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ON THE THEORY OF COUPLED πNN-NN SYSTEMS

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1. INTRODUCTION

The problem of the relativistic generalization of the Faddeev equations emerged immediately after the appearance of the method of the integral three-body equations [1]. The general form of such equations and references to many original papers can be found, e.g., in refs. [2,3]. However, a detailed analysis of the relativistic three-body equations including the effects of particle absorption and emission has only recently been initiated [3,10] in connection with the studies of pion scattering on a deuteron at low and medium energies, i.e., in the case where the kinetic energy of the incident pion is below the production threshold of the second pion. At present this problem is intensively investigated both theoretically and experimentally since it can shed light on many aspects of pion-nuclear, pion-nucleon and nucleon-nucleon interactions [3,11].

The first attempt to take into account the effect of true pion absorption in the elastic collision problem based on the nonrelativistic three-body equation was made in ref. [4], by using the model of the bound πN state in the P_{11} wave. The consistent formulation of the scattering problem including pion absorption on the basis of the three-body equations was given in refs. [5,6], in which the effective interaction Hamiltonians of the πNN system, with an arbitrary number of particles in intermediate states, were constructed using the method of Feshbach-Okubo projection operators [12] and the pr lem of overcounting of pions in similar equations has been solved. Analogous three-body equations constructed with no relativistic reduction techniques and Taylor's graphical method [13] have been obtained in ref. [7] and employed to cal culate the πd scattering reaction in the (3.3) resonance region. In ref. [8], in constructing the three-body equations to describe the πd scattering processes, the coupling of the πNN channel to the NN and $\pi\pi NN$ channels was explicitly taken into account on the basis of the pion-nucleon Hamiltonian used in the Chew-Low theory and the role of disconnected diagrams that arise from including the three-body forces in equations for the πNN system has been explored. The most general and convenient form of the relativistic three-body equations for the πNN system with particle absorption and production was derived, on the basis of generalized many-body Bethe-Salpeter equations of the relativistic field theory[13], in papers [9,10]. In these equations, in addition to including the true pion absorption, there have been obtained the coupled sets of equations for the amplitudes of the NN–πNN and πd–πNN channels and all the diagrams necessary for preserving two-body and three-body unitarity. In addition, in ref. [9b] it has been demonstrated that such equations are equivalent to the equations given in ref. [9a], which were obtained using the method of projection operators of Feshbach-Okubo. We note that all the above three-body equations were solved numerically by using the separable or isobar models of two-particle interactions, which does not allow one to separate the contributions coming from the pole and non-singular parts of the two-particle t-matrices to the channel amplitudes of interest. The problem of the separate inclusion of the pole and non-singular terms of the two-body t-matrices or the appropriate Green functions incorporated in the non-relativistic three-body equations was investigated in refs. [14,15]. Such studies, however, are very complicated because of the necessity to solve integral equations with two variables. The relativistic quantum field theory offers a different possibility of singling out the main singularities of the three-body equations. This possibility consists of describing, through the single-particle local field, a composite particle (deuteron or Δ isobar in our case), as well as other "elementary" particles (nucleons and a pion). As a result of such a treatment of the deuteron and the Δ isobar, the term describing the intermediate transition into the single-particle deuteron, Δ-isobar or nucleon state, i.e., the pole term of Green's two-body function, is separated from the nucleon-nucleon and pion-nucleon complete Green function. As will be demonstrated below, this separation of single-particle intermediate states provides the possibility of calculating individually contributions coming from the pole and non-pole terms of the two-body Green function into the amplitudes of the πd and NN scattering processes in question.

The present paper consists of two parts. The first one deals with the formal derivation of relativistic two-body and three-body equations. The potentials of these equations are written down in a form convenient for further use and the renormalization of multiparticle propagators has been carried out. In the second part, the three-body equations derived are applied to the problem of the πNN interaction with pion absorption and emission included. The final equations take into

account the three-body forces completely; in addition, the possibility of incorporating the singular terms of three-body potentials [8] is proposed which does not require an increase in the dimensionality of the integral equations. In other words, in contrast to ref. [8], there is no need in introducing auxiliary and non-observable amplitudes for the purpose of regularizing the iteration series of the set of the three-body equations. In the set of equations derived a certain approximation leads, in the (3.3) resonance region, to singling out the subsystem of relativistic two-body equations, which permits the calculation in a unified manner of characteristics of the πd, NN and NΔ interactions by using the vertex functions of the NN–d, Nπ–Δ and Nπ–N particles. A method of constructing the vertex function of the NN-d or Δ–Nπ particles is suggested. This method is based on the previously obtained formal relations between the one-body and two-body Green functions.

2. GENERAL RELATIONS

We shall proceed from the existence of the relativistic many-body Green function Γ_{mn} involving n particles in the initial state and m particles in the final state. In the relativistic quantum field theory one can take for such functions the following expression

$$\Gamma_{mn} = \langle 0|T(\psi^{+}_{\alpha_1}(x_1)\psi^{+}_{\alpha_2}(x_2)\dots\psi^{+}_{\alpha_m}(x_m)\psi_{\beta_1}(y_1)\dots\psi_{\beta_n}(y_n))|0\rangle, \quad (1)$$

where T denotes the time ordering of the $\psi_{\alpha_i}(x_i)$ and $\psi_{\alpha_j}(x_j)$ single-particle local fields of the i-th and j-th dressed particles $(i = 1,2,\dots,m,\ j = 1,2,\dots,n)$ and $|0\rangle$ is the state vector of the physical vacuum.

Furthermore, taking into account the fact that the functions Γ_{mn} describe all possible processes $n \to m$ with infinite multiplicity of k-particle intermediate states, we present this function as a sum of the same Green functions over all k-particle intermediate states, i.e.,

$$\Gamma_{mn} = \delta_{mn} G_n + G_m \sum_k \mathcal{H}_{mk} \Gamma_{kn}, \quad (2a)$$

$$= \delta_{mn} G_n + \sum_k \Gamma_{mk} \mathcal{H}_{kn} G_n, \quad (2b)$$

where $\delta_{mn} = \begin{cases} \hat{1} & m=n \\ 0 & m \neq n \end{cases}$ is the known Kronecker symbol, G_m is Green's function for m free particles, which can be defined as the product of the Feynman propagators $\Delta_F(i)$ of particles $i = 1,\dots,m$.

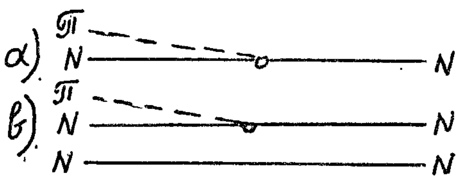

Fig.1. The simplest vertex functions $\mathcal{H}_{21}$ (a) and $\mathcal{H}_{32}$ (b) in pion-nucleon interactions.

From relations (2a,b) it is seen that $\mathcal{H}_{mk}$ describes the particle interactions preset by the original Hamiltonians or Lagrangians and leading to the transition k→m. In terms of the graphical diagram method of Feynman $\mathcal{H}_{km}$ is describable by simplest vertex diagrams, of both connected and disconnected type, with k-incoming and m-outcoming lines. For example, Fig.1a shows the vertex diagram $\pi N\text{-}N(\mathcal{H}_{21})$, whereas the simplest disconnected vertex diagram $\pi NN\text{-}NN(\mathcal{H}_{32})$ is given in Fig.1b. In both cases the vertex functions are determined from the original Hamiltonians of the πN interactions, for example, for $\mathcal{H}_{21}=f\gamma_5\tau$ the Hamiltonian $H_I = f\bar{\psi}\gamma_5\tau\psi\phi$. If we incorporate other reducible diagrams into $\mathcal{H}_{mk}$, then, according to relations (2a,b), these diagrams will arise repeatedly in calculating the Green function τ_{mn} and this is eliminated from the beginning.

Equation (2a) can be presented in a more compact form if one explicitly singles out one-particle and two-particle states and includes the remaining many-particle states in the "effective potentials" W^2_{mn}, i.e.,

$$\tau_{22'} = \delta_{22'} G_2 + G_2 W^2_{21}\tau_{12'} + G_2 W^2_{22''}\tau_{2''2}, \tag{3a}$$

$$\tau_{12'} = G_1 W^2_{11'}\tau_{1'2'} + G_1 W^2_{12}\tau_{22'}, \tag{3b}$$

$$\tau_{21} = G_2 W^2_{21'}\tau_{1'1} + G_2 W^2_{22'}\tau_{2'1}, \tag{4a}$$

$$\tau_{1'1} = \delta_{1'1} G_1 + G_{1'} W^2_{1'1''}\tau_{1''1} + G_{1'} W^2_{1'2}\tau_{21}, \tag{4b}$$

where primes at subscripts 1 and 2 denote different one-particle and two-particle states, and here and below is supposed a summation of all corresponding quantim numbers and variables by identical indices of two neighbouring expressions; the W^2_{mn} quantities are given by the following expressions

$$W^2_{mn} = \mathcal{H}_{mn} + \sum_{k>2}\sum_{\ell>2}\mathcal{H}_{mk}\,[(\Delta^2)^{-1}]_{k\ell}\,\mathcal{H}_{\ell n}, \qquad m,n = 1,2 \tag{5a}$$

$$(\Delta^2)_{k\ell} = \delta_{k\ell}\, G_\ell^{-1} - \mathcal{H}_{k\ell}, \qquad k,\ell = 3,4,\ldots \tag{5b}$$

The superscript over the W^2_{mn} effective potentials means that the Feynman diagrams describing these functions in intermediate states include more than two particles, that is they can be regarded as two-body irreducible diagrams.

In calculating Green's functions τ_{mn} it is more convenient to use, instead of Green's functions G_n for n non-interacting particles, the renormalized propagators $\mathcal{G}_n$, which can be expressed as follows

$$\mathcal{G}_n = \prod_{i=1}^{n}\Delta'_F(i) = G_n + G_n F_n \mathcal{G}_n, \tag{6a}$$

where

$$\Delta'_F(i) = \langle 0|T(\psi^+_{a_i}(x_i)\psi_{a_i}(y_i))|0\rangle \equiv \mathcal{G}_1(i) \equiv \tau_{11}(i) \tag{6b}$$

is the Feynman propagator of the i-th dressed particle, $F_n = \prod_{i=1}^{n} f_1(i)$, $f_1(i) = [\Delta'_F(i)]^{-1} - [\Delta_F(i)]^{-1}$ is the self-energy part of the propagator, well known in the quantum field theory.

Clearly this substitution of the propagators will lead to changes in the definitions of function $\mathcal{H}_{mk}$ since we have included part of the reducible diagrams in the $\mathcal{G}_m$ functions. We impose the condition that the forms of eqs. (2a,b) and the function should not change, i.e.,

$$\tau_{mn} = \delta_{mn}\mathcal{G}_n + \sum_k \mathcal{G}_m h_{mk}\tau_{kn}. \tag{7}$$

Comparing relations (7) and (2a) and using formula (6b) one can easily see that $\mathcal{H}_{mn} = h_{mn} + \delta_{mn}F_n$. In other words, if we succeed in calculating the self-energy part of the propagators $f_1(i)$ we shall be able to construct function h_{mk} in relations (7).

Then, from relations (7) it is possible to derive sets of equations for Green's functions τ_{mn} which would be similar to eqs. (3a,b), (4a,b) and include renormalized propagators $\mathcal{G}_n$. In place of W^2_{mn}, we shall have new functions w^2_{mn}, which are defined by the same formulas (5a,b), where $\mathcal{H}_{mn}$ and G_n are substituted by functions h_{mn} and $\mathcal{G}_n$. In this case eqs. (4a,b) will assume the following form

$$\tau_{21} = \mathcal{G}_2 w^2_{21}\mathcal{G}_1 + \mathcal{G}_2 w^2_{22'}\tau_{2'1}, \tag{8a}$$

$$0 = \mathcal{G}_1 w^2_{11'}\mathcal{G}_1 + \mathcal{G}_1 w^2_{12}\tau_{21'}. \tag{8b}$$

The condition (8b) is the consequence of our choice of $\tau_{11} \equiv \mathcal{G}_1$ as initial "free" propagators in relations (7). It means that we have already included in the propagator all possible off-shell corrections incorporated in the self-energy part f_1. Therefore, it is legitimate to assume all the combinations of kind (8b) or $\mathcal{G}_1 f_1 \mathcal{G}_1$ to be equal to zero although, clearly, $f_1 \neq 0$. We shall see below that the equalities (8a,b) will prove useful for the further analysis of the three-body equations. In particular, condition (8b) will enable us to write in a more compact form the equations for the πNN system with the three-body forces included.

In order to derive a set of equations for three interacting particles we single out in relations (7) the one-, two- and three-particle states from those involving a large number of particles by including the latter into the effective potentials w^3_{mn} (m, n = 1, 2, 3) which will be presented by the sum of all three-body irreducible Feynman diagrams. For the τ_{mn} (m, n = 1, 2, 3) Green functions of interest we have

$$\tau_{m2} = \delta_{m2}\mathcal{G}_2 + \mathcal{G}_m \sum_k w^3_{mk}\tau_{k2}, \tag{9}$$

$$\tau_{m3} = \delta_{m3}\mathcal{G}_3 + \mathcal{G}_m \sum_k w^3_{mk}\tau_{k3} \tag{10}$$

where

$$w^3_{mn} = h_{mn} + \sum_{k>3}\sum_{\ell>3} h_{mk}[(\Delta^3)^{-1}]_{k\ell}\, h_{\ell n}, \tag{11a}$$

$$(\Delta^3)_{k\ell} = \delta_{k\ell}\,\mathcal{G}^{-1}_\ell - h_{k\ell}, \qquad \ell, k = 4, 5 \ldots . \tag{11b}$$

The set of eqs. (9) and (10) describes the three-particle interaction with particle absorption and emission included in the framework of the quantum field theory. As far as we know, such equations were first derived in ref. [18] on the basis of the analysis of the Feynman diagrams. Subsequently many authors, in different formulations and approximations (refs [2,19,20]), obtained the three-body equations including particle absorption and emission. The most common methods for deriving such equations are that involving the analysis of the Feynman diagrams by using the first (or last) cut lemma [13] and the method of constructing the effective Hamiltonian by the Feshbach-Okubo projection operators [12]. In refs. [5,6,9,10] these methods have just been used to obtain the three-body equations for the πNN system of particles. One can easily see that if we take for Green's functions τ_{mn} the two-time (quasipotential) Green function [21] by assuming $\tilde{\tau}_{mn} = \tau_{mn}\,\big|_{x_{0i}=t_1,\ y_{0j}=t_2}$ or Green's function τ_{mn} constructed with the effective Hamiltonians for interaction of three particles, as used in refs. [5,6,9a], or the functions τ_{mn} from refs. [7,9,10,21] then, despite the fact that we shall have different prescriptions for constructing the functions h_{mn}, $\mathcal{G}_m$ and w^3_{mn}, the final structure of the two-body and three-body equations (3), (4), (9) and (10) does not change. For concreteness, we recall that in the quantum field theory Green's functions τ_{mn} and $\mathcal{G}_m$ are derived by formulas (1) and (6a).

In the present paper we shall use the quantum-mechanical description of a composite particle through the one-particle local field. For example, according to refs. [16,17], one can introduce for deuteron the annihilation operator $a_f(P_d, \alpha_d, t)$ which, at $t \to \pm\infty$, behaves exactly in the same manner as the annihilation operator for one particle in asymptotic states and is constructed by means of the deuteron wave function and the time ordering of one-nucleon local fields. In this treatment, in describing the πNN system there will arise Green's functions τ_{31} and τ_{13} as the functions of the intermediate transition to the one-particle deuteron state, whereas the πd and NΔ scattering processes will be regarded as the 2-2 process, rather than 3-3, as was accepted in refs. [3-10].

3. EQUATIONS FOR COUPLED πNN, πd, NN, NΔ SYSTEMS

We apply the three-body equations (9) and (10) to describe the processes occurring in the πNN system of particles. According to the generally accepted formulation of composite particles, at low and medium energies ($T_\pi < 300$ MeV) the following two-particle states $2 = \{\pi d, NN, N\Delta\}$ and only one three-particle state $3 = \pi NN$ will explicitly occur in the initial and final states. Below the two-particle states indicated will be denoted by Greek letters $\eta, \nu, \sigma = 1,2,3 = \{\pi d, NN, N\Delta\}$; the three-particle state is denoted by 0 and finally the possible single-particle state of the deuteron will be denoted by letter d. To simplify the further calculation we shall below use the renormalized $\mathcal{G}_n$, w^3_{mn} functions and assume that all the h_{mn} functions entering into eqs. (7) and (11a,b) are equal to zero if the particle number m differs from n by more than unity, i.e.,

$$h_{mn} = 0 \quad \text{if} \quad |m - n| > 1. \tag{12}$$

This assumption, according to the definition of functions w^k_{mn}, gives

$$w^3_{11'} = w^2_{11'} = 0 \qquad w^2_{12} = w^3_{12} = h_{12}$$
$$w^2_{21} = w^3_{31} = h_{21} \qquad w^3_{13} = w^3_{31} = 0. \tag{13}$$

After that, the sets of eqs. (9) and (10) for the processes of the πNN system of interest can be written down as follows

$$\tau_{00'} = \delta_{00'}\mathcal{G}_0 + \mathcal{G}_0 K_{00''}\tau_{0''0''} , \quad (14)$$

$$\tau_{\eta\nu} = \mathcal{G}_\eta \delta_{\eta\nu} + \mathcal{G}_\eta \sum_\sigma K_{\eta\sigma}\tau_{\sigma\nu} + \mathcal{G}_\eta w^3_{\eta 0}\tau_{0\nu} = G_{\eta\nu} + \sum_\sigma G_{\eta\sigma} w^3_{\sigma 0}\tau_{0\nu} , \quad (15a)$$

$$\tau_{0\nu} = \mathcal{G}_0 \sum_\eta w^3_{0\eta}\tau_{\eta\nu} + \mathcal{G}_0 w^3_{00'}\tau_{0'\nu} , \quad (15b)$$

where $\mathcal{G}_0 \equiv \mathcal{G}_{\pi NN}, \mathcal{G}_\nu$ are Green's functions for the non-interacting particles of the πNN and ν systems (πd, NN, NΔ),

$$K_{\eta\nu} = \omega^3_{\eta\nu} + h_{\eta d}\mathcal{G}_d h_{d\nu} , \quad (16a)$$

$$G_{\eta\nu} = \delta_{\eta\nu}\mathcal{G}_\nu + \mathcal{G}_\eta \sum_\sigma K_{\eta\sigma} G_{\sigma\nu} , \quad (16b)$$

$$K_{00'} = w^3_{00'} + \sum_{\eta\nu} w^3_{0\eta} G_{\eta\nu} w^3_{\nu 0'} \equiv w^3_{00'} + v_0 , \quad (17a)$$

$h_{\nu d}$ and $h_{d\nu}$ are the vertex functions of the ν pair of particles and deuteron; $\omega^3_{\eta\nu}$, $\omega^3_{\eta 0}$ and $\omega^3_{00'}$ are the potential functions which, according to (11a), have not less than four particles in intermediate states between the corresponding combinations of particles. As at the beginning we took into account only the π, N, Δ and d particles, by analogy with ref. /9/, it is possible to include into the w^3 potentials the exchange interactions involving any number of heavy $\rho, \omega, \ldots$ mesons, N$\bar{\text{N}}$ pairs, etc..

To derive the final form of the three-body relativistic equations with particle absorption and emission included, it is important to take into account the disconnected parts of the $w^3_{\eta 0'}$, $w^3_{0\eta}$ and $w^3_{00'}$ potentials so that the iteration series of similar equations should not contain the products of the two neighbouring disconnected diagrams of the same type. Let us denote the connected and disconnected parts of the w^3 potentials by w^C and w^D, respectively. From Figs.2,3 and 4 we see that if $w^D_{0\nu}$ and $w^D_{\nu 0}$ (see Fig.2) contain only one or two terms, then $w^D_{00'}$ incorporates not only the terms describing the nucleon-nucleon v_{NN} (Fig.3a) and pion-nucleon $v_{\pi N}$ (Fig.3b,c) interactions in the three-body πNN space, but also the disconnected parts (Fig.4a) v_{S1} and v_{S2} (Fig.4b) of the three-body interactions, considered in ref. /8/. Note that the v_{NN} and $v_{N\pi}$ potentials are describable by the sum of all possible two-particle irreducible diagrams, in the presence of

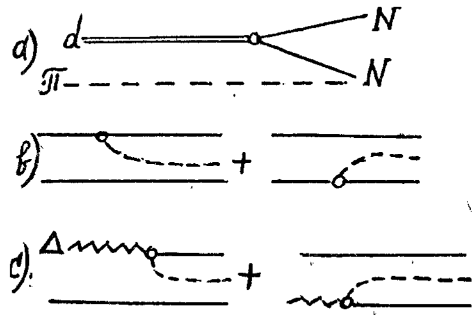

Fig.2. The disconnected diagrams of the $w^3_{\nu 0}$ potential.

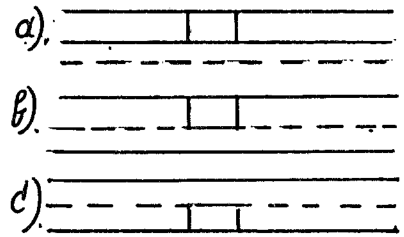

Fig.3. The disconnected parts of the $w^3_{00'}$ potential, corresponding to the NN and πN subsystems.

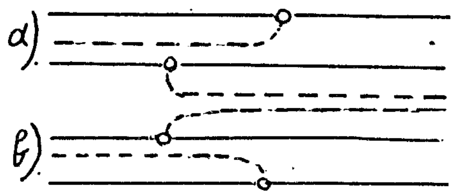

Fig.4. The disconnected three-body interactions entering into the $w^3_{00'}$ potential.

a third, non-interacting particle (see Fig.3), i.e., v_{NN} and $v_{\pi N}$ are the $w^2_{22'}$ -potentials (5a) in the three-body πNN space. For example, for the v_{NN} potential, if we express $w^2_{22'}$ (NN), by using a set of eqs. (9), through potentials w^3_{mn} (m, n = 1,2,3), we obtain

$$v_{NN} = w^2_{22'}(NN)[\Delta'_F(\pi)]^{-1} , \quad (18a)$$

$$= [w^3_{22'}(NN) + w^3_{NN,0}\, g^3_{00'}\, w^3_{0',NN}][\Delta'_F(\pi)]^{-1} , \quad (18b)$$

$$g^3_{00'} = \delta_{00'}\mathcal{G}_0 + \mathcal{G}_0 w^3_{00''} g^3_{0''0'} . \quad (19)$$

Let us consider the problem of including the three-body forces $w^C_{00'}$, v_{S1} and v_{S2}. To do so we introduce the following combinations of potentials

$$V_1 = v_{NN} + w^C_{00'} , \quad (20a)$$

$$V_2 = v_{\pi N_1} + v_{S1} , \quad (20b)$$

$$V_3 = v_{\pi N_2} + v_{S2} . \quad (20c)$$

After that formula (17a) can be rewritten in the following form

$$K_{00'} = \sum_{i=1}^{3} V_i + V_0 \,. \tag{17b}$$

Using the procedure of constructing the three-body equations for transition matrices[22] we obtain that in dividing the $K_{00'}$ potential into four parts according to eq. (17b) equations (14) will be equivalent to the following set of equations

$$O_{i0'} = \mathcal{G}_0^{-1} + \sum_{j=1}^{3} \bar{\delta}_{ij}\, t_j\, \mathcal{G}_0 O_{j0'} + t_0 \mathcal{G}_0 O_{00'} \,, \tag{21a}$$

$$O_{00'} = \sum_{j=1}^{3} t_j\, \mathcal{G}_0\, O_{j0'} \,, \tag{21b}$$

where $\bar{\delta}_{ij} = 1-\delta_{ij}$; and the O_{ab} transition matrices are connected with Green's functions $r_{00'}$ in the following way

$$r_{00'} = \delta_{ab}\, g_a + g_a O_{ab}\, g_b \,, \qquad a,b = 0,\ i = 0,1,2,3 \tag{22}$$

$$g_a = \mathcal{G}_0 + \mathcal{G}_0 V_a g_a = \mathcal{G}_0 + \mathcal{G}_0 t_a \mathcal{G}_0 \,. \tag{23}$$

In this case, according to the definition of the v_{NN}, $v_{\pi N}$ and V_j potentials, Green's function g_j contains only the non-pole part of the complete nucleon-nucleon (j=1) or pion-nucleon (j=2,3) Green's functions.

In fact, from eqs. (18a), (20a), and (23) for the g_1 function we have

$$g_1 = g^2_{22'}\,(NN)\Delta'_F(\pi) + g^2_{22'}\,(NN)\Delta'_F(\pi)\, w^C_{00'}\, g_1 \,, \tag{24}$$

where Green's function $g^2_{22'}$ is determined by means of the potential, i.e.,

$$g^2_{22'} = \delta_{22'}\, \mathcal{G}_2 + \mathcal{G}_2\, w^2_{22''}\, g^2_{2''2'} \tag{25}$$

and corresponds to the discontinuous part of the spectrum of complete two-body Green's function $r_{22'}$. This can be easily seen if eqs. (3a,b), (4a,b) and (8a,b) are used to present Green's function $r_{22'}$ in the following form

$$r_{22'} = \delta_{22'}\, \mathcal{G}_2 + \mathcal{G}_2\, h_{21}\, r_{12'} + \mathcal{G}_2\, w^2_{22''}\, r_{2''2'} \,, \tag{26a}$$

$$= g^2_{22'} + g^2_{22''}\, h_{2''1}\, r_{12'} \,, \tag{26b}$$

$$= g^2_{22'} + \chi_{21}\, \mathcal{G}_1\, \chi_{12} \,, \tag{26c}$$

where $\chi_{21} = r_{21} \frac{1}{\mathcal{G}_1}$. In a similar manner one can demonstrate that functions g_2 and g_3 contain the non-pole part $g^2_{22'}(\pi N)$ of complete pion-nucleon Green's function $r_{22'}(\pi N)$.

Now, combining eqs. (14) and (15a,b) one can express Green's functions $r_{\eta\nu}$ and $r_{0\nu}$ through function $r_{00'}$ and using relation (22) we obtain

$$r_{0\nu} = \sum_{\sigma} g_i\, O_{i0}\, g_0 w^3_{0\sigma}\, G_{\sigma\nu} \,, \tag{27a}$$

$$r_{\eta\nu} = G_{\eta\nu} + \sum_{\sigma\sigma'} G_{\eta\sigma} w^3_{\sigma 0}\, (g_0 \delta_{00'} + g_0 O_{00'} g_{0'}) w^3_{0'\sigma'}\, G_{\sigma'\nu} \,. \tag{27b}$$

After that, on the basis of equations (21a,b) and relations (27a,b) and using some algebraic transformations it is possible to deduce (see Appendix 1) the following equations for the $U_{\eta\nu}$ and $U_{i\nu}$ matrices of the transition from the ν two-particle state to the two-particle η or three-particle $0 \equiv \pi NN$ state, namely,

$$U_{i\nu} = w^3_{0\nu} + \sum_{\eta} w^3_{0\eta}\, \mathcal{G}_\eta\, U_{\eta\nu} + \sum_{j=1}^{3} \bar{\delta}_{ij}\, t_j\, \mathcal{G}_0 U_{j\nu} \,. \tag{28a}$$

$$U_{\eta\nu} = M_{\eta\nu} + \sum_{\sigma} M_{\eta\sigma} \mathcal{G}_\sigma U_{\sigma\nu} + w^3_{\eta 0}\, \mathcal{G}_0 \sum_{j=1}^{3} t_j\, \mathcal{G}_0 U_{j\nu} \,, \tag{28b}$$

where $U_{i\nu}$ and $U_{\eta\nu}$ transition matrices are connected with Green's functions $r_{0\nu}$ and $r_{\eta\nu}$ in the following way

$$r_{0\nu} = g_1 U_{i\nu}\, \mathcal{G}_\nu \,, \tag{29a}$$

$$r_{\eta\nu} = \delta_{\eta\nu}\, \mathcal{G}_\eta + \mathcal{G}_\eta\, U_{\eta\nu} \mathcal{G}_\nu \,. \tag{29b}$$

The potentials $M_{\eta\nu}$ entering into the set of equations (28a,b), in addition to $K_{\eta\nu}$ (16a), contain also the single-particle exchange interaction $Z_{\eta\nu}$ between the η and ν pairs of particles (see Fig.5) and the disconnected diagrams $[w^3_{\eta 0}\, \mathcal{G}_0 w^3_{0\nu}]^D$, which are shown in Fig.6, i.e.,

$$M_{\eta\nu} = K_{\eta\nu} + w^3_{\eta 0} \mathcal{G}_0\, w^3_{0\nu} \,, \tag{30a}$$

$$= K_{\eta\nu} + Z_{\eta\nu} + [w^3_{\eta 0}\, \mathcal{G}_0\, w^3_{0\nu}]^D \,. \tag{30b}$$

In Appendix 2 it is demonstrated that the disconnected parts $M_{\eta\nu}$ of the kernel of eq. (28a,b), as well as the v_{S1}, v_{S2} potentials entering into V_2, V_3 (20b,c), do not lead to the appearance of divergent terms in the iteration series of eq. (28a,b). In this case, essential are relations (8b) which, after taking into account conditions (12) and (13), will take the following form

$$\mathcal{G}_1 h_{12} \tau_{21'} = 0, \qquad (31a)$$

$$\mathcal{G}_1 h_{12} g^2_{22'} h_{2'1'} \mathcal{G}_{1'} = 0. \qquad (31b)$$

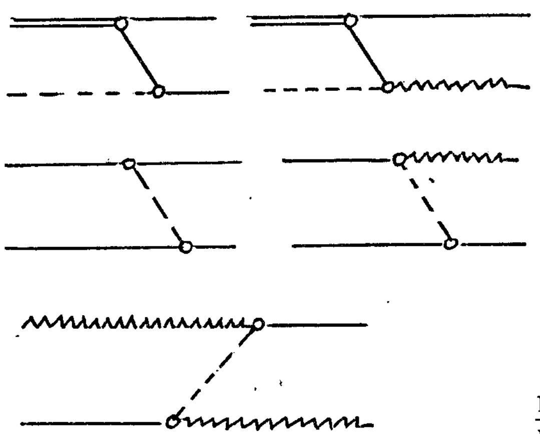

Fig.5. One-particle exchange potentials.

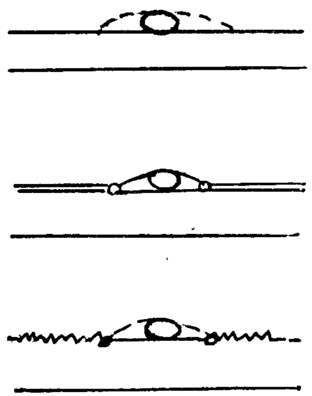

Fig.6. The disconnected parts of the $M_{\eta\nu}$ potential entering into $w^3_{\eta 0}\mathcal{G}_0 w^3_{0\nu}$ (30a,b).

By using relations (31a,b) one can demonstrate that all the diagrams leading to the renormalization of the already renormalized single-particle propagators cancel out in calculating the iteration series for the $U_{i\nu}$ and $U_{\eta\nu}$ transition matrices. Therefore, the disconnected diagrams entering into $M_{\eta\nu}$ (30a,b) and those occurring due to potentials v_{S1} and v_{S2} in the iteration series of equations (28a,b) can be omitted from the beginning.

The three-particle equations (28a,b) describing the πd, NN and NΔ scattering processes including pion absorption and emission fully take into account the contributions coming from the three-body forces $w^C_{00'}$, v_{S1}, v_{S2}, $w^C_{\eta 0}$ and $w^C_{0\eta}$. It is noteworthy that, in contrast to ref. [8] we did not need to introduce any auxiliary transition matrices for the purpose of including the disconnected potentials v_{S1} and v_{S2}. The main difference between equations (28a,b) and similar equations from refs. [9,10] lies in the fact that in deriving eq. (28a,b) the deuteron and Δ isobar were regarded as single-particle states. On the one hand, this leads to the appearance of new two-particle amplitudes and potentials of the interaction of the πd, NN and NΔ particles. In particular, instead of one 3-2 matrix of the πd-NN transition and three three-particle amplitudes 3-3 used in refs. [9,10] to describe the πd scattering processes, in the set of eqs. (28a,b) three two-particle amplitudes are present which describe the processes $\pi d \to \begin{matrix}\pi d\\ NN\\ N\Delta\end{matrix}$, and three $U_{i\nu}$ transition matrices each of which corresponds to a transition from the two-particle state ν to the three-particle state $\pi NN \equiv 0$. Moreover, in our case there have arisen additional potentials of interaction between the particle pair NΔ and the off-mass shell potential of the intermediate transition from the NN state to the deuteron single-particle state (see the second term of the potential (16a)). On the other hand, the treatment of the deuteron and the Δ isobar as single-particle states leads to the fact that in the (3.3) resonance region it becomes possible to make the following approximation. Omitting the three-body forces and neglecting the non-pole term of pion-nucleon Green's function, i.e., assuming $g_2 = g_3 = 0$, we obtain the following two-body equations

$$U_{\eta\nu} \approx U^{(1)}_{\eta\nu} = m_{\eta\nu} + \sum_\sigma m_{\eta\nu} \mathcal{G}_\sigma U^{(1)}_{\sigma\nu}, \qquad (32)$$

where

$$m_{\eta\nu} = M_{\eta\nu} + w^3_{\eta 0} \mathcal{G}_0 t_1 \mathcal{G}_0 w^3_{0\nu} \qquad (33a)$$

$$= w^3_{\eta\nu} + h_{\eta d} \mathcal{G}_1(d) h_{d\nu} + w^3_{\eta 0} g_1 w^3_{0\nu} \qquad (33b)$$

and Green's function g_1 connected with the t-matrix by relation (23) is defined by formula (24).

Potentials $M_{\eta\nu}$ and $m_{\eta\nu}$ differ from the complete two-particle potential, e.g., from NN potential ($\eta = \nu = 2$) in that in $M_{22'}(NN)$ the term $I_{22'}(NN) = w^3_{NN,0} \mathcal{G}_0 w^3_{00'} g^3_{00'} w^3_{0,NN}$ entering into the potential $w^2_{22'}(NN)$ (18a,b) is absent and in $m_{22'}(NN)$ this term is substituted by its part $w^3_{NN,0}\mathcal{G}_0 v_{NN} g\, w^3_{0,NN}$. We note that the microscopic calculation of the two-body nucleon-nucleon potentials $w^2_{22'}(NN)$ or v_{NN} (18a,b) is complicated because the total inclusion of all intermediate three-particle $0 \equiv \pi NN$ states in these potentials requires taking into account the term $I_{22'}(NN)$ which contains three-particle Green's function $g^3_{00'}$ (19). Function $g^3_{00'}$ in turn contains, through the potential v_{NN}, also the function $I_{22'}$(NN), i.e., the construction of the complete nucleon-nucleon potential with all three-particle πNN states included is a nonlinear problem. A similar conclusion concerning the $v_{\pi N}$ potentials can be arrived at if one takes into account the three-particle states $\pi\pi$N, this property being inherent in the corresponding v_{NN} and $v_{\pi N}$ potentials from refs. [5,6,7,9,10], which seems to be characteristic of the potentials constructed in the quantum field theory with an infinite number of degrees of freedom.

Equations (32) allow us to calculate in a unified manner the two-body processes of πd, NN and NΔ scattering on the basis of the simplest vertex functions N-Nπ, Δ-Nπ and d-NN. In calculating similar processes one can include also the heavy meson contribution by including the $w^3_{\eta\nu}$ potentials (16a) in the corresponding diagrams. Another difference of eqs. (28a,b) and (30) from similar three-body equations from refs. [9,10] lies in the fact that in deriving eqs. (28a,b) and (30) we did not use the separable model of the two-body interactions for the NN potentials and this enables us in calculating (30) to employ non-separable, microscopic potentials NN and NΔ, e.g., the NN and NΔ potentials constructed e.g., in the one-boson exchange model.

In the low-energy region it is known that the contribution from the Δ resonance is small and other partial waves of the πN interactions are significant. Therefore, one should not expect eqs. (30) to describe well the πd scattering processes. To include the non-resonance partial waves of the πN interactions in eqs. (28a,b) it is simplest to use the separable model of two-body interactions. In this case, if the two-term separable model [23] is employed for the resonance πN t-matrices and the common single-term separable model for the remaining t-matrices, then the final equations obtained from eqs. (28a,b) will coincide with analogous equations from refs. [9,10]. A difference will lie in the fact that it is not difficult to include the separable potentials v_{S1} and v_{S2} [8] by using eqs. (28a,b) and it is not necessary at all to use the separable model of NN and N interactions to construct the vertex functions d-NN and Δ-Nπ.

In the consistent microscopic calculation of kernels of integral equations (28a,b) or (30) it is necessary to set the simplest vertex functions h_{21} and h_{12} at least. In practice, the situation however is different. Namely, it is commonly believed that the potentials of two-body interactions are set (for example, by solving the inverse scattering problem or in the one-boson exchange model for the NN interactions, etc.).

and functions h_{21} and h_{12} should be found on the basis of these potentials. Such a problem is substantiated by the circumstance that the vertex functions d-NN cannot be defined like the vertex function N-πN from the known Lagrangians of nucleon-pion system interactions since the deuteron state is formed on the basis of the NN-interactions and the vertex functions d-NN should be constructed with the help of the given NN potential.

In order to construct the functions h_{21} through the two-body interaction potentials we make use of eq. (8a) which, taking into account conditions (12) and (13), can be presented in the following form

$$h_{21} = (\mathcal{G}_2^{-1}\delta_{22'} - w^2_{22'})\Gamma_{2'1}\frac{1}{\mathcal{G}_1} \qquad (34a)$$

or explicitly, for the NN vertex function in the momentum representation we have

$$h_{21}(p_1p_2;Q) = \int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\{\delta^4(p_1-q_1)(\hat{p}_1-m_N-\Sigma(p_1))\times \qquad (34b)$$

$$\times\delta^4(p_2-q_2)(\hat{p}_2-m_N-\Sigma(p_2)) - w^2_{22'}(p_1p_2;q_1q_2)\}\Gamma_{21}(q_1q_2;Q)\frac{1}{\mathcal{G}_1(Q)},$$

where $\mathcal{G}_1(Q)$ is the renormalized deuteron propagator; Σ is the nucleon mass operator; p, q and Q denote the four-momenta of individual nucleons and deuterons.

Then, bearing in mind the definition of the deuteron wave function in the quantum field theory, the deuteron function on mass shell $Q^2 = m_d^2$ and $p_1+p_2=Q$ can be presented in the following form

$$\delta^4(p_1+p_2-Q)\psi_Q(p_{12}) = \Gamma_{21}(p_1p_2;Q)\frac{1}{\mathcal{G}_1(Q)} = \chi_{21}(p_1p_2;Q). \qquad (35)$$

The two-body irreducible potential $\omega^2_{22'}$ in eqs. (34a,b) can be replaced by the complete two-particle potential $K_{22'} = w^2_{22'} + h_{21}\mathcal{G}_1 h_{12}$ if the condition (31a) is employed. Then it becomes clear that h_{21} is defined by the off-shell behaviour of the potential $w^2_{22'}$, and on the mass and energy surface, when $Q^2=m_d^2$ and $p_1+p_2=q_1+q_2=Q$, we have $h_{21}(p_1p_2;Q)=0$ since the right-hand part of eqs. (34a,b) in this case is the Bethe-Salpeter equation (or the Schrödinger equation, or a quasipotential two-body equation) for the deuteron wave function. Thus if we have the nucleon-nucleon potential $w^2_{22'}$ preset, eqs. (34a,b) permit the construction of the vertex function NN-d. In a similar manner one can construct the vertex functions h_{21} and h_{12} of the Δ-$N\pi$ or N-$N\pi$ particle systems through the corresponding potentials $w^2_{22'}$.

In conclusion the author expresses his deep gratitude to T.I.Kopaleishvili for permanent interest in the present work and numerous stimulating discussions. Thanks are also due to I.V.Amirkhanov, V.B.Belyaev, M.Kh.Khankhasayev and A.A.Khelashvili for discussions and I.L.Bogolubsky and L.V.Pashkevich for their help in preparing this manuscript.

APPENDIX 1

Let us follow the derivation of eqs. (28a,b) for the transition matrices O_{i0} and $O_{00'}$ from eqs. (21a,b) for the auxiliary matrices $\tau_{0\nu}$ and $\tau_{\eta\nu}$. Using formulas (27a,b) and (29a,b) defining the coupling between Green's functions $\tau_{0\nu}$ and $\tau_{\eta\nu}$, on the one hand, and the O_{i0}, $O_{0'0}$, $U_{j\nu}$ and $U_{\eta\nu}$ matrices, on the other, from eqs. (21a,b) we obtain

$$g_i U_{i\nu} \mathcal{G}_\nu = g_i \mathcal{G}_0^{-1} g_0 \sum_\sigma w^3_{0\sigma} G_{\sigma\nu} + g_i \{ \sum_{j=1}^{3} \bar{\delta}_{ij} V_j g_j U_{j\nu} \mathcal{G}_\nu + \sum_\sigma w^3_{0\sigma} (\mathcal{G}_\sigma U_{\sigma\nu} \mathcal{G}_\nu + \delta_{\sigma\nu} \mathcal{G}_\nu - \mathcal{G}_{\sigma\nu}), \tag{Ap1-1a}$$

$$\mathcal{G}_\eta U_{\eta\nu} \mathcal{G}_\nu + \delta_{\eta\nu} \mathcal{G}_\nu - \mathcal{G}_{\eta\nu} = \sum_\sigma G_{\nu\sigma} w^3_{\sigma 0} g_0 (\sum_{j=1}^{3} V_j g_j U_{j\nu} \mathcal{G}_\nu), \tag{Ap1-1b}$$

where we have introduced new Green's function

$$\mathcal{G}_{\eta\nu} = G_{\eta\nu} + \sum_{\sigma,\sigma'} G_{\eta\sigma} w^3_{\sigma 0} g_0 w^3_{0\sigma'} G_{\sigma'\nu} \tag{Ap1-2}$$

and used the known equalities following from the definition of functions V_a, g_a and t_a (17a,b), (20a,b) and (23), namely

$$V_i g_i = t_i \mathcal{G}_0 ; \quad V_0 g_0 = t_0 g_0 = \sum_{\eta,\nu} w^3_{0,\eta} G_{\eta\nu} w^3_{\nu,0} g_0 . \tag{Ap1-3}$$

For further calculations we write in a more explicit form the t_0 -matrix which will be sought in the following form

$$t_0 = \sum_{\eta,\nu} w^3_{0,\eta} x_{\eta\nu} w^3_{\nu,0} . \tag{Ap1-4}$$

Then, after using the explicit form of potential V_0 from equations for the t_0 -matrix we have

$$x_{\eta\nu} = G_{\eta\nu} + \sum_{\sigma,\sigma'} G_{\eta\sigma} (w^3_{\sigma,0} \mathcal{G}_0 w^3_{0,\sigma'}) x_{\sigma'\nu} , \tag{Ap1-5a}$$

$$x_{\eta\nu} = \delta_{\eta\nu} \mathcal{G}_\nu + \mathcal{G}_\eta \sum_\sigma M_{\eta\sigma} x_{\sigma\nu} , \tag{Ap1-5b}$$

where the explicit form of potential $M_{\eta\nu}$ is given by formulas (16a) and (30a,b). Then, combining eqs. (Ap1-5),(Ap1-4) and (Ap1-2) one can easily see that

$$x_{\eta\nu} = \mathcal{G}_{\eta\nu} , \tag{Ap1-6a}$$

$$\mathcal{G}_0^{-1} g_0 \sum_\sigma w^3_{0\sigma} G_{\sigma\nu} - \sum_\sigma w^3_{0\sigma} \mathcal{G}_{\sigma\nu} = 0 , \tag{Ap1-6b}$$

$$\sum_\sigma G_{\eta\sigma} w^3_{\sigma 0} g_0 = \sum_\sigma \mathcal{G}_{\eta\sigma} w^3_{\sigma 0} \mathcal{G}_0 . \tag{Ap1-6c}$$

Substituting relations (Ap1-6a,b,c) in eqs. (Ap1-1a,b) it is easily to obtain eqs. (28a,b) sought.

APPENDIX 2

We consider the disconnected diagrams describing the kernels of the set of integral equations (28a,b) and occurring in the calculation of the iteration series of these equations. In addition to the disconnected parts $w^3_{0\sigma}$ and t_1 functions shown in Figs.2a,b,c and 3a, the disconnected diagrams arise also in calculating the t_2, t_3, $M_{\eta\nu}$ and $w^3_{\nu 0} \mathcal{G}_0 t_j$ kernels of integral equations (28a,b). From defining the scattering t_2 -matrix (20b) and (23) we have

$$\{ t_2 \}^{\mathrm{Disc.}} = t_{\pi N_1} + (1 + t_{\pi N_1} \mathcal{G}_0) v_{S1} , \tag{Ap2-1a}$$

$$\{ w^3_{20} \mathcal{G}_0 t_2 \}^{\mathrm{Disc.}} = w^{D1}_{20} \mathcal{G}_0 \{ t_{\pi N_1} + (1 + t_{\pi N_1} \mathcal{G}_0) v_{S1} \} , \tag{Ap2-2a}$$

$$\{ w^3_{20} \mathcal{G}_0 t_3 \}^{\mathrm{Disc.}} = w^{D2}_{20} \mathcal{G}_0 \{ t_{\pi N_2} + (1 + t_{\pi N_2} \mathcal{G}_0) v_{S2} \} , \tag{Ap2-2b}$$

where the scattering $t_{\pi N_1}$ matrix is constructed on the basis of the potential $v_{\pi N_1}$ (see Fig.3b) and the disconnected parts w^{D1}_{20} and w^{D2}_{20} of the w^3_{20} potential are shown in Fig.2b. Analogous disconnectednesses arise in calculating the $w^3_{30} \mathcal{G}_0 t_2$, $w^3_{30} \mathcal{G}_0 t_3$ and $w^3_{10} \mathcal{G}_0 t_1$ kernels of integral equations (28a,b). Therefore all the conclusions that we shall make regarding the disconnectedness given in (Ap2-2a,b) will be valid for the remaining terms $w^3_{\nu 0} \mathcal{G}_0 t_j$ as well. Taking into account that $w^{D1}_{20} \sim h_{12} \equiv h_{N_1, N_1 \pi}$ and $v_{S1} \sim h_{21} \equiv h_{N_1 \pi, N_1}$ and according to the definition of Green's function $g^2_{22} = \mathcal{G}_2 + \mathcal{G}_2 t_{\pi N} \mathcal{G}_2$ and following condition (31b) we have

$$\{w_{20}^{3}\,\mathcal{G}_0 t_2\}^{\mathrm{Disc.}} = w_{20}^{D1}\,\mathcal{G}_0 t_{\pi N_1}\,, \qquad \text{(Ap2-3a)}$$

$$\{w_{20}^{3}\,\mathcal{G}_0 t_3\}^{\mathrm{Disc.}} = w_{20}^{D2}\,\mathcal{G}_0 t_{\pi N_2}\,. \qquad \text{(Ap2-3b)}$$

In exactly the same manner, using the conditions (31a,b) it is obtained that the v_{S1}, v_{S2} potentials [8] shown in Fig.4 do not lead to additional disconnected diagrams in the iteration series of eq. (28a,b). In particular,

$$\{t_2\mathcal{G}_0 t_3\}^{\mathrm{Disc.}} = (1+t_{\pi N_1}\mathcal{G}_0)v_{S1}\,\mathcal{G}_0 t_{\pi N_2} +$$

$$+(1+t_{\pi N_1}\mathcal{G}_0)\{v_{S1}\mathcal{G}_0(1+t_{\pi N_2}\mathcal{G}_0)v_{S2}\} = (1+t_{\pi N_2}\mathcal{G}_0)v_{S1}\mathcal{G}_0 t_{\pi N_2}\,, \qquad \text{(Ap2-4)}$$

where in the second term of the first equality (Ap2-4) we again used formula (31b). Both components of this equality are shown in Fig.7, where we see that in eq. (Ap2-4) the term that leads to the renormalization of the single-particle nucleon propagator is equal to zero. It is easy to deduce that the remaining disconnected terms in $t_2\mathcal{G}_0 t_3$ and $w_{\nu 0}^{3}\,\mathcal{G}_0 t_j$ vanish in calculation of the following iteration, i.e., $\{t_2\mathcal{G}_0 t_3\mathcal{G}_0 t_2\}^{\mathrm{Disc.}}=0$, $\{t_3\mathcal{G}_0 t_2\mathcal{G}_0 t_3\}^{\mathrm{Disc.}}=0$ and $\{w_{\nu 0}^{3}\,\mathcal{G}_0 t_j\,\mathcal{G}_0 t_i\,\bar{\delta}_{ij}\}^{\mathrm{Disc.}}=0$, and this again is the consequence of the conditions (31a,b).

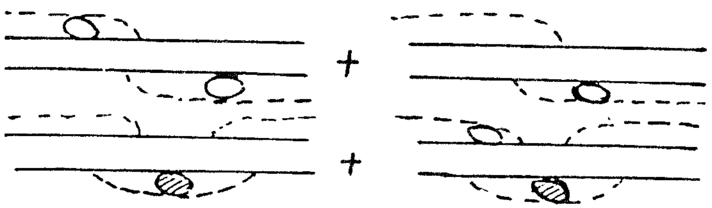

Fig.7. The disconnected diagrams occurring in calculating $t_2\mathcal{G}_0 t_3$. The πN scattering t-matrices and Green's functions $g_{22'}^{2}(\pi N)$ are indicated by open and shaded circles, respectively.

Disconnected diagrams appear also in the calculation of the $M_{\eta\nu}$ kernels of the integral equation (28a,b) (see Fig.6), as well as in the combination $w_{\eta 0}^{3}\,\mathcal{G}_0 t_j\, w_{0\nu}^{3}$ of iteration terms for the amplitude $U_{\eta\nu}$. However, substituting equation (28a) into (28b) it is possible to demonstrate that these disconnected diagrams cancel

$$U_{\eta\nu} = M_{\eta\nu} + \sum_{\sigma} M_{\eta\sigma}\mathcal{G}_\sigma U_{\sigma\nu} + w_{\eta 0}^{3}\,\mathcal{G}_0\sum_{j=1}^{3} t_j\,\mathcal{G}_0\, w_{0\nu}^{3} +$$

$$+ w_{\eta 0}^{3}\,\mathcal{G}_0\sum_{j=1}^{3} t_j\,\mathcal{G}_0\sum_{\sigma=1}^{3} w_{0\sigma}^{3}\,\mathcal{G}_\sigma U_{\sigma\nu} + w_{\eta 0}^{3}\,\mathcal{G}_0\sum_{j=1}^{3} t_j\,\mathcal{G}_0\sum_{i=1}^{3}\delta_{ij}\, t_i\,\mathcal{G}_0 U_{i\nu}\,. \qquad \text{(Ap2-5)}$$

The disconnected diagrams in eq. (Ap2-5) occur in the following expressions

$$\{M_{\eta\nu}\}^{\mathrm{Disc.}} = \delta_{\eta\nu}\, w_{\eta 0}^{D}\,\mathcal{G}_0\, w_{0\nu}^{D}\,, \qquad \text{(Ap2-6a)}$$

$$\{w_{\eta 0}^{3}\,\mathcal{G}_0\sum_{j=1}^{3} t_j\,\mathcal{G}_0 w_{0\nu}^{3}\}^{\mathrm{Disc.}} = \delta_{\eta\nu}\,\delta_{\nu j}\; w_{\eta 0}^{D}\,\mathcal{G}_0 t_j\,\mathcal{G}_0\, w_{0\nu}^{D}\,. \qquad \text{(Ap2-6b)}$$

After summing these disconnected terms, according to eq.(31b), we see that they cancel. The same conclusion can be drawn also in including the disconnected terms $\{w_{0\eta}^{3}\,\mathcal{G}_\eta M_{\eta\nu}\}^{\mathrm{Disc}}$ in the iteration series for the $U_{i\nu}$ matrices, substituting eq. (28b) into (28a) and using again formulae (Ap2-5) and (Ap2-6a,b).

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К теории связанных πNN–NN систем

На основе описания дейтрона и Δ изобары в виде одночастичного состояния получен один из вариантов релятивистских уравнений для взаимосвязанных πNN- и NN-систем. Пренебрегая в области /3.3/ резонанса неполюсной частью пион-нуклонной функции Грина, автор свел трехчастичные уравнения к системе уравнений для двухчастичных амплитуд переходов между πd-, NN- и NΔ-каналами.

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On the Theory of Coupled πNN–NN Systems

Proceeding from the description of the deuteron and Δ isobar as a one-particle state, a version of relativistic equations for coupled πNN and NN systems is obtained. It is demonstrated that, if one neglects the non-pole term of the pion-nucleon Green function in the (3.3) resonance region, the three-body equations reduce to a set of equations for the two-body amplitudes of transitions between the πd, NN and NΔ channels.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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