сообщения
объединенного
института
ядерных
исследований
дубна

E4-82-192

V.B.Belyaev, S.E.Brener, E.M.Gandyl,
A.L.Zubarev

ANALYSIS OF SPECIFIC PROPERTIES OF TWO-CHANNEL DESCRIPTION OF $dT\mu$-MESOMOLECULE

1982

I. INTRODUCTION

Quantum-mechanical systems, the interaction in which may be described by two potentials of essentially different ranges, have become a subject of inclusive study in the recent years[1]. In this case three situations related to the short-ranges potential properties can be distinguished: 1) The short-range potential does not contain levels close to zero or resonances, its influence on the system spectrum may be treated within perturbation theory. This picture is realised, e.g., in π-mesoatoms. 2) The short-range potential has a level near zero or a resonance; a potential of this type leads to the so-called Zel'dovich effect[2] which consists in an essential rearrangement of the spectrum of the whole system as compared to the spectrum in the long-range potential. Obviously, the perturbation theory in strong interaction fails to work here. A possible example is the $p\bar{p}$-atom. 3) The short-range potential is complex (i.e., transitions are possible into other channels) with a large imaginary part, and it has a resonance at an energy the real part of which is near zero on the nuclear scale. In the system with such a potential there may appear a new (as compared to the spectrum of levels of the unperturbed problems) level which cannot be described in the framework of perturbation theory; and the Zel'dovich phenomenon does not occur. A possible example is the $dT\mu^-$-mesomolecule [3]. In this interesting case the state produced is neither pure nuclear, nor molecular; both the types of forces are equally responsible for its formation. In this paper we shall be interested in the spectrum of $dT\mu$-mesomolecule in the state with the total angular momentum $L = 0$.

2. TWO-CHANNEL PROBLEM

Strong dT-interaction in the mesomolecule will be described by the two-channel system of Schrödinger equations with two channel open at a given energy

$$dT \begin{cases} \to dT \\ \to {}^4He + n. \end{cases}$$

The two-channel description is a model because of the absence of the corresponding microscopic calculations. Consideration of this problem as a five-body problem is itself of a considerable interest since the transition $dT \to {}^4He + n$ occuring due to the tensor interaction could be a source of a new information on the nature of this interaction.

Thus, the two-channel system of equations for the strong dT-interaction has the form

$$[H^{\circ}_{dT} + \frac{1}{r} + V_{11}]\psi_1 + V_{12}\,\psi_2 = E\psi_1 \,,$$
$$[H^{\circ}_{He\,n} + V_{22}]\psi_2 + V_{21}\,\psi_1 = (E+Q)\psi_2 \,, \qquad (1)$$

where H°_{dT} is the operator of the kinetic energy of relative motion of dT with the angular momentum $\ell_{dT} = 0$, $H^{\circ}_{He\,n}$ is the operator of the kinetic energy of relative motion of ${}^4He\,n$ $Q = 17.58$ MeV.

Boundary conditions for (1) correspond to the two open channels we are interested in. Interactions V_{11}, $V_{12} = V_{21}$, and V_{22} were chosen as potentials of the type of square well:

$$V_{ij} = V^{\circ}_{ij}\,\theta(r_c - r), \quad \text{except } i = j = 2,$$
$$V_{22} = V^{\circ}_{22}\,\theta(r_c - r) + \frac{6}{r^2}\theta(r - r_c). \qquad (2)$$

Parameters of the potentials (2) were defined so that the solutions of equations (1) would describe the experimental data on reaction $dT \to {}^4He + n$ at $0 \le E_{dT} \le 500$ keV [4] and d-phase of elastic $n\,{}^4He$-scattering at the neutron energy $E_n \sim 18$ MeV [5]. For a potential of the form (2) the available experimental data admit a slight variation of the parameters. The calculation was carried out with the following sets of parameters:

Set I	Set II
$V^{\circ}_{11} = -1.3020$ MeV,	$V^{\circ}_{11} = -1.2927$ MeV,
$V^{\circ}_{22} = -2.9193$ MeV,	$V^{\circ}_{22} = -2.9353$ MeV,
$V^{\circ}_{12} = -2.4032$ MeV,	$V^{\circ}_{12} = -2.3652$ MeV.

The cross section of reaction $dT \to {}^4He\,n$ is shown in Fig. 1: the solid curve is theory, * - experiment. As is seen from the Figure, both the sets give indistinguishable curves which fit experiment with a good accuracy. For the $n\,{}^4He$ phase at $E_n = 18$ MeV we have

$$\delta_2^{theor} \simeq 5.5^{\circ}, \qquad \delta_2^{exp\,/5/} \simeq (5.8 \pm 0.3)^{\circ}.$$

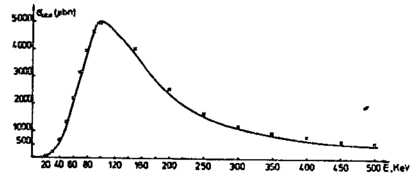

Fig. 1. Results of the fit of the cross section of reaction $dT \to {}^4He\,n$ (solid curve) and experimental data. (***).

To consider the $dT\mu$ mesomolecule with the strong dT-interaction taken into account, it is necessary to solve the two-channel system of three-particle equations of the form

$$[H^{\circ}_{dT}(\vec r) + V_{11}(r) + V_{\mu d}(\vec r,\vec\rho) + V_{\mu T}(\vec r,\vec\rho) + \frac{1}{r} + h^{\circ}_{\mu}(\vec\rho)]\psi_1(\vec r,\vec\rho) +$$
$$+ V_{12}(r)\psi_2(\vec r,\vec\rho) = E\psi_1(\vec r,\vec\rho),$$
$$[H_{He\,n}(\vec r) + V_{22}(r) + h_{\mu}(\vec r,\vec\rho)]\psi_2(\vec r,\vec\rho) + V_{21}(r)\psi_1(\vec r,\vec\rho) = \qquad (3)$$
$$= (E+Q)\psi_2(\vec r,\vec\rho),$$

where $h^{\circ}_{\mu}(\rho)$ is the muon free Hamiltonian, $V_{\mu A}$ is the muon interaction with nucleus A, $h_{\mu} = h^{\circ}_{\mu} + V_{He\mu}$. The system of three-particle equations (3) will be changed to a system of effective two-particle equations with the use of the following approximations: 1) In the first channel the muon will be assumed to give rise to a certain effective dT-interaction V_{μ}. Formally, this can always be done exactly, however, a specific choice of the potential $V_{\mu}(r)$ certainly originates some model representations. In this paper we use a Morse potential:

$$V_{\mu}(r) = D\,(e^{-2\alpha x} - 2e^{-\alpha x}), \quad x = r - r_0, \quad r \ge r_c \qquad (4)$$

with the parameters $D = 0.102$ a.u. $= 0.57 \cdot 10^{-3}$MeV, $\alpha = 0.743$ a.u.$^{-1} = 2.89 \cdot 10^{-3}$fm^{-1}, $r_0 = 2.1$ a.u. $= 539.8$ fm chosen for the spectrum [6] of the $d t\mu$-mesomolecule in the state with $L = 0$.

We stress that with an effective potential of the type (4) no question arises concerning its adequacy to the exact interaction at $r < r_c$, while in the region $r \ge r_c$ this question remains still open. 2) In the second channel one may change with a good accuracy the muon Hamiltonian h_{μ} by some average muon energy E_{μ} [3]. As a results, we arrive at the following system of equations

$$[-(2\mu_{dT})^{-1}\frac{d^2}{dr^2} + V_{\mu}(r) + V_{11}(r)]\psi_1(r) + V_{12}(r)\psi_2(r) = (E - E_1)\psi_1(r) \qquad (5)$$
$$[-(2\mu_{He\,n})^{-1}\frac{d^2}{dr^2} + V_{22}(r)]\psi_2(r) + V_{21}(r)\psi_1(r) = (E + Q - E_{\mu})\psi_2(r).$$

Here E_1 is the energy of the $t\mu$-atom ground state.

3. RESULT

The system of equations (5) was solved with boundary conditions of the form

$$\psi_1(r) \underset{r\to\infty}{\simeq} e^{ik_1 r}, \quad \mathrm{Im}\,k_1 > 0, \quad k_1 = \sqrt{(E - E_1) 2\mu_{dT}}, \qquad (6)$$

$$\psi_2(r) \underset{r\to\infty}{\simeq} f e^{ik_2 r}, \quad \mathrm{Re}\,k_2 > 0, \quad k_2 = \sqrt{(E + Q - E_\mu) 2\mu_{Hen}}, \qquad (7)$$

which ensure a bound state in the first channel and an outgoing wave in the ^{4}He n channel. As is seen from the form of equations (5), all the quantities here are determined except for the muon energy E_μ, the variation limits of which are fixed by the energy conservation law only. Thus, the eigenvalues of the energy $E - E_1$ of the system $dT\mu$ will, in general, be functions of this parameter. The numerical calculations have shown that the system of equations (5) has solutions corresponding to the eigenvalues of the mesomolecule problem with $L = 0$ in the potential V_μ. Wave functions of these states are plotted in Fig.2.

Besides these solutions, known from calculations which do not take into account the strong interaction[6], in the system there develop two more levels with essentially different widths. In Fig. 3 we present the μ-energy dependence of the real part of the energy of the level with large width for two sets of parameters of the strong dT-interaction. The width of this level depends weakly on the muon energy and equals $\Gamma/2 \simeq 13.25$ keV for set I and $\Gamma/2 \simeq 12.85$ keV for set II. As one can observe from the Figure, the strong interaction in system $dT\mu$ may initiate a new state with the energy real part near zero and with a large width only at $E_\mu \sim 1$ MeV. Note that this phenomenon remains stable with respect to the variation of parameters of the strong dT-interaction. In Fig. 4 we show the wave function of this state $\psi_1(r)$ at $E_\mu = 0.9$ MeV and $E_R = 0$ for the first set of parameters. It is seen that this function concentrates mainly in the range of the strong dT-interaction, i.e., at $r \simeq 7$ fm. The second level has a small width $\Gamma/2 \sim 10^{-2} - 10^{-3}$ eV and a positive real part of the energy $E_R \sim 6$-7 keV. Both the real and imaginary parts of energy of this level depend slightly on the muon energy and strong-interaction parameters. If such a level would exist in the realistic $dT\mu$ system, this could cause the S-wave resonance in deuteron scattering on $t\mu$ atoms at an energy of the relative motion ~ 6-7 keV.

The quantity α characterizing the distribution of the function ψ_1

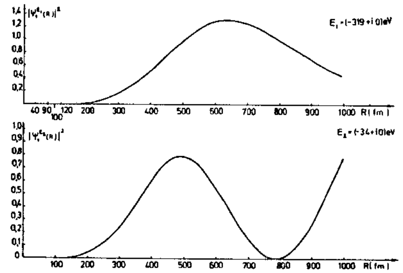

Fig. 2. Nonnormalized wave function of the ground state of mesomolecule $dT\mu$ with $\epsilon_1 = 319$ eV. $\psi_1^{\epsilon_2}$ -nonnormalized wave function of the excited state with $\epsilon_2 = 34$ eV.

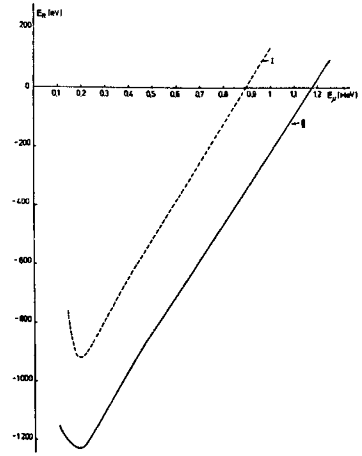

Fig. 3. Dependence of the real part E_R of the new-state energy on the muon energy E_μ for set II of the strong dT-interaction, (– – –), and the same for set I (———).

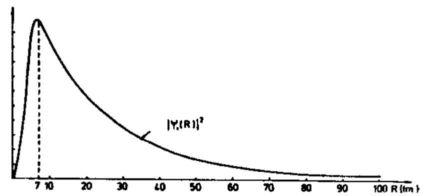

Fig. 4. The r-dependence of the squared modulus of the function $|\psi_1(r)|^2$ of the system in the first channel.

$$\alpha = \frac{\int_0^{r_c} |\psi_1(r)|^2 dr}{\int_0^{\infty} |\psi_1(r)|^2 dr}$$

turns out to equal ~0.25 for this state that exceeds by several orders the analogous ratio for states of the Coulomb nature.

CONCLUSION

We have considered the model of $dT\mu$-molecule which reproduces all the properties of the molecule and interactions of its constituents. In this model, two types of new states are found, which are absent when the pure Coulomb problem is solved. It is shown that the decay of states with a large width and a real part of the energy in the interval $-200 \text{ eV} \leq E_R \leq 0$ is accompanied by the emission (escape) of hard μ-mesons with an energy considerably larger than characteristic mesoatomic energies.

REFERENCES

1. Кудрявцев А.Е., Макрушин В.Е., Шапиро И.С. ЖЭТФ, 1978, 74, с.432; Попов В.С., Кудрявцев А.Е., Мур В.Д. ЖЭТФ, 1979, 77, с.1727; Simonov Yu.A. et al. "Strong coupled channel..." Groningen University Press, 1981, to be publ. Phys.Rep. v.78 (1982).
2. Зельдович Я.Б. ФТТ, 1959, 1, с.1637.
3. Беляев В.Б. и др. ОИЯИ, Р4-8172, Дубна, 1981.
4. Liskien H., Paulsen. Nucl. Data Tables, 1973, v.11, No. 7, p.602.
5. Bond J.E., Rirk F.W. Nucl.Phys., 1977, A287, p.317.
6. Виницкий С.И. и др. ОИЯИ, Р4-13036, Дубна, 1980.

Received by Publishing Department
on March 16 1982.

SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Беляев В.Б. и др. Исследование особенностей двухканального описания мезомолекулы $dT\mu$ E4-82-192

Исследуется спектр мезомолекулы $dT\mu$ в состоянии с $L=0$ при различных параметризациях сильного dT-взаимодействия. В рассматриваемой модели $dT\mu$ -молекулы изучаются состояния, которые не могут быть получены в рамках теории возмущений по сильному взаимодействию. Показано, что распад состояний, обладающих, большой шириной и реальной частью энергии, лежащей в интервале -200 эВ $\leq E_R \leq 0$ эВ, сопровождается вылетом жестких μ-мезонов с энергией, значительно превышающей характерные мезоатомные энергии.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1982

Belyaev V.B. et al. Analysis of Specific Properties of Two-Channel Description of $dT\mu$ -Mesomolecule E4-82-192

The spectrum of $dT\mu^-$ -mesomolecule is investigated in a state with $L=0$ for different parametrizations of the strong dT-interaction. In the considered model of $dT\mu$ -molecule the states are studied, which cannot be found within the perturbation theory in strong interaction.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982