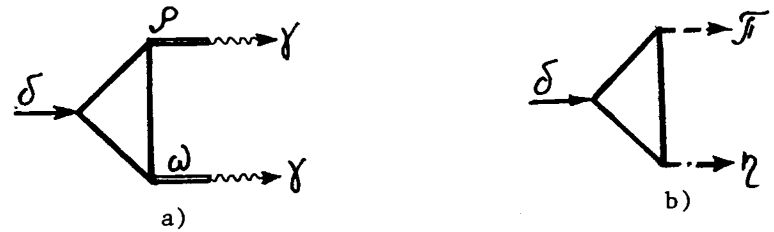

Fig.2

Let us now describe the δ-meson decays. The $\delta\to2\gamma$ process is also described by the finite triangle diagram which is not anomalous but of the usual type (fig.2a). Such diagrams were calculated in paper[10]. For the amplitude of the $\delta\to2\gamma$ decay we get the expression

$$T_{\delta\to2\gamma} = \frac{2\alpha}{3\pi F}(g^{\mu\nu}q_1q_2 - q_1^\nu q_2^\mu)\,\epsilon_1^\mu\,\epsilon_2^\nu\,, \quad (5)$$

where $\alpha = \frac{e^2}{4\pi}$, q_1, q_2 and ϵ_1, ϵ_2 are the momenta and photon polarizations. The width of the $\delta\to2\gamma$ process equals

$$\Gamma_{\delta\to2\gamma} = \frac{m_\delta}{\pi}\left(\frac{\alpha m_\delta}{12\pi F}\right)^2 = 1.3\text{ keV}. \quad (6)$$

The $\delta\to\eta\pi$ process differs from the previous decays as the corresponding triangle diagram is defined by the logarithmically divergent integral (fig.2b). In paper[1] this integral denoted by I_2 was estimated from the width of $\rho\to2\pi$ process

$$I_2 = -i\frac{3}{(2\pi)^4}\int\frac{d^4k}{(k^2-m_q^2)^2} = \frac{1}{4g^2}.$$

As a result, the amplitude of $\delta^\circ\to\eta\pi^\circ$ process can be written in the form

$$T_{\delta\to\eta\pi} = \frac{4}{\sqrt{3}}Fg^2(\cos\theta - \sqrt{2}\sin\theta). \quad (7)$$

We use the Goldberger-Treiman identity here

$$g = \frac{m_q}{F_\pi}, \quad (8)$$

where m_q = 240 MeV is the effective quark mass. Then for the width of $\delta\to\eta\pi$ process we get

$$\Gamma^{(\theta=-11^\circ)}_{\delta\to\eta\pi} = \frac{(g^2F)^2}{3\pi m_\delta^3}(\cos\theta - \sqrt{2}\sin\theta)^2\sqrt{[m_\delta^2-(m_\eta+m_{\pi^\circ})^2][m_\delta^2-(m_\eta-m_{\pi^\circ})^2]} = 40\text{ MeV}, \quad (9)$$

while the experimental value is [9]

$$\Gamma^{exp.}_{\delta \to \eta\pi} = (52 \pm 8) \text{ MeV} \qquad (10)$$

The agreement is quite good.

3. THE DECAY $\eta \to \pi^{o}\gamma\gamma$

So, we have all components necessary to describe the decay $\eta \to \pi^{o}\gamma\gamma$. This process takes place with the participation of three groups of diagrams mainly *.

The first group is the triangle diagrams with the participation of the intermediate ρ^{o} and ω mesons (fig.3a). The contribution of such diagrams was estimated in paper [11]. The second group of diagrams is the box diagrams (fig.3b). Finally, the third group is again the triangle diagrams but with the intermediate δ-meson (fig.3c). In the theory with chiral symmetry the last two groups of diagrams completely cancel out. The contribution from the first group remains only [4,11]. As a result, the calculated width is about three times as small as the experimental value. However, if we consider the σ-particle to be the real δ-meson, the decompensation of the last two groups occurs and the theoretical prediction for the width of the decay $\eta \to \pi^{o}\gamma\gamma$ is considerably improved. The contribution from the first group of diagrams to the amplitude of $\eta \to \pi^{o}\gamma\gamma$ process is (fig.3a)

$$T_{(\rho,\omega)} = \frac{\sqrt{3}\alpha\alpha_{\rho}}{(2\pi F)^{2}}(\cos\theta - \sqrt{2}\sin\theta)\left[\frac{1}{(p-q_1)^2 - m_{\rho}^2} + \frac{1}{(p-q_1)^2 - m_{\omega}^2}\right] \times$$
$$\times[(p^2 - pq_1)(g^{\mu\nu} q_1 q_2 - q_1^{\nu} q_2^{\mu}) + pq_1 p^{\nu} q_2^{\mu} + pq_2 p^{\mu} q_1^{\nu} - p^{\mu} p^{\nu} q_1 q_2 -$$
$$- g^{\mu\nu}(pq_1)(pq_2)]\epsilon_1^{\mu}\epsilon_2^{\nu} + (q_1 \leftrightarrow q_2). \qquad (11)$$

Here p, q_1 and q_2 are the momenta of η-meson and photons, ϵ_i -the photon polarization and $\alpha_{\rho} = \frac{g_{\rho}^2}{4\pi} = 3$. The second term in (11) follows from the first by interchanging the photon momenta. This part of the amplitude gives the following contribution to the width of the decay $\eta \to \pi^{o}\gamma\gamma$ [11].

$$\Gamma_{(\rho,\omega)} = 0{,}36 \text{ eV}. \qquad (12)$$

* The fourth group of diagramms is connected with pion loops. It was studied in paper [5], where it was shown that the contribution of such diagrams to the width of the decay $\eta \to \pi^{o}\gamma\gamma$ is very small.

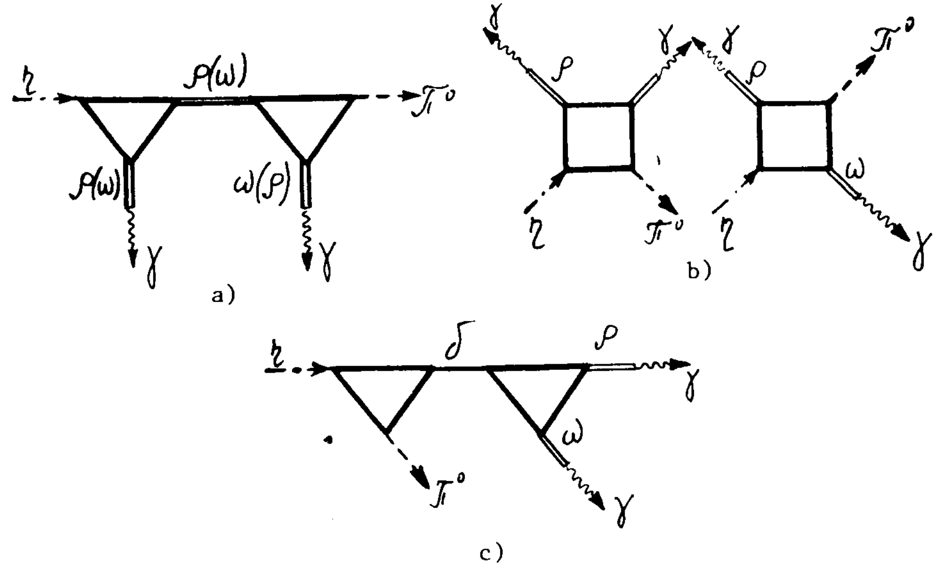

Fig.3

The contribution to the amplitude from the box diagrams (fig.3b) is

$$T_{(\Box)} = \frac{2\alpha}{3\sqrt{3}\pi F^2}(\cos\theta - \sqrt{2}\sin\theta)(q_1^{\nu} q_2^{\mu} - g^{\mu\nu} q_1 q_2)\epsilon_1^{\mu}\epsilon_2^{\nu}. \qquad (13)$$

And, finally, the contribution from triangle diagrams with the intermediate δ meson (fig.3c) is

$$T_{(\delta)} = -\frac{8\alpha}{3\sqrt{3}\pi}\cdot\frac{g^2}{(m_{\delta}^2 - 2q_1 q_2)}(q_1^{\nu} q_2^{\mu} - g^{\mu\nu} q_1 q_2)\epsilon_1^{\mu}\epsilon_2^{\nu}. \qquad (14)$$

Using the Goldberger-Treiman identity, the combined contribution to the amplitude from the second and third groups of diagrams (13) and (14) can be rewritten in the form

$$T_{(\Box)} + T_{(\delta)} = \frac{2\alpha(\cos\theta - \sqrt{2}\sin\theta)}{3\sqrt{3}\pi F^2}\left[1 - \frac{4m_q^2}{m_{\delta}^2 - 2q_1 q_2}\right](q_1^{\nu} q_2^{\mu} - g^{\mu\nu} q_1 q_2)\epsilon_1^{\mu}\epsilon_2^{\nu}. \qquad (15)$$

It is easy to see from the obtained expression that in the case of nonlinear chiral theory the contribution from the last two groups of diagrams to the amplitude vanishes. Really, in the linear σ-model, obtained in [1] in the one-loop approximation with quark loops, the σ-particle plays the role of the δ-resonance. For it the following mass relation takes place

$$m_\sigma^2 = m_\pi^2 + 4m_q^2 . \tag{16}$$

Then the bracket in (15) becomes

$$[1 - \frac{4m_q^2}{4m_q^2 + m_\pi^2 - 2q_1q_2}] . \tag{17}$$

In order to deduce the nonlinear chiral model from the σ-model, it is necessary to make the mass of the σ-particle to tend to infinity. It is seen from (16) that this limit is achieved by turning the effective quark mass to infinity. But in the limit $m_q \to \infty$ the bracket (17) vanishes and the last two groups of diagrams do not contribute to the amplitude.

In the more realistic physical case, when the σ-meson is used instead of the δ-particle the amplitude (15) gives the following contribution to the width of the decay

$$\Gamma_{(\Box+\delta)} = \frac{3[a(\cos\theta-\sqrt{2}\sin\theta)]^2}{4\pi m_\eta (8\pi F)^4} \int_{m_\pi}^{\frac{m_\eta^2+m_\pi^2}{2m_\eta}} d\omega \sqrt{\omega^2-m_\pi^2}\,(m_\eta^2+m_\pi^2-2m_\eta\omega)^2 \times \tag{18}$$

$$\times[1-\frac{4m_q^2}{m_\delta^2-m_\eta^2-m_\pi^2+2m_\eta\omega}] = 0.27\,\text{eV}.$$

The interference of the first group of diagrams with the two other groups gives further considerably contribution to the width of the studied process

$$T_{(\rho,\omega)}(T_{(\Box)}+T_{(\delta)}) = \frac{2a^2a_\rho(\cos\theta-\sqrt{2}\sin\theta)^2}{3\pi^3F^4}(q_1q_2)^2\,[1-\frac{4m_q^2}{m_\delta^2-2q_1q_2}][\frac{m_\rho^2}{m_\rho^2-(p-q_1)^2}-1]. \tag{19}$$

Hence it follows

$$\Gamma_{(int)} = \frac{a_\rho[a(\cos\theta-\sqrt{2}\sin\theta)]^2}{12(2\pi)^8F^4m_\eta}(J_1-J_2), \tag{20}$$

where

$$J_1 = \frac{m_\rho^2}{m_\eta}\int_{m_\pi}^{\frac{m_\eta^2+m_\pi^2}{2m_\eta}} d\omega (m_\eta^2+m_\pi^2-2m_\eta\omega)^2[1-\frac{4m_q^2}{m_\delta^2-m_\eta^2-m_\pi^2+2m_\eta\omega}]\times$$

$$\times\ln\frac{m_\rho^2-m_\eta(\omega-\sqrt{\omega^2-m_\pi^2})}{m_\rho^2-m_\eta(\omega+\sqrt{\omega^2-m_\pi^2})} = 4m_\rho^2m_\pi^3m_\eta\cdot 0.084, \tag{21}$$

$$J_2 = 2\int_{m_\pi}^{\frac{m_\eta^2+m_\pi^2}{2m_\eta}} d\omega\sqrt{\omega^2-m_\pi^2}\,(m_\eta^2+m_\pi^2-2m_\eta\omega)^2\,[1-\frac{4m_q^2}{m_\delta^2-m_\eta^2-m_\pi^2+2m_\eta\omega}] = 8m_\pi^4m_\eta^2\cdot 0.028.$$

Substituting the obtained values for J_1 and J_2 into (20) we get

$$\Gamma_{int} = 0{,}35 \text{ eV}. \tag{22}$$

Summing (12), (18) and (22) we come to the final value for the total width of the process

$$\Gamma_{\eta\to\pi^o\gamma\gamma} = 0{,}98 \text{ eV}. \tag{23}$$

This estimate is in good agreement with experimental data, obtained recently /3/

$$\Gamma^{exp.}_{\eta\to\pi^o\gamma\gamma} = (0{,}81\pm0{,}22) \text{ eV}. \tag{24}$$

4. CONCLUSION

The calculation performed shows that in the σ-model, which emerges in the consideration of four-quark interactions /1,2/, σ-particles should not be treated as fictitious particles which must be eliminated from the final variant of the theory (for instance, by turning to the nonlinear chiral theory). They should be identified with well-known scalar resonances $\epsilon(0^+)0^{++}$ and $\delta(1^-)0^{++}$. The breakdown occurring here of mass formulae following from the chiral symmetry of theory ($m_\sigma^2 = m_\pi^2 + 4m_q^2$) should be considered as a breakdown of this symmetry, which does exists in Nature.

So we arrive at the theory, describing only really existing particles. The theory is completely renormalizable and gives results in a good agreement with many experimental data, in particular, for the decay $\eta\to\pi^o\gamma\gamma$ which is difficult to explain with the previous positions.

In subsequent publications it will be shown how the decays of scalar, pseudoscalar, vector, pseudo-vector, and tensor mesons are explained by our model.

In conclusion authors express their gratitude to Ivanov A.N. for useful discussions. In his paper with Troitskaya N.I. the decay $\eta\to\pi^o\gamma\gamma$ is also described in some other model /6,12/. Their result differs from our result ($(\Gamma_{\eta\to\pi^o\gamma\gamma}$ =0.7 eV).

REFERENCES

1. Volkov M.K., Ebert D. JINR, P2-81-836, Dubna, 1981. Ebert D., Volkov M.K. Berlin-Zeuthen Preprint, PHE 82-3, 1982.
2. Eguchi T. Phys.Rev., 1976, 14D, p. 2755.
3. Binon F. et al. Yad.Fiz., 1981, 33, p. 1534.
4. Volkov M.K., Kreopalov D.V. JINR, P4-81-697, Dubna, 1981.
5. Volkov M.K., Ebert D. Yad.Fiz., 1979, 30, p. 1420.
6. Ivanov A.N., Shehter V.M. Yad.Fiz., 1980, 31, p. 530.
7. Appel V.D. et al. Yad.Fiz., 1979, 30, p. 366.
8. Andrews D.E. et al. Phys.Rev.Lett., 1977, 38, p. 198.
9. Particle Data Group. Rev.Mod.Phys., 1980, 52, N2.
10. Volkov M.K., Pervushin V.I. Yad.Fiz., 1975, 22, p. 346.
11. Oppo G., Oneda S. Phys.Rev., 1967, 160, p. 1397.
12. Ivanov A.N., Troitskaya N.I. Yad.Fiz., 1982, 36, N2.

Received by Publishing Department
on May 17 1982.

Креопалов Д.В., Волков М.К. Распад $\eta \to \pi^{\circ}\gamma\gamma$ в модели мезонов с кварковыми петлями E4-82-355

С помощью модели, основанной на рассмотрении четырехкварковых взаимодействий, описываются распады мезонов, идущие через кварковые петли. Вычислены распады $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, $\eta' \to \rho\gamma$, $\eta' \to \omega\gamma$, $\delta \to \gamma\gamma$, $\delta \to \eta\pi$ и $\eta \to \pi^{\circ}\gamma\gamma$. Предполагается, что роль фиктивных σ-частиц в киральной σ-модели играют физические резонансы ϵ и δ. Предсказание для ширины распада $\eta \to \pi^{\circ}\gamma\gamma$ хорошо соответствует последним экспериментальным данным.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Kreopalov D.V., Volkov M.K. The Decay $\eta \to \pi^{\circ}\gamma\gamma$ in the Model of Mesons with Quark Loops E4-82-355

Decays of mesons, proceeding through the quark loop are described within the model based on the consideration of four-quark interactions. The decays $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, $\eta' \to \rho\gamma$, $\eta' \to \omega\gamma$, $\delta \to \gamma\gamma$, $\delta \to \eta\pi$, $\eta \to \pi^{\circ}\gamma\gamma$ are calculated. It is supposed, that the role of fictitious σ-particles is played by the physical ϵ and δ resonances. The prediction for the width of the decay $\eta \to \pi^{\circ}\gamma\gamma$ is in good agreement with recent experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1982