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## $2974 / 82$

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## PION POLARIZABILITIES

## FROM BACKWARD AND FIXED-u

 SUM RULES[^0]The possibility of investigating the pion Compton effect and the pion polarizabilities in the radiative scattering of high energy pions on nuclear Coulomb fields/1/ or in the radiative single pion photoproduction on protons ${ }^{\prime 2 /}$ has been recently stressed out. At the same time, information on the process $\gamma \gamma \rightarrow \pi \pi \quad$ may be obtained by studying the colliding beam reaction $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \pi \pi$ and indeed interesting results have already been found (see, for instance, ref. ${ }^{/ 3 /}$ ). A natural theoretical framework which simultaneously involves quantities relevant to both the $\gamma \pi \rightarrow \gamma \pi$ and $\gamma \gamma \rightarrow \pi \pi$ channels is provided by sum rules for the pion polarizabilities derived from backward or fixed-u dispersion relations. Such sum rules have been firstly put Eorward/ $/ 4 /$ and used $/ 4-7 /$ in connection with the difference $a-\beta$ between the electric ( $a$ ) and magnetic ( $\beta$ ) polarizabilities of the proton. Their analogs in the pion case are particularly appealing since for pions, unlike the nucleons, $\alpha-\beta$ is the most important combination (on quite general grounds one expects $\left.\left.{ }^{18\rangle}(\alpha+\beta)_{\pi}^{\ll \mid(\alpha-\beta)}\right)_{\pi} \mid\right)$.

For a review on previous calculations of the pıon polarizabilities within various approaches (quantum field theoretical, quark models, forward dispersion relations) we send the reader to the review article $/ 9 /$. Here we recall only that predictions based on forward finite energy sum rules (FFESR) are strongly model dependent because of difficulties in evaluating reliably the high energy asymptotic contributions.

In this note we shall present some simple numerical estimates of the pion polarizabilities using backward and fixed $\mathrm{u}=\mu^{2}$ ( $\mu=$ pion mass, $\mathbf{u}=$ the usual Mandelstam variable) sum rules. In some sense our approach looks complementary to that of FFESR since the annihilation channel exchanges are now taken into account directly, mainly through $\gamma \gamma, \pi \pi$ amplitudes, rather than indirectly by means of kegge parameters describing the asymptotics. While the s-channel contributions expressed by integrals over cross-sections for photoabsorption on the pion can be more or less reliably computed using the known radiative and strong width of vector, axial vector and tensor meson resonances, the evaluation of the amnihilation channel contributions still remains largely affected by model dependence despite important clarifications brought recently by the experimental study of the pion pair production in photon-photon collisions. Although in our procedure the model depen-

dence problem appears so merely shifted rather than much mitigated, one has at least the advantage of starting with a convergent (subtraction free) dispersion representation for $(a-\beta)_{\pi}$ and there is also a realistic hope that further better knowledge of ( $\gamma, \gamma \rightarrow$ hadrons) - processes shall help reducing the existing ambiguities in the determination of the pion polarizabilities.

We shall deal with the following two slightly different sum rules for $(a-\beta)_{\pi}^{/ 10 /}$ :

1) fixed angle $\theta=180^{\circ}$ sum rule:

$$
\begin{equation*}
(\alpha-\beta)_{\pi} \approx \cdot(\alpha-\beta)_{\pi}^{(\mathrm{s})}+(\alpha-\beta)_{\pi}^{(\mathrm{t})} \tag{1}
\end{equation*}
$$

$$
\begin{align*}
& (\alpha-\beta)_{\pi}^{(\mathrm{s})}=-\frac{1}{8 \pi^{2} \mu} \int_{4 \mu^{2}}^{\infty} \mathrm{ds} \frac{\mathrm{~S}+\mu^{2}}{\mathrm{~s}\left(\mathrm{~s}-\mu^{2}\right)} \mathrm{M}^{(\mathrm{s})}\left(\mathrm{s}, \mathrm{t} \pi-\frac{\left(\mathrm{s}-\mu^{2}\right)^{2}}{\mathrm{~s}}\right),  \tag{2}\\
& (a-\beta)_{\pi}^{(\mathrm{t})}=-\frac{1}{8 \pi^{2} \mu_{\mu}}: \int_{4 \mu^{2}}^{\infty} \frac{\mathrm{dt}}{\mathrm{t}} \mathrm{M}^{(\mathrm{t})}\left(\mathrm{t}, \mathrm{u}=\mu^{2}-\frac{\mathrm{t}}{2} \pm \frac{1}{2}\left[\mathrm{t}\left(\mathrm{t}-4 \mu^{2}\right)\right]^{1 / 2}\right)  \tag{3}\\
& \text { 2) fixed } \mathrm{u}=\mu^{2} \\
& \text { sum rule: }
\end{align*}
$$

$$
(\alpha-\beta)_{\pi}=(\alpha-\beta)_{\pi}^{[\dot{\mathrm{s}}]}+(\alpha-\beta)_{\pi}^{[\mathrm{t}]}
$$

$$
\begin{align*}
& (\alpha-\beta)_{\pi}^{[\mathrm{s}]}=-\frac{1}{8 \pi^{2} \mu} \int_{4 \mu^{2}}^{\infty} \frac{\mathrm{ds}}{\mathrm{~s}-\mu^{2}} \mathrm{M}^{(\mathrm{s})}\left(\mathrm{s}, \mathrm{u}=\mu^{2}\right),  \tag{-}\\
& (\alpha-\beta)_{\pi}^{[\mathrm{t}]}=--\frac{1}{8 \pi^{2} \mu} \int_{4 \mu^{2}}^{\infty} \frac{\mathrm{dt}}{\mathrm{t}} \mathrm{M}^{(\mathrm{t})}\left(\mathrm{t}, \mathrm{u}=\mu^{2}\right)
\end{align*}
$$

$M^{(s)}, M^{(t)}$ are the $s$-and $t$-channel absorptive parts of the amplitude

$$
\begin{equation*}
\mathrm{M}=2 \mathrm{~A}+\left(\frac{\mathrm{t}}{4}+-\mu^{2}\right) \mathrm{B} \equiv \frac{4 \mathrm{f}_{++}}{\mathrm{t}} \tag{4}
\end{equation*}
$$

where $A, B$ are the invariant amplitudes (free of kinematical problems ${ }^{/ 11 /}$ ) specifying the pion Compton scattering $S$-matrix element and $f_{++}$is the helicity amplitude describing transitions with photon helicities $+1,+1$ in the $\gamma \gamma \rightarrow \pi \pi$ channel:

$$
\begin{aligned}
& \left\langle\gamma\left(\mathrm{k}^{\prime} \cdot\right), \pi\left(\mathrm{p}^{\prime}\right)\right| \dot{\gamma}(\mathrm{k}), \pi(\mathrm{p})>=\delta_{\mathrm{f}, \mathrm{i}}+\mathrm{i}(2 \pi)^{-2}\left(16 \mathrm{k}_{0}^{\prime} \mathrm{k}_{0} \mathrm{p}_{0}^{\prime} \mathrm{p}_{0}\right)^{-1 / 2} \times \delta^{4}(\ldots) \\
& \quad \times \epsilon_{\mu}^{+}\left(\mathrm{k}^{\prime}\right) \mathrm{T}_{\mu \nu}\left(\mathrm{p}^{\prime} ; \mathrm{k}^{\prime} ; \mathrm{p}, \mathrm{k}\right) \epsilon_{\nu}(\mathrm{k}), \\
& \mathrm{T}_{\mu \nu}\left(\mathrm{p}^{\prime}, \mathrm{k}^{\prime} ; \mathrm{p}, \mathrm{k}\right)=\mathrm{A}(\mathrm{~s}, \mathrm{t}, \mathrm{u})\left(\mathrm{k} \cdot \mathrm{k}^{\prime} \mathrm{g}_{\mu \nu} \cdot-\mathrm{k}_{\mu} \mathrm{k}_{\nu}^{\prime}\right)- \\
& -\mathrm{B}(\mathrm{~s}, \mathrm{t}, \mathrm{u})\left[\mathrm{k} \cdot \mathrm{k}^{\prime} \mathrm{P}_{\mu} \mathrm{P}_{\nu}-(\mathrm{PK})\left(\mathrm{P}_{\mu} \mathrm{k}_{\nu}^{\prime}+\mathrm{P}_{\nu} \mathrm{k}_{\mu}\right)+\mathrm{g}_{\mu \nu}(\mathrm{PK})^{2}\right],
\end{aligned}
$$

$\mathrm{P}=\frac{\mathrm{p}+\mathrm{t}^{\prime}}{2}, \quad \mathrm{~K}=\frac{\mathrm{k}+\mathrm{k}^{\prime}}{2}$,

$$
\begin{equation*}
\mathrm{s}=(\mathrm{p}+\mathrm{k})^{2}, \quad \mathrm{t}=\left(\mathrm{k}-\ddot{\mathrm{k}}^{\prime}\right)^{2}, \mathrm{~s}+\mathrm{t}+\mathrm{u}=2 \mu^{2} \tag{5}
\end{equation*}
$$

$s=\mu^{2}+2 \mu \omega \quad\left(\omega=\right.$ the incident photon energy in the laboratory ${ }^{\text {. }}$ system).

As is shown in ref. ${ }^{10 /}$ one can put the above sum rules in the form

$$
\begin{align*}
& (\alpha-\beta)_{\pi}^{(8)}=\frac{1}{2 \pi^{2}} \int_{\omega_{0}=8 \mu / 2 \omega}^{\infty} \frac{\mathrm{d} \omega}{\left(1+\frac{\omega}{\mu}\right)[\sigma(\mathrm{YES})-\sigma(\mathrm{NO})], ~}  \tag{2a}\\
& (a-\beta)_{\pi}^{(t)}=-\frac{1}{2 \pi^{2}{ }_{\mu}} \int_{4 \mu^{2}}^{1 B \mu^{2}}-\frac{d t}{t^{2}} \sum_{\mathrm{J}=\mathrm{OV} \text { ©n }}(2 \mathrm{~J}+1)\left[\frac{\mathrm{t}\left(\mathrm{t}-4 \mu^{2}\right)}{16}\right]^{\mathrm{J} / 2} \mathrm{E}_{+}^{\mathrm{J}}(\mathrm{t}) \mathrm{h}_{\mathrm{J}}^{*}(\mathrm{t})+ \tag{3a}
\end{align*}
$$

+ higher annihilation channel contributions;

$$
\begin{aligned}
& (a-\beta)_{\pi}^{[\mathrm{s}]}=\frac{1}{2 \pi^{2} \mu_{\mu}}: \int_{\omega_{0}=3 \mu / 2}^{\infty}-\frac{\mathrm{d} \omega}{\omega} \sum_{\ell=1}^{\infty} \mathrm{d}_{1,-1}^{\ell}\left(\mathrm{x}=-\frac{\mathrm{s}+\mu^{\ell}}{\mathrm{s}-\mu^{2}}\right)\left[\sigma_{\mathrm{E}_{\ell}}(\omega)-\sigma_{\mathrm{M}_{\ell}}(\omega)\right],\left(2 \mathrm{a}^{-}\right) \\
& (a-\beta)_{\pi}^{[\mathrm{t}]}=-\frac{1}{2 \pi^{2} \mu_{\mu}^{2}} \int_{4 \mu^{2}}^{1 \theta \mu^{2}}-\frac{\mathrm{dt}}{\mathrm{t}^{2}} \sum_{\mathrm{J}=\mathrm{even}}^{\sum}(2 \mathrm{~J}+1)\left[\frac{\mathrm{t}\left(\mathrm{t}-4 \mu^{2}\right)}{16}\right]^{\mathrm{J} / 2} \mathrm{~g}_{+}^{\mathrm{J}}(\mathrm{t}) \mathrm{h}_{\mathrm{J}}^{*}(\mathrm{t}) \mathrm{P}_{\mathrm{J}}(\cos \psi)\left(3 a^{-}\right)
\end{aligned}
$$

+ higher annihilation channel contributions.
$\sigma$ (YES) and $\sigma$ (NO) stand for the sum of the photoabsorption cross-sections containing respectively the parity flip and non-flip mutipoles:

$$
\begin{align*}
& \sigma(\mathrm{YES})=\sum_{\ell=\text { odd }}^{\sum} \sigma_{\mathrm{E} \ell}(\omega)+\sum_{\ell=\text { even }}^{\sum} \sigma_{M_{\ell}}(\omega),  \tag{6}\\
& \sigma \cdot(\mathrm{NO})=\sum_{\ell=\text { even }}^{\sum} \sigma_{\mathrm{E} \ell}(\omega)+\sum_{\ell=\text { odd }}^{\sum} \sigma_{\mathrm{M}_{\ell}}(\omega) ; \tag{7}
\end{align*}
$$

$h_{J}(t)=\exp \left[i \delta_{J}(t)\right] \cdot \sin \delta_{J}(t)$ and $g_{+}^{J}(t) \quad$ denote respectively the $\pi \pi \rightarrow \pi \pi$ and $\gamma \gamma \rightarrow \pi \pi$ partial waves (for $t$ in the elastic unitarity region $4 \mu^{2} \leq t \leq 16 \mu^{2}$ they have (modulo $\pi$ ) the same phase $\delta_{\mathrm{J}}(\mathrm{t}) ; \mathrm{d}_{1,-1}(\mathrm{x})$ and $\mathrm{P}_{\mathrm{J}}(\cos \psi)$ are the usual rotation group functions and Legendre polynomials ( $\psi=$ the $t$-channel centre of mass (c.m.) angle, $\cos \psi=(u-s) /\left[t\left(t-4 \mu^{2}\right)\right]^{1 / 2} ; x=$ the cosine of the c.m. angle for $\dot{y} \pi \rightarrow \gamma \pi$ ).).

We start now discussing the evaluation of the (more re-
 sorption channels with two and three pions are retained; the process $\gamma \pi \rightarrow \pi \pi$ is considered in the $p$-resonance approxima-
tion while for $\gamma \pi \rightarrow \pi \pi \pi$ we take only $\omega$, $\phi, A_{1}, A_{2}$ resonance contributions to $y \pi \rightarrow \pi \rho$. So we retain only $\mathrm{E}_{1}$, M1 and M2 transitions (corresponding, respectively, to the $A_{1}(\rho, \omega, \phi)$ and $A_{2}$ resonances) and integrate the Breit-Wigner forms

$$
\begin{equation*}
\sigma_{\mathrm{J}}(\mathrm{~s})=\frac{2 \pi \mathrm{~S}}{\left(\mathrm{~s}-\mu^{2}\right)^{2}}(2 \mathrm{~J}+1) \frac{\Gamma_{\mathrm{i}} \Gamma_{\mathrm{f}}}{\left(\sqrt{\left.\overline{\mathrm{~s}}-\mathrm{M}_{\mathrm{R}}\right)^{2}+\Gamma_{\text {total }}^{2} / 4}\right.} \tag{8}
\end{equation*}
$$

with corresponding angular momentum factors included in $\Gamma_{f}$ to ensure correct threshold behaviour, etc. Masses, strong and total widths are taken from ref. $12 /$. The following radiative widths are used (see refs. ${ }^{\prime 9,12 /}$ ) : $\Gamma(\rho \rightarrow \pi \gamma)=0,063$ $\Gamma(\omega \rightarrow \pi \gamma)=0.88, \quad \Gamma(\phi \rightarrow \pi \gamma)=0.57 \times 10^{-2} \quad, \Gamma\left(\mathrm{~A}_{1} \rightarrow \pi \gamma\right)=0.60, \Gamma\left(\mathrm{~A}_{2} \rightarrow \pi \gamma\right)=$ $=0.45$ (all values in MeV ). Numerical integration leads then to the results (for polarizabilities the units of $10^{-4} \mathrm{fm}{ }^{3}$ are employed throughout this paper):

$$
\begin{gathered}
(a-\beta){ }_{\pi}^{(\mathrm{s})} \pm=-0.98+2.13+1.37 \approx 2.5, \\
\quad(\rho) \quad\left(\mathrm{A}_{1}\right) \quad\left(\mathrm{A}_{2}\right)
\end{gathered} \quad \begin{gathered}
(a-\beta){ }_{\pi^{\circ}}^{(\mathrm{s})}=-0.98-14.37-0.06 \approx-15.4 .
\end{gathered}
$$

$$
(\phi)
$$

Analogously one finds for the $s$-channel contribution in the fixed $u=\mu^{2}$ sum rule

Below we display for comparison the results of the evaluation of $(\alpha-\beta)_{\pi}^{(s),[s]}$ in a narrow width resonance approximation:

$$
\begin{aligned}
& (\alpha-\beta){ }_{\pi}^{(\mathrm{s})}=\frac{2}{\pi \mu}\left[-\frac{g_{\rho}^{2}\left(\mathrm{M}_{\rho}^{2}+\mu^{2}\right)}{\mathrm{M}_{\rho}^{2}-\mu^{2}}+\frac{\mathrm{g}_{\mathrm{A}_{1}}^{2}\left(\mathrm{M}_{\left.\mathrm{A}_{1}+\mu^{2}\right)}^{2}\right.}{\mathrm{M}_{\mathrm{A}_{1}}^{2}-\mu^{2}}+\frac{5}{3} \frac{\mathrm{~g}_{\mathrm{A}_{2}}^{2}\left(\mathrm{M}_{\mathrm{A}_{2}}^{2}+\mu^{2}\right)}{\mathrm{M}_{\mathrm{A}_{2}}^{2}-\mu^{2}}\right]= \\
& \quad \simeq(-1.0+3.2+2.3)=4.5, \\
& \quad(a-\beta)_{\pi_{0}^{0}}^{(\mathrm{s})}=-\frac{2}{\pi \mu} \sum_{\mathrm{R}=\rho, \omega, \phi} \frac{\mathrm{g}_{\mathrm{R}}^{2}\left(\mathrm{M}_{\left.\mathrm{R}^{+}+\mu^{2}\right)}^{2}\right.}{\mathrm{M}_{\mathrm{R}}^{2}-\mu^{2}}=(-1.0-14.2-0.03)=-15.2 ;
\end{aligned}
$$

$$
\begin{align*}
& (\alpha-\beta){ }_{\pi^{ \pm}}^{[\mathrm{s}]} .  \tag{*}\\
& (\rho) \quad\left(A_{1}\right) \quad\left(A_{2}\right) \\
& (a-\beta)_{\pi^{\circ}}^{[\mathrm{s}]}=\begin{array}{ccc} 
& -0.94 & -13.91 \\
(\rho) & -0.06 & (\omega) \\
& (\phi)
\end{array} \tag{-}
\end{align*}
$$

$$
\begin{aligned}
& (\alpha-\beta)_{\pi \pm}^{[\mathrm{s}]}=\frac{2}{\pi \mu}\left[-: \frac{\mathrm{g}_{\rho}^{2} \mathrm{M}_{\rho}^{2}}{\mathrm{M}_{\rho}^{2}-\mu^{2}}+\frac{\mathrm{g}_{\mathrm{A}_{1}}^{2} \mathrm{M}_{\mathrm{A}_{1}}^{2}}{\mathrm{M}_{\mathrm{A}_{1}-\mu^{2}}^{2}}+\frac{5}{3} \cdot \frac{\mathrm{~g}_{\mathrm{A}_{2}}^{2} \mathrm{M}_{\mathrm{A}_{2}}^{2}}{\mathrm{M}_{\mathrm{A}_{2}-\mu^{2}}^{2}} \cdot \frac{\mathrm{M}_{\mathrm{A}_{2}}^{2}+3 \mu^{2}}{\mathrm{M}_{\mathrm{A}_{2}-\mu^{2}}^{2}}\right] \simeq 4.7,\left(11^{\prime}\right) \\
& (a-\beta)_{\pi^{\circ}}^{[\dot{s}]}=-\frac{2}{\pi \mu} \quad \sum \quad \sum \quad \frac{g_{\mathrm{R}=\rho, \omega, \phi}^{2} \mathrm{M}_{\mathrm{R}}^{2}}{\mathrm{M}_{\mathrm{R}}^{2}-\mu_{2}^{2}}=-15.1 ; \\
& g_{R}=\left[\frac{6 \pi M_{R}^{3} \Gamma(R \rightarrow \pi \gamma)}{\left(M_{R}^{2}-\mu^{2}\right)^{3}}\right]^{1 / 2}, \quad R=\rho, \omega, \phi, A_{1}, A_{2} .
\end{aligned}
$$

The $A_{1}$ contribution has been computed using for the vertex $\mathrm{A}_{1} \pi \gamma$ the expression ${ }^{13 /} 2 \mathrm{~g}_{\mathrm{A}_{1}} \mathrm{~F}_{\mu \nu} \mathcal{F}^{\mu \nu} \phi_{\pi}$, where $\phi_{\pi}, \mathrm{F}_{\mu \nu}$ and $\mathscr{F}^{\mu \nu}$ stand for the pion, the electromagnetic and the $A_{1}$ fields. Before discussing the $t$-channel pieces $(a-\beta)_{\pi}^{(t)}[t]$ we mention that saturation of the known sum rule

$$
\begin{equation*}
\alpha+\beta=\frac{1}{2 \pi^{2}} \int_{\omega_{0}=3 \mu / 2} \frac{\sigma_{\mathrm{T}}(\omega) \mathrm{d} \omega}{\omega^{2}} \tag{12}
\end{equation*}
$$

( $\sigma_{\mathrm{T}}$ the total cross-section for photoabsorption on pions) by retaining only E1, M1, M2 transitions and proceeding as we did above in connection with $(\alpha-\beta)_{\pi}^{(s),[s]}$, yields the (almost certainly underestimated) values

$$
\begin{equation*}
(\alpha+\beta)_{\pi^{ \pm}} \approx 0.2, \quad(\alpha+\beta)_{\pi^{\circ}} \approx 0.5 \tag{13}
\end{equation*}
$$

Saturation of Eq. (12) in narrow width approximation (with the same set of intermediate states) leads to

$$
(a+\beta)_{\pi^{ \pm}} \simeq 0.2, \quad(\alpha+\beta)_{\pi^{\circ}}=0.9
$$

The evaluation of the $t$-channel contributions $(\alpha-\beta)_{\pi}^{(t),[ }$ is a much more delicate task and at least reasonable knowledge of the appearing $\gamma y \rightarrow \pi \pi$ amplitudes is needed. The $\gamma \gamma \rightarrow \pi \pi$ process has been investigated by several groups from measurements of the colliding beam reaction $\mathrm{e}^{+} \mathrm{e}^{-\rightarrow} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \pi \pi$ (see the review $/ 3 /$ ). The dominant feature observed is a strong signal from, the $\mathrm{f}(127.0 \mathrm{MeV})\left[\mathrm{I}^{\mathrm{G}}\left(\mathrm{J}^{\mathrm{P}}\right) \mathrm{C}_{\mathrm{n}}=0^{+}\left(2^{+}\right)_{+}\right]$meson; so far no trace of the $\epsilon(\approx 700 \mathrm{MeV})$ meson seems to appear; the TASSO group provides the limit $\Gamma(\epsilon \rightarrow y y)<1.5 \mathrm{keV}$ for the radiative width of the $\epsilon$ meson ${ }^{14 /}$. The Crystal Ball group at SPEAR has found by studying the angular distribution of the $\mathrm{f} \rightarrow \pi^{\circ} \pi^{\circ}$ decay that the production of the $\mathrm{f}(1270 \mathrm{MeV})$ meson in $\gamma \gamma$ scattering is strongly dominated by photon pairs with opposite helicity $(\Gamma(f) \gamma \gamma)=3 \mathrm{keV}, \Gamma[\mathrm{f} \rightarrow \gamma(+) \gamma(+)] \ll \Gamma[\mathrm{f} \rightarrow \gamma(+) \gamma(-)]$, confirming so previous theoretical expectations ${ }^{15 /}$. Since the
helicity channel of interest to us is that with both protons of helicity +1 (see Eq. (4)), the general scheme which seems to be required for our purposes is to consider $(\alpha-\beta)_{\pi}^{(t)},[t]$ as dominated by $\mathrm{I}=\mathrm{J}=0$ contributions and employ a model for the ( $\mathrm{I}=\mathrm{J}=0$ ) yy $\rightarrow \pi$ amplitude which does not deviate too much from its corresponding Born (quantum electrodynamical) expression. The simplest way of satisfying these demands would be to use for the absorptive part $\mathrm{M}^{(t)}$ in Eqs. (3), $\left(3^{*}\right)$ an effective $\epsilon\left(0^{+}\right)$Breit-Wigner model assuming a large total width for this resonance $/ 16 /$ :

$$
\begin{align*}
& M_{(t)(I=J=0)}^{(t)}=\frac{3}{2} g_{\epsilon \gamma \gamma} g_{\epsilon \pi \pi}\left(-\frac{4}{M_{\epsilon}^{2}}\right) \frac{\left(\frac{\mathrm{t}-4 \mu^{2}}{M_{\epsilon}^{2}-4 \mu^{2}}\right)}{\left(M_{\epsilon}^{2}-t\right)^{2}+M_{\epsilon}^{2} \Gamma^{2}\left(\frac{\mathrm{t}-4 \mu^{2}}{M_{\epsilon}^{2}-4 \mu^{2}}\right)} \\
& \mathrm{g}_{\epsilon \gamma \gamma}=4\left[\pi M_{\epsilon} \Gamma(\epsilon+\gamma \gamma)\right]^{1 / 2}  \tag{14}\\
& g_{\epsilon \pi \pi}=4 M_{\epsilon}\left[\frac{2 \pi}{3}: \frac{\Gamma(\epsilon \rightarrow \pi \pi)}{\left(M_{\epsilon}^{2}-4 \mu^{2}\right)^{1 / 2}}\right], \quad \Gamma \simeq \Gamma(\epsilon \rightarrow \pi \pi) .
\end{align*}
$$

Recalling the, relationship between charge and isospin labels

$$
\begin{align*}
& M^{\left(\pi^{ \pm}\right)}=\frac{2}{3}\left[M^{(\mathrm{I}=0)}+\frac{1}{2} M^{(\mathrm{I}=\ell)}\right]  \tag{15}\\
& M^{\left(\pi^{g}\right)}=\frac{2}{3}\left[M^{(\mathrm{I}=0)}-\mathrm{M}^{(\mathrm{I}=2)}\right]
\end{align*}
$$

taking $M \stackrel{3}{=} 660 \mathrm{MeV}, \Gamma_{\text {total }}=\Gamma(\epsilon \rightarrow \pi \pi)=640 \mathrm{MeV}, \Gamma(\epsilon \rightarrow \gamma \gamma)=1.3 \mathrm{keV}$ and integrating over t in Eqs. (3), (3') from $4 \mu^{2}$ to $\infty$, one finds

$$
\begin{equation*}
(a-\beta) \stackrel{(t),[t] ;(\epsilon)}{\pi^{ \pm}, \pi^{\circ}}=8.3 \tag{16}
\end{equation*}
$$

To what extent this value is representative for the actual $(\alpha-\beta)_{\pi}^{(t),[t]}$ is hard to say. In the following we shall confine ourselves to the more modest task of computing the ( $\mathrm{I}=\mathrm{J}=0$ ) contribution to $(a-\beta)_{\pi}^{(t)},[t]$ coming only from the elastic unitarity region $4 \mu^{2} \leq t \leq 16 \mu^{2}$ of the t-channel cut, leaving so open the question of higher waves and higher than $\pi \pi$ states contributions to the unitarity sum. The partial waves $\mathrm{g}_{+}^{(\mathrm{I}=\mathrm{J}=0 \mathrm{f})}(\mathrm{t})$ and $h^{(1 \times J=0)}(t)$ entering Eqs. (3a), (3a') are taken as given by the resonance model devised in the last of refs. ${ }^{\prime 6 /}$ (see for details ref. ${ }^{17 /}$ ). We recall that although very crude, this model incorporates (approximately) the experimental knowledge of the $\pi \pi \rightarrow \pi \pi(\mathrm{I}=\mathrm{J}=0)$ phase shift and worked apparently well in connection with the proton polarizabilities and proton Compton scattering. We have found

$$
\begin{align*}
& (a-\beta)_{\pi}^{(\mathrm{t}), \pi \pi(\mathrm{I}=\mathrm{J}=\mathrm{C} 0)}=(a-\beta)_{\pi}^{[\mathrm{t}], \pi \pi(\mathrm{I}=\mathrm{J}=0)}=  \tag{17}\\
& =-\frac{1}{2 \pi^{2} \mu_{4}} \int_{4 \mu^{2}}^{16 \mu^{2}} \frac{\mathrm{dt}}{\mathrm{t}^{2}} \mathrm{~g}_{+}^{(\mathrm{I}=\mathrm{J}=0)}(\mathrm{t}) \mathrm{h}^{*}(\mathrm{I}=\mathrm{J}=0) \\
& (\mathrm{t})=3.7
\end{align*}
$$

and hence, taking into account Eqs. (15),

$$
\begin{equation*}
(a-\beta) \underset{\pi^{ \pm}, \pi^{\circ}}{(\mathrm{t}),[\mathrm{t}]} \quad(\pi \pi ; \text { s-wave } ; 4 \mu \leq \mathrm{t} \leq 16 \mu) \simeq 2.5 \tag{18}
\end{equation*}
$$

Although it has to be regarded with some caution, this value for the $s$-wave contribution of the elastic unitarity portion of the $t$-channel cut should be typical for situations in which the amplitude $g_{+}^{(I=J=0)}(\mathrm{t})$ does not differ too much from its Born approximation and does not have a zero in the immediate vicinity of the threshold. The amplitude $g_{+}$actually employed by us develops a zero at $t \simeq 21 \mu^{2}$; in order to keep the whole amplitude $\mathrm{g}_{+}(\mathrm{t})$ close to its Born approximation, an arbitrary subtraction constant appearing in the N/D equations which determine it has been fixed by demanding that at threshhoid ( $t=4 \mu \quad)^{g_{+}} \simeq g_{+}^{\text {Born }}$

The authors of ${ }^{+}$ref. ${ }^{i 18 /}$ remove an analogous ambiguity by relating the subtraction constant to ( $\alpha-\beta$ ) proton in the context of a backward sum rule for the latter. Their resulting partial wave $g_{+}(t)$ in the region $4 \mu^{2} \leq t \leq 16 \mu^{2}$ does not seem to differ too much from ours.

Strictly speakinng, since the ( $I=J=0$ ) t-channel piece is the same in both the fixed $\theta=180^{\circ}$ and fixed $u=\mu^{2}$ sum rules (Eqs. (3a) and (3a")) other waves should also be included in $(\alpha-\beta)^{(t)}$ and $(\alpha-\beta)^{[\mathrm{t}]}$ to avoid inconsistencies between the two. sum rules (as seen from Eqs. (9), ( $9^{\circ}$ ), (10), ( $10^{\circ}$ ), ( $\left.a-\beta\right)^{(3)}$ and $(\alpha-\beta)[s]$ although practically equal for $\pi^{ \pm}$differ somewhat in the $\pi^{\circ}$ case). If the large uncertainties affecting the $I=J=0 \quad$ contribution could be removed, one may try to use simultaneously the two sum rules in order to constrain less reliable contributions from higher waves.

The large model dependence of the $t$-channal contributions in the sum rules discussed here has as its corrospondent in the FFESR approach presentad in rop. ${ }^{\prime \prime}$ tho oqually large uncertainties affecting the high onorgy noymptotic contributions which account for tho samo annihilation channcl qefects by means of Reggo polas oxchangoo. It la worth noting that for the combination (ol t-channol lsoopin lal $(a-\beta) \pi^{t}-(a-\beta) \pi^{\circ}$, almost entirely depondont only upon B -channal affacts, both our results and that of rof. 0 agroo romarkably well with each other. Indcod, ono Pinds $(a-\beta)_{n} \pm-(n-\beta)_{\pi^{\circ}} \simeq 18$ (from Eqs. (9), (10)) $=17.5$ (from Eqs. (9 $),\left(10^{\circ}\right)$ ) $\simeq 20$ (from Eqs. (11),
(11')) in this work while Table 6 of ref. ${ }^{\prime 9} /$ gives the value 20.5
We conclude with the remark that for a reliable calculation of the annihilation channel contributions to the pion polarizabilities which would permit a good prediction not only for $a_{\pi^{ \pm}-a_{\pi^{\circ}}}$ but for $a_{\pi}^{ \pm}$and $a_{\pi}^{\circ}$ separately, further more detailed experimental investigation of the $y \gamma \rightarrow \pi \pi, y y \rightarrow \pi \pi \pi \pi \quad, \gamma y \rightarrow \mathbf{k} \overline{\mathrm{k}}$, $y y \rightarrow \mathrm{p} \overline{\mathrm{p}}$ reactions is needed in order to obtain the necessary information on the helicity channe $\lambda=0$ (both photons with equal helicity) of interest in this context.

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Received by Publishing Department
on March 191982.

Фильков Л.В., Гуяшу И., Радеску Е.Е.
Поляризуемость пиона из правил сумм назад и при фиксированном и
Различные вкпады в полнриауемость пиона оцениваются с помощью правил сумм, полученных из дисперсионных соотношений назад и при фиксированном $\mathbf{u}=\mu^{2} / \mu$ - масса пиона/ для соответствующих комптоновских амплитуд. Вклад 8 -канала вычислиется достаточно надежно с использованием известных сильных и радиационных щирин меяоных резонансов. Однако модельная зависимость в оценке t -канальной части остается сильной, несмотря на важные прояснения, которые принесли недавно экспериментальные данные по процессу у $\gamma \rightarrow \pi \pi$.

Работа выполнена а Лаборатории теоретической физики оияи.

Сообщение Объединенного института ядерных исследований. Дубна 1982

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E4-82-133
Pion Polarizabilities from Backward and Fixed u Sum Rules
Various contributions to the pion polarizabilities are estimated using sum rules obtained from backward and fixed $\mathbf{u}=\mu^{2}$ ( $\mu$ - pion mass) dispersion relations for the relevant pion Compton scattering amplitude. While the $s$-channel part can be quite reliably computed in terms of the strong and radiative widths of known meson resonances, the evaluation of the $t$ channel piece remains highly model dependent despite important clarifications provided recently by measurements of the $y \gamma \rightarrow \pi \pi$ reaction.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.


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