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INVESTIGATION OF SECOND-ORDER OPTICAL POTENTIAL FOR ELASTIC $\pi\,^4$He SCATTERING

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1. INTRODUCTION

Several papers have been reported lately in which elastic pion-nucleus scattering has been analysed in the framework of an optical model (OM) with a second-order potential $U^{(2)}$ /1-4/. These investigations were stimulated by, at least, two circumstances: 1) a first-order OM potential $U^{(1)}$ provides a bad description of experimental data at low energies ($T_\pi < 100$ MeV), 2) the inclusion of a second-order potential $U^{(2)}$ allows one to study the influence of two-particle nucleon-nucleon correlations on elastic πA -scattering. The main results of the above-mentioned papers, from our point of view, are the following: they have shown a strong energy dependence of $U^{(2)}$ and it has turned out that the inclusion of $U^{(2)}$ is more important at low energies. At the same time, it was shown that the influence of short-range N-N correlations on the elastic πA-scattering is quite weak.

The main goal of our investigation is to find the effects which determine the gross features of $U^{(2)}$ and its dependence on energy. We have analyzed the different physical processes which may contribute to $U^{(2)}$: N-N correlations, double spin- and isospin-flip of nucleons in the intermediate states and the role of correction to the impulse approximation (IA).

As for N-N correlations we study the long-range N-N correlations due to the recoil of a nucleus. In our region of small transferred momenta ($T_\pi \sim 25$-250 MeV) just these long-range correlations determine the main features of the two-particle correlation function. Nevertheless, it turned out that the influence of these long-range correlations on the elastic $\pi\, ^4$He scattering is quite small.

We have found that the dominant contribution to $U^{(2)}$ at low energies is given by the process of pion charge-exchange in the intermediate states. The correction of the impulse approximation is extremely large, too. But its contribution to $U^{(2)}$ is to a large extent cancelled out with correction of the coherent approximation.

From a methodical point of view the advantage of our calculations is in a complete consideration of the spin-isospin structure of the pion-nucleon amplitude $f_{\pi N}$. (Lee and Chakravarti /1/ did not take into account the spin terms in the $f_{\pi N}$).

We also calculated $U^{(2)}$ with just the same approximations as for $U^{(1)}$. We have not assumed the static approximation in evaluating $U^{(2)}$, i.e., we have not neglected the difference between the pion-nucleus (ACM) and pion-nucleon (2CM) centre of mass systems as Wakamatsu[2] did.

All calculations were performed within the framework of two approaches in the multiple scattering theory: the Kerman-McManus-Thaler formalism[5] and that of Watson (see, for example, ref.[6]). The comparison of the Kerman-McManus-Thaler (KMT) and Watson (W) approaches shows that the alterations caused by the $U^{(2)}$ are smaller in the KMT-model.

The article is organized as follows: in section 2 we discuss the physical contents of the $U^{(2)}$ and the main assumptions of our model. In section 3 we consider the influence of correlations, the processes involving double spin (isospin)-flip and the correction of IA. The results derived within the frameworks of two formalisms, of KMT and W, are compared. Concluding remarks will be given in section 4.

2. OPTICAL MODEL WITH A SECOND-ORDER POTENTIAL $U^{(2)}$

2.1. Physical Contents of $U^{(2)}$

We shall briefly remind the main relationships of the optical model. For definiteness we shall use here the Watson approach (W). Thus, a many-body equation is to be solved for the π-nucleus T-matrix

$$T = U + U\, G(E)\,\hat{P}\, T, \qquad (1)$$

where $\hat{P}$ is the projection operator on the nuclear ground state $\hat{P} = |0\rangle\langle 0|$, and $G(E)$ is the pion-nucleus Green function $G(E) = (E - H_A - K_\pi + i\epsilon)^{-1}$. The optical potential U is defined by the equation

$$U = A\,\tau + (A-1)\,\tau\, G(E)\,\hat{Q}\, U, \qquad (2)$$

where $\hat{Q} = \sum_{n\neq 0} |n\rangle\langle n|$ and τ is the many body operator for the scattering on a bound nucleon:

$$\tau(E) = u + u\, G(E)\,\hat{Q}\,\tau(E). \qquad (3)$$

u is the pion-nucleon potential.

The impulse approximation (IA) consists in the replacement of τ by two-particle operator t, where

$$t(\omega) = u + u\, g(\omega)\, t(\omega), \qquad g(\omega) = (\omega - K_\pi - K_N + i\epsilon)^{-1}. \qquad (4)$$

From (3) and (4) it follows that

$$\tau(E) = t(\omega) + t(\omega)\,[G(E)\hat{Q} - g(\omega)]\,\tau(E). \qquad (5)$$

Let us consider that the second order optical potential $U^{(2)}$ is a sum of all terms quadratic in the t-matrix. It is clear, that such terms appear, first, in the iteration of eq. (2) (called usually a correction of the coherent approximation (CA)) and second, due to the iteration of eq. (5), that is a correction of the IA. Therefore

$$U^{(2)} = U^{(2)}_{CA} + U^{(2)}_{IA}, \qquad U^{(2)}_{CA} = A\,(A-1)\, t\, G(E)\,\hat{Q}\, t, \qquad (6)$$

$$U^{(2)}_{IA} = A\, t\,[G(E)\hat{Q} - g(\omega)]\, t. \qquad (7)$$

Assuming that $G_{nn}(E) = \langle n|G(E)|n\rangle \simeq G_{00}(E)$ and using the closure approximation, the matrix element $U^{(2)}$ may be rewritten as

$$\langle \vec{Q}'\, 0|U^{(2)}|0\,\vec{Q}\rangle = A(A-1)\langle t\, G_{00}(E)\, t\rangle_{00} \times C_{00}(\vec{Q}' - \vec{Q}'', \vec{Q}'' - \vec{Q}) - A\,\langle t\rangle_{00} G_{00}(E)\, F_{00}(\vec{Q}' - \vec{Q}'')\, F_{00}(\vec{Q}'' - \vec{Q})\langle t\rangle_{00}, \qquad (8)$$

where $F_{00}(q)$ is the nuclear form factor and $C_{00}(\vec{q}_1\,\vec{q}_2)$ is the Fourier-transform of the two-particle nuclear density

$$C_{00}(\vec{q}_1\vec{q}_2) = \langle 0|\exp(-i\vec{q}_1\vec{r}_1)\exp(-i\vec{q}_2\vec{r}_2)|0\rangle. \qquad (9)$$

The complete two-particle correlation function is

$$D(\vec{q}_1\vec{q}_2) = C_{00}(\vec{q}_1\vec{q}_2) - F_{00}(q_1)\, F_{00}(q_2). \qquad (10)$$

It is clear from eqs. (8)-(10) that taking into account of $U^{(2)}$ allows one to study the sensitivity of πA elastic scattering to the nucleon correlations in a nucleus. Besides that, $U^{(2)}$ contains a correction of the IA. Below we shall examine in greater detail the expressions from eq. (8).

2.2. Correlation Function and Form Factors

Nuclear wave functions were constructed using the single-particle harmonic oscillator basis. Then, in the case of ^4He, the correlations due to the Pauli exclusion principle vanish (see ref.[7]). At the same time, the short-range N–N correlations are not important in the elastic πA scattering. Therefore, we take into account only long-range recoil correlations. Then the nuclear form factor is

$$F(q) = \exp\left(-\frac{A-1}{4A} q^2 a_0^2\right),$$

where $a_0^2 = \frac{2}{3}\frac{A}{A-1}(R_{ch}^2 - r_{ch}^2)$, R_{ch} and r_{ch} are the r.m.s. charge radii of ^{4}He and the proton. Then matrix element (9) takes the following form

$$C_{00}(\vec{q}_1 \vec{q}_2) = \exp\left(-\frac{a_0^2}{4}\frac{A-1}{A}(q_1^2+q_2^2)\right)\exp\left(-\frac{a_0^2\, \vec{q}_1 \vec{q}_2}{2A}\right). \qquad (11)$$

2.3. Pion-Nucleon t-Matrix and the Choice of ω

The t-matrix of pion scattering on a free nucleon is connected with the amplitude $f_{\pi N}$

$$f_{\pi N}(\vec{k}_f \vec{k}_i\, \omega(k_0)) = A_0(\vec{k}_f \vec{k}_i) + (\vec{t}\vec{\tau}) A_T(\vec{k}_f \vec{k}_i) +$$

$$+ i\vec{\sigma}[\vec{n}_f \times \vec{n}_i]\,(A_S(\vec{k}_f \vec{k}_i) + (\vec{t}\vec{\tau}) A_{ST}(\vec{k}_f \vec{k}_i)), \qquad (12)$$

$$\vec{n}_i = \vec{k}_i / |\vec{k}_i|, \qquad \vec{n}_f = \vec{k}_f / |\vec{k}_f|,$$

where $\omega(k_0)$ is the total energy in the 2CM system and the values A_0, A_S, A_T, A_{ST} are as usually related to the pion-nucleon phase shifts.

At low ω, and on the energy shell, $(k_f = k_i = k_0)$ we used for the πN phase shifts the parametrization made by Salomon et al.[8] and for ω above the inelastic threshold in πN scattering we used the CERN-TH phase shifts[9]. We assume that the off-shell behaviour of the $f_{\pi N}$ is of a separable type:

$$f_{\pi N}(\vec{k}_f \vec{k}_i\, \omega(k_0)) = \sum_\ell \frac{g_\ell(k_f)\; g_\ell(k_i)}{g_\ell^2(k_0)}\; f_{\pi N}^{(\ell)}(\vec{k}_0' \; \vec{k}_0 \; \omega(k_0)).$$

The form factors $g_\ell(k)$ were obtained by solving the inverse scattering problem of πN-scattering as did Londergan et al.[10].

For construction of the optical potential the choice of connection between the energy E in ACM and ω in 2CM in which the $f_{\pi N}$ is calculated is very important. We adopted the following energy choice:

$$\omega(k) = E(Q_0) - (A-1)M - \frac{1}{8M}\frac{\mu}{\mathfrak{M}}\frac{A-1}{A}(\vec{Q}+\vec{Q}')^2, \qquad (13)$$

where M is the mass of a nucleon. μ and $\mathfrak{M}$ are reduced masses of πN and πA systems, respectively. $\vec{Q}$ and $\vec{Q}'$ are the pion momenta in ACM in which the potential (8) is calculated. $E(Q_0)$ is the total energy in ACM.

The relationship (13) has been discussed in detail by Mach[11] and we will only mention that (13) provides for the optical potential being Galileo-invariant. At the same time, the choice (13) allows one to minimize the error of the factorization approximation.

2.4. Initial Equation for $U^{(2)}$.

The Lippman-Schwinger eq.(1) is conveniently rewritten for the amplitude of elastic scattering:

$$F(\vec{Q}'\vec{Q}_0) = V(\vec{Q}'\vec{Q}_0) - \frac{1}{(2\pi)^2}\int \frac{V(\vec{Q}'\vec{Q}'')\,F(\vec{Q}''\vec{Q}_0)\,d\vec{Q}''}{[E(Q_0) - E(Q'') + i\epsilon]\,\mathfrak{M}(Q'')}, \qquad (14)$$

where $\vec{Q}_0$ is the momentum of the initial pion in ACM, and

$$V(\vec{Q}'\vec{Q}) = -\mathfrak{M}(Q)\, U(\vec{Q}'\vec{Q})\,/(2\pi).$$

If in (8) we pass from $t_{\pi N}$ to $f_{\pi N}$, and average it over the ground state and expand in partial waves, we obtain the following expression for $v_\ell^{(2)}$:

$$v_\ell^{(2)}(Q'Q) = \frac{1}{4\pi}\int \frac{dQ''\, dx_1\, dx_2}{[E(Q_0) - E(Q'') + i\epsilon]\,\mathfrak{M}(Q'')} \times$$

$$\times \{ (A-S)^2 P_\ell^0(x_1) P_\ell^0(x_2) A_0(\vec{k}_1\vec{p}_1) A_0(\vec{k}_2\vec{p}_2) F_{00}(q_1) \times$$

$$\times F_{00}(q_2) + (A-1)(A-S)\sum_{m=-\ell}^{\ell} e^{-\frac{a^2 R}{4}} (-1)^m \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(x_1) \times \qquad (15)$$

$$\times P_\ell^m(x_2)\,[\, I_m(z) A_{0T} - B_{ST}(I_{m+1}(z) + I_{m-1}(z))] \},$$

where

$$A_{0T} = -A_0(\vec{k}_1\vec{p}_1) A_0(\vec{k}_2\vec{p}_2) + \frac{2}{3} A_T(\vec{k}_1\vec{p}_1) A_T(\vec{k}_2\vec{p}_2). \qquad (16)$$

$$B_{ST} = \frac{1}{6} \sin\theta_1 \sin\theta_2 [A_S(\vec{k}_1\vec{p}_1) A_S(\vec{k}_2\vec{p}_2) + 2A_{ST}(\vec{k}_1\vec{p}_1) \times \times A_{ST}(\vec{k}_2\vec{p}_2)] . \qquad (17)$$

Here

$$\vec{k}_1 = \vec{Q}' - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}'+\vec{Q}''), \quad \vec{k}_2 = \vec{Q} - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}''+\vec{Q}), \qquad (18)$$

$$\vec{p}_1 = \vec{Q}'' - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}'+\vec{Q}''), \quad \vec{p}_2 = \vec{Q}'' - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}''+\vec{Q}).$$

Relation (18) between the momenta in 2CM system and ACM arises from our method of taking into account the Fermi-motion of nucleons (see ref./11/). Note that the static approximation, carried out by Wakamatsu/2/, corresponds to neglecting the terms with $\frac{A-1}{2A}\frac{\mu}{M}$ in (18). Further,

$$z = \frac{a^2}{6} Q'Q'' \sin\theta_1 \sin\theta_2, \quad \vec{q}_1 = \vec{Q}' - \vec{Q}'', \quad \vec{q}_2 = \vec{Q}'' - \vec{Q},$$

$$R = Q'^2 + Q^2 + \frac{8}{3}(Q''^2 - Q'Qx_1 - Q''Qx_2) + \frac{2}{3}Q'Qx_1x_2,$$

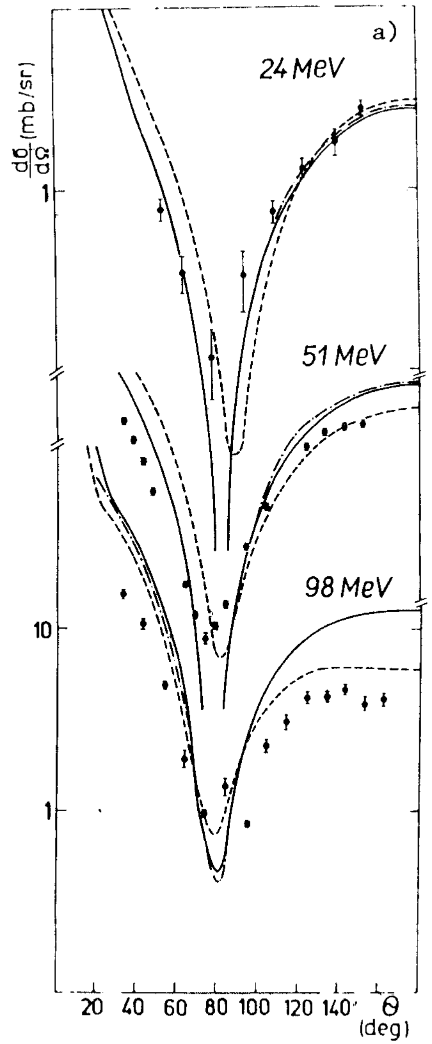
$$a^2 = a_0^2 \frac{A-1}{A}, \quad x_1 = \frac{\vec{Q}'\vec{Q}''}{|\vec{Q}'\vec{Q}''|}, \quad x_2 = \frac{\vec{Q}\vec{Q}''}{|\vec{Q}\vec{Q}''|}.$$

$I_m(z)$ is the modified Bessel function. $P_\ell^m(x)$ is associated Legendre polynomial. The coefficient S distinguished between the Watson (S=0) and KMT(S=1) multiple scattering formalisms. Equation (14) was solved in momentum space by matrix inversion/12/. The Coulomb effects were taken into account according to the method of Vincent, Phatak/13/.

3. NUMERICAL RESULTS AND DISCUSSION

3.1. The Influence of $U^{(2)}$ on Elastic Scattering. Principal Trends

All calculations were performed for the elastic scattering of π^- mesons on ^4He.



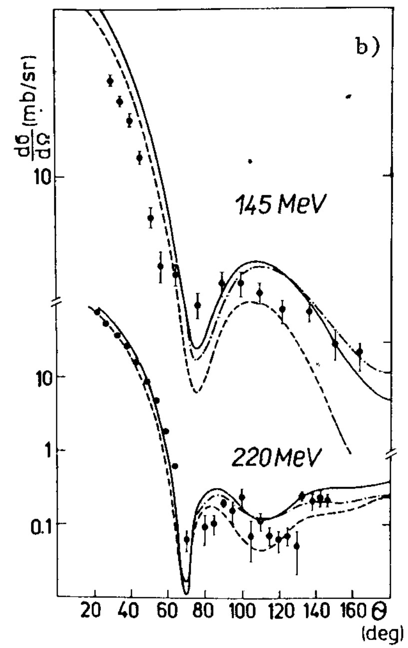


Fig.1. The influence of the $U^{(2)}$ on the differential cross sections of the elastic $\pi\,^4He$ scattering (W-approach). The dashed, full and dot-dashed lines correspond to calculations with $U^{(1)}$, $U^{(1)}+U^{(2)}$ and $U^{(1)}+U^{(2)}$ with recoil correlations "turned off", respectively. The experimental data are from/14-17/.

In Figures 1,2 we show the differences obtained through taking into account the second order potential $U^{(2)}$ for differential and total cross sections of $\pi\,^4He$ scattering. One can see that at low energies the addition of $U^{(2)}$ decreases strongly the $d\sigma/d\Omega$ for the forward scattering and significantly deepens the $d\sigma/d\Omega$ in the region of the minimum. Total cross section is also decreased almost by half. As pion energy increases, the addition of $U^{(2)}$ deteriorates the agreement with the experimental data: differential cross sections

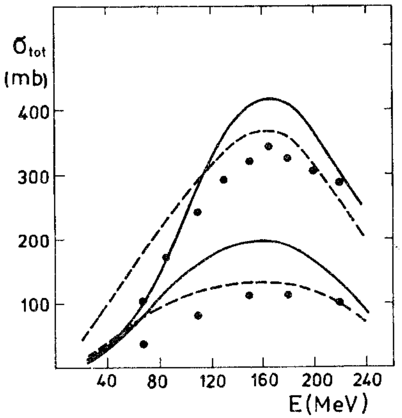


Fig.2. The influence of the $U^{(2)}$ on the total and total elastic cross sections of the elastic $\pi\ ^4He$ scattering (W -approach). The dashed line corresponds to calculations with $U^{(1)}$, the full line - to $U^{(1)}+U^{(2)}$. The experimental data are from [15,18].

as well as the total one increase as compared with the results of calculations with first order potential $U^{(1)}$ only. Such energetic dependence is a common feature for all calculations with the second order potential. We shall try to explain quantitatively this fact in section 4. Now we only note that the strong decrease of $d\sigma/d\Omega$ at the minimum (Fig.1) and the fall of the total cross section (Fig.2) at low energies are closely connected. The point is that at low energies the $d\sigma/d\Omega$ in the deep depends on the imaginary parts of the partial amplitudes $f_\ell^{\pi\ ^4He}$ only. That is due to the fact that at low energies the real parts of the partial amplitudes in S- and P -waves have the opposite signs and in the deep region their contributions to $d\sigma/d\Omega$ are negligible. Therefore, the deepening of the minimum in $d\sigma/d\Omega$ is tantamount to a decrease of σ_{tot}.

Table 1 shows the S- and P -phase shifts at two typical energies.

Table 1

Phase shifts δ_S and δ_P at $T_\pi = 51$ and 145 MeV from the calculations with different potentials

T_π (MeV)	51		145	
Phases (deg)	δ_S	δ_P	δ_S	δ_P
$U^{(1)}$	-5.3	14.	-8.5	54.
$U^{(1)}+U^{(2)}$	-6.5	11.	-13.	66.
From PSA Nichitiu et al.[19]	-8.6 $\pm$0.28	8.25 $\pm$0.10	-18.4 $\pm$1.2	21.3 $\pm$3.2

It is evident that the same trend is observed: at low energies $U^{(2)}$ moves the phase shifts in right direction, though, it is not enough to give agreement with data of the phase shifts analysis. From Table 1 one can conclude that the reason for the increasing disagreement at high energies is the abnormal increase of the P -wave phase shift.

3.2. The Role of Recoil Correlations

The term $C_{00}(\vec{q}_1\vec{q}_2)$ (eqs. (9) and (11)) appears as a result of taking into account the recoil long-range correlations. To study the sensitivity of the elastic $\pi\ ^4He$ scattering to such correlations we performed a calculation in which the recoil effects were "turned off". For that we take $C_{00}(\vec{q}_1\vec{q}_2)$ in the following form

$$C_{00}(\vec{q}_1\vec{q}_2) = F_{00}(q_1)F_{00}(q_2) \tag{19}$$

Then the correlation function $D(\vec{q}_1\vec{q}_2)$ in (10) is zero. (However, we did not change the form factors and, in this sense, the effect of recoil did not "turn off" entirely). The obtained results correspond to the dash-dotted lines in Figure 1. One can see that the lack of long-range correlations practically does not change the differential cross sections at low energies. In the resonance region, there is an effect at the backward scattering only.

Therefore, one can conclude that by no way does the space correlations determine the main features of the $U^{(2)}$. It is clear, then, that taking into account the short-range correlations, which change the $D(\vec{q}_1\vec{q}_2)$ at high q only, must also not be important in the $\pi\ ^4He$. That was demonstrated by Lee, Chakravarti [1] and Wakamatsu [2].

3.3. The Effects of Charge Exchange and Double Spin Flip in the Intermediate State

From eqs. (15)-(17) one can see that when the space correlations are "turned off" (in the sense of eq. (19)) the terms in $U^{(2)}$ which depend on the scalar part of $f_{\pi N}$ (i.e., the terms of $A_0(\vec{k}_1\vec{p}_1)A_0(\vec{k}_2\vec{p}_2)$) disappear. In the KMT approach this cancellation is complete but in the W-formalism a part of scalar terms is not vanished. Further we shall use the KMT approach.

Figure 3 presents the results of calculations when the recoil correlations are "turned off" and the terms $A_T(\vec{k}_1\vec{p}_1)A_T(\vec{k}_2\vec{p}_2)$

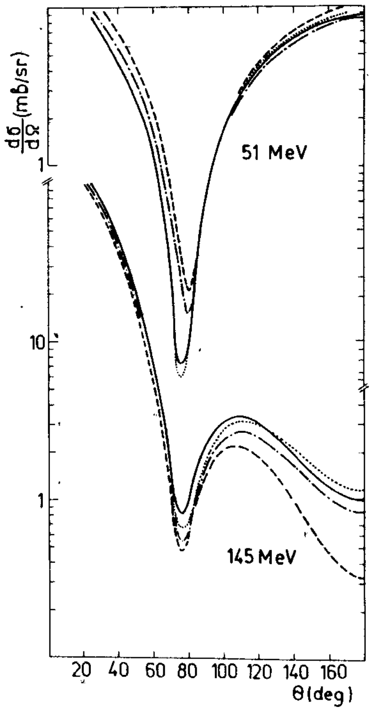


Fig.3. The role of different effects in the $U^{(2)}$ (KMT-approach). The dashed line corresponds to calculations with $U^{(1)}$ only. The full line – to $U^{(1)}+U^{(2)}$, the dotted line – to $U^{(1)}+U^{(2)}$ without recoil correlations, the dash-dotted line – to $U^{(1)}+U^{(2)}$ without recoil correlations and without terms with $A_T A_T$.

are omitted (dash-dotted lines). It is seen that at low energies these results are very similar to those, when only $U^{(1)}$ was taken into account. The deep in $d\sigma/d\Omega$ is shallow and at small angles the cross sections increase appreciably. The remaining difference between the dashed and the dash-dotted lines is due to the term of B_{ST} (see eq. (17)) and corresponds to the effects of double spin-flip in the intermediate states. One can conclude that at low energies the main contribution in $U^{(2)}$ is made by the terms that are proportional to the production of $A_T A_T$.

It is seen from Figure 3 that at high energies all effects give approximately equal contributions in the $U^{(2)}$. Nevertheless, the role of the double spin (isospin) flip terms increase and at the backward scattering their contribution becomes significant.

The appearance of $A_T A_T$ terms is associated with the double isospin-flip processes $\pi^{\pm} \to \pi^{\circ} \to \pi^{\pm}$ in the intermediate states, i.e., with virtual excitation of $T=1$ nuclear states. In our model we assume from the beginning that all intermediate states degenerate and their energies are equal to the energy of the ground state. Nevertheless, our conclusion about the importance of the influence of the excitation of states with $T=1$ on the elastic $\pi\, ^4He$ scattering at low energies is confirmed by the results of the paper by Gmitró, Mach/20/.These authors studied the influence of the excitation of different nuclear states on the elastic $\pi\, ^4He$ scattering in the framework of coupled channels method. They conclude that the contribution of the states with $T=1$ is the most significant one.

3.4. The Correction to the Impulse Approximation (IA)

As we noted earlier, the potential of $U^{(2)}$ includes a correction of the IA (see eq. (7)). The physical meaning of (7) is a double pion rescattering on the same nucleon. In general, the rescattering of all orders is exactly taken into account by the operator τ (see eq. (3)). However, then τ is replaced by t, this property of the τ-matrix is lost and it is necessary to control in some way the validity of IA. When writing down (7) in detail with closure approximation and neglecting the energy of nucleon K_N in the $g(\omega)$ (see eq. (4)) we obtain:

$$U^{(2)}_{IA}(\vec{Q}',\vec{Q}) = \frac{(A-S)(A-1)}{A}\,\frac{1}{(2\pi)^3}\int \frac{d\vec{Q}''}{[E(Q)-E(Q'')+i\epsilon]\,\mathfrak{M}(Q'')} \times \quad (20)$$

$$\times \{\langle t\, G_{00}(E)\, t\rangle_{00}\,[C_{00}(\vec{q}_1\vec{q}_2) - F_{00}(\vec{q}_1+\vec{q}_2)]\},$$

where $\vec{q}_1 = \vec{Q}' - \vec{Q}''$, $\vec{q}_2 = \vec{Q}'' - \vec{Q}$.

It is seen that the term with $F_{00}(\vec{q}_1+\vec{q}_2)$ corresponding to the rescattering on the same nucleon, is independent of the integration variable Q''. It leads to the convergence of the respective integral at large Q'' being provided for only by a decrease of pion-nucleon form factors $g_\ell(k)$ off the mass shell. Therefore, the calculations with $U^{(2)}_{IA}$ demonstrate a strong dependence on the off-shell behaviour of $f_{\pi N}$. But it is impossible to choose a too fast decrease of $f_{\pi N}$ off the mass shell because it leads to a rather strong increase of the nonlocality region of πN interaction. For studying the dependence of $U^{(2)}_{IA}$ on the off-shell behaviour of $f_{\pi N}$ we used simple Yamaguchi form factors:

$$g_\ell(k) = \frac{k^\ell}{(1+\Lambda a^2 k^2)^2},$$

where $a^2 = 0.056\ fm^2$.

Usually $\Lambda=1$. We used $\Lambda=4$, then the $g_\ell(k)$ are not much different from those which are determined in the approach of Londergan et al./10/, at least in the resonance P_{33}-partial wave. Nevertheless, the calculations of $U^{(2)}_{IA}$ at low energies show that this correction is so large that it breaks down the unitarity, i.e., a still faster decreasing of $g_\ell(k)$ is needed. At high energies the unitariry is not violated but the correction is still large. That is illustrated in Figures 4 and 5.

It is important to note, that as has been shown by Feshbach et al./7/, the main part of the correction of IA (the term

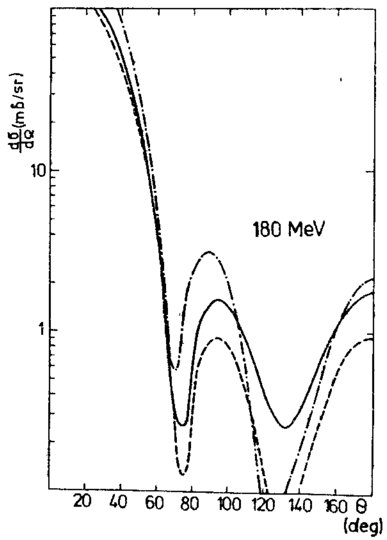


Fig.4. The role of the correction to the impulse approximation on the differential cross sections of the elastic $\pi\, ^4He$ scattering (KMT). The dashed, full and dash-dotted lines correspond to calculations with $U^{(1)}$, $U^{(1)}+U^{(2)}$ and $U^{(1)}+U^{(2)}_{IA}$.

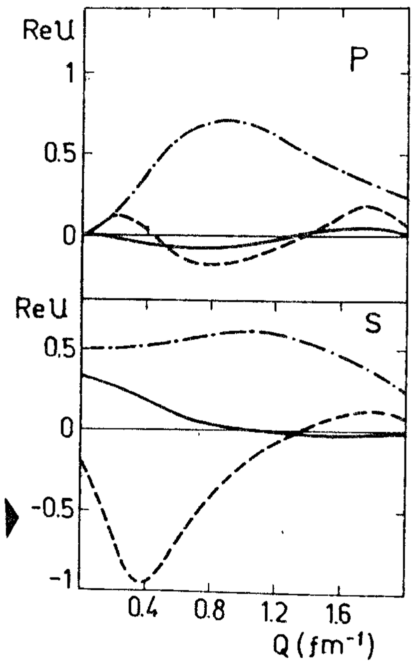


Fig.5. The real part of the optical potential U(Q, Q) in the S- and P-waves. The dashed line corresponds to the $U^{(1)}$, the full line - to the $U^{(2)}$ only, the dash-dotted line - to the $U^{(2)}_{IA}$. $T_\pi = 180$ MeV.

with $F_{00}(\vec{q}_1+\vec{q}_2)$) is cancelled out exactly with a corresponding term from the correction of the coherent approximation if the closure approximation is assumed. It may be proved, that such cancellation appears in any order of the optical potential. Therefore, though the correction of IA is large, it is cancelled out, to a large extent, with the correction of the coherent approximation. Moreover, if one works with the full expression for $U^{(2)} = U^{(2)}_{CA} + U^{(2)}_{IA}$ then the terms which appear from solving the Lippman-Schwinger equation (1) with $U^{(1)}$ only and correspond to the double scattering on non-correlated nucleons are exactly cancelled out with the part of $U^{(2)}$ which is proportional to $F_{00}(q_1)F_{00}(q_2)$. This cancellation does not occur if only $U^{(2)}_{IA}$ is taken into account and in this case

there appears a strange mixture of terms that describe double scattering on the correlated and non-correlated nucleons.

3.5. The Comparison of the KMT and W Approaches

A detailed analysis of the advantages and shortcomings of both approaches can be found, for example, in the papers/21,22/ Here we note only that both approaches lead to the exactly the same results if either eq. (1) is solved exactly or (1) is solved with the first order potential, but the latter depends on the exact τ-matrix. When impulse approximation is performed, the KMT and W models predict different results. In principle, one can consider this difference as a measure of deviation from the exact solution of the problem of elastic πA-scattering. It is interesting to study the change of this difference due to addition of $U^{(2)}$. The results of calculation are summarized in the Table 2.

Table 2

The comparisons of the results of the KMT and W approaches*

T (MeV)	24	51	145	180	220
$\delta^{(1)}/\delta^{(2)}$	1.	2.4	4.4	2.9	1.5
$\Delta_W/\sigma^{(1)}_{tot}$ (%)	60	59	11	17	17
$\Delta_{KMT}/\sigma^{(1)}_{tot}$ (%)	54	46	6	9	8

*) $\delta = |\sigma_{KMT} - \sigma_W|$, $\Delta = |\sigma^{(1)} - \sigma^{(2)}|$, where $\sigma^{(1)}$ and $\sigma^{(2)}$ were calculated with the potentials $U^{(1)}$ and $U^{(1)}+U^{(2)}$, respectively. The ratio in the first line is for σ_{el}.

It is seen that the addition of the $U^{(2)}$ decreases significantly the difference between KMT and W results. Moreover, in the KMT-model taking into account of $U^{(2)}$ does not change the cross sections as much as in the W-approach. One can conclude, with some caution, that this fact may indicate that the convergence of the series for an optical potential is better in the KMT-approach.

4. CONCLUSIONS

As is seen from Figures 1 and 2 the addition of the second order potential $U^{(2)}$ leads to better agreement with experimental data at low energies ($T_\pi < 100$ MeV) and deterioration at the resonance region. That peculiarity may be understood on the basis of the following quantitative arguments. Let us assume that the amplitude $f_{\pi N}$ includes only the resonant wave, omit the term with the principal-value integral in the Green function in (14) $G(E) \simeq -i\pi\delta(E(Q_0') - E(Q''))$* and leave in the $U^{(2)}$ only the terms that are proportional to $A_T A_T$. From the discussions in section 3 one can draw the conclusion that these approximations do not change significantly the main features of $U^{(2)}$. When

$$U^{opt} \simeq U^{(1)} + U^{(2)} \simeq (A-1)\, t_{\pi N} F_{00}(q) - i\pi (A-1)^2 D_{00}(q_1 q_2)(t_{\pi N})^2 ,$$

where $t_{\pi N} \simeq t_{33}$.

In the Δ_{33} -region the t_{33} is a large imaginary quantity, i.e., $t_{33} \simeq \epsilon + iC$. Then the addition of $U^{(2)}$ leads to a further increase of the imaginary part of U^{opt} by quantity $\sim C^2$. Therefore, the σ_{tot} must be increased, too. At $T_\pi \sim 70$ MeV, on the contrary, the potential $U^{(1)}$ is a large real quantity $U^{(1)} = C + i\epsilon$ and the addition of $U^{(2)}$ leads, in general, to a decrease of the imaginary part of the total U^{opt}. Therefore, the cross sections must be decreased.

These oversimplified arguments are not to be taken literally. They indicate only that the strong energy dependence of $U^{(2)}$ is not caused by some approximations in the model, but is a reflection of a resonant structure of the pion-nucleon scattering.

Figures 1,2 and Table 2 show that taking into account of $U^{(2)}$ is more important at low energies. In this region the behaviour of $U^{(2)}$ is determined mainly by the terms connected with pion charge exchange in the intermediate states. The influence of the N-N space correlations due to the recoil is rather small in the elastic $\pi\,^4He$ scattering. In the resonance region the effects of the double spin-flip in the intermediate states become important, too.

The study of the correction of the impulse approximation involved in the $U^{(2)}$ shows that though the correction itself is rather big, it is largely cancelled out with the correction of the coherent approximation. Such a mechanism of the mutual cancellation, probably, makes applicable the impulse approximation in the pion-nucleus scattering.

Finally, the comparison of the KMT and W approaches shows that the alterations caused by the $U^{(2)}$ are smaller in the KMT-model.

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*We checked that such approximation did not change the gross features of the results of calculations with $U^{(2)}$.

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Изучение эффектов, связанных с учетом оптического потенциала второго порядка в упругом $\pi\,^4$He-рассеянии

Выполнены расчеты упругого $\pi\,^4$He -рассеяния в рамках оптической модели с потенциалом второго порядка. Изучена роль эффектов, связанных с отдачей ядра, с перезарядкой и двойным спин /изоспин/-флипом в промежуточном состоянии. Исследована коррекция к импульсному приближению. Проводится сравнение между подходами Кермана-МакМануса-Талера и Ватсона.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Mach R., Sapozhnikov M.G. E4-82-189

Investigation of Second-Order Optical Potential for Elastic $\pi\,^4$He Scattering

The calculations of elastic $\pi\,^4$He scattering within the framework of the optical model with a second-order potential were performed. The effects of recoil correlations, charge exchange and double spin (isospin) flip in the intermediate states are studied. The correction of the impulse approximation is investigated. Comparison between Kerman-McManus-Thaler and Watson formalisms is made.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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