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**SIMPLE CONFIGURATIONS  
IN THE CAPTURE STATE  
AND THE GENERAL PICTURE  
OF NUCLEAR STATE COMPLICATIONS**

**1974**

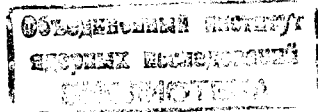
**ЛАБОРАТОРИЯ  
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ**

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V.G.Soloviev

**SIMPLE CONFIGURATIONS  
IN THE CAPTURE STATE  
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Простые конфигурации в захватных состояниях и усложнение структуры состояний с ростом энергии возбуждения

Изложены основные положения модели для описания фрагментации, основанной на учете взаимодействия квазичастиц с фононами. Приведены результаты расчетов по фрагментации одноквазичастичных состояний.

Изложены общие положения подхода, основанного на операторной форме волновой функции высоковозбужденного состояния, и оценен вклад отдельных простых конфигураций в волновых функциях нейтронных резонансов. Продемонстрировано, когда можно использовать модель валентного нейтрона. Обсуждена структура компаунд-состояний.

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Simple Configurations in the Capture State  
and the General Picture of Nuclear State  
Complications

See the Abstract in the text.

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It is asserted that the effective nuclear forces and the methods of solving the many-body problem may serve as a basis for describing states of low, intermediate and high excitation energy. It is indicated that it is important to study nuclear structure complications with increasing excitation energy and fragmentation of single-particle and many-particle states.

The foundations of the model for describing fragmentation which is based on the account of the quasiparticle-phonon interaction are presented. The results of calculations of fragmentation for the  $631\text{h}$ ,  $640\text{h}$ ,  $620\text{h}$  and  $600\text{h}$  one-quasiparticle states in  $^{239}\text{U}$  are given. It is shown that the state strength is distributed over a wide energy interval, the distribution maximum is shifted down with respect to the single-particle energy. The degree of fragmentation is shown to depend strongly on the position of the single-particle level with respect to the Fermi level. A modified version of the model is given for treating highly excited states of the type of giant resonances and for studying their influence on the structure of states of intermediate excitation energy.

The general assumptions of the approach based on the operator form of the wave function of highly excited states are presented, and the contribution of individual simple configurations to the neutron resonance wave functions is estimated. It is shown in which case the valence neutron model can be employed. The compound-state structure is discussed.

I. The general regularities of the behaviour of the excited states of complex (medium-weight and heavy) nuclei are rather well studied. They reflect the universal properties of the atomic nuclei and are the following:

1) In all the nuclei there are low-lying quasiparticle and vibrational collective states. In deformed nuclei they generate rotational bands. These states are rather simple, they are well studied and are correctly described in terms of quasiparticles and phonons, e.g., ref. <sup>1</sup>.

11) The highly excited states are very complex and are defined by a large number of degrees of freedom. The highly excited

state wave functions contain thousands of various components. Random distribution of most degrees of freedom permits interpreting these compound states within the framework of the statistical model.

iii) Among the highly excited states are states with rather large few-quasiparticle and one-phonon components. These are the isobar analogue states, the isospin of which is by unity higher than that of the surrounding states and the collective states which are given the name of giant resonances. There is a large group of giant resonances corresponding to different multipoles with isoscalar and isovector parts. Among them the giant E1-resonances are the most completely studied ones.

iv) With increasing excitation energy the level density grows and the level structure becomes more complicated. Thus, a transition proceeds from simple low-lying states to more complicated states with intermediate excitation energy and then to very complicated highly excited states. The structure complication proceeds in different ways in magic, vibrational, transition and deformed nuclei. However, in all the medium and heavy nuclei at the neutron binding energy  $B_n$  the states are so complicated that they are usually described within the framework of the statistical model.

One has long been showing an increasing interest in the study of the structure of highly excited states. Progress is associated with the construction of the resonance reaction theory<sup>2</sup> and the development of neutron spectroscopy, e.g., ref.<sup>3</sup>. Of much importance is also the description of highly excited states within the framework of the particle-hole approach<sup>4</sup> and its further development. Valuable experimental information about highly excited states is extracted from charged particles and heavy ion induced reactions. Of a special importance are electron-nucleus scattering and photonuclear reactions, e.g., ref.<sup>5</sup>.

One of the main universal properties of nuclei, namely complication of the state structure with increasing excitation energy, is worst studied. The present report is mostly devoted to some attempts to find a general approach to the study of this problem.

The detailed study of nuclear structure shows that theoretically the transition from low-lying states to states of intermediate excitation energy is predominant. However, the complex experimental study of intermediate states by means of nuclear reactions and mass separators on particle beam advances slowly. A large amount of progress has been made in the field of neutron resonances. A special situation with neutron resonances is due to the

availability of complete and exact experimental data rather than to their physical meaning. The wave functions of neutron resonances are very complex, and therefore we may only hope to find some of their few-quasiparticle components. The problem of extracting from experimental data some information about a number of simplest configurations in the neutron resonance wave functions is also discussed in the present report.

2. The low-lying states of complex nuclei are rather well described within the framework of the semi-microscopic approach. The Hamiltonian describing the interaction between the nucleons in the nucleus is of the form

$$H = H_{av} + H_{pair} + H_R + H' \quad (I)$$

In other words, the effective nuclear interaction is represented in the form of the average field described by the Saxon-Woods potential, the interactions leading to superconducting pairing correlations, the multipole-multipole and other interactions. In the semi-microscopic description one calculates the relative values rather than the absolute ones, for example, the energies of excited states with respect to the ground state or changes of the nucleus energy with increasing deformation parameters.

An analysis performed in ref.<sup>6</sup> shows that the effective nuclear interactions (I) and the methods of solving nuclear many-body problem may serve as a basis for describing low, intermediate and highly excited states of atomic nuclei.

In the understanding of the structure of highly excited states and their description in the language of quasiparticles and phonons in which the low-lying states are also treated the main role is attributed to the process called fragmentation of fractionation. By fragmentation we mean the distribution of the strength of a one-, two- or many-particle states over many nuclear levels. In other words, fragmentation is responsible, for example, for the distribution of the strength of the single-particle state being the solution of the Schrödinger equation with the Saxon-Woods potential over some nuclear levels.

There are two main causes leading to complications of the state structure with increasing excitation energy: the first one is the interaction of the single-particle and collective motions described as the interaction of quasi-particles with phonons; the second cause is the coupling of the intrinsic and rotational motions described via the Coriolis interaction.

The quasiparticle-phonon interaction is of much importance in the process of fragmentation. It leads to mixing of the compo-

nents differing by one phonon or by two quasiparticles. As a result of accounting for the quasiparticle-phonon interaction, the wave function has the form of a sum of one-, two-, three- and higher phonon components in the case of doubly even nuclei.

The quasiparticle-phonon interaction strongly affects the structure of low-lying states of deformed<sup>1,7,8</sup> and spherical<sup>1,9</sup> nuclei. It results in a fragmentation of the strength of one-phonon, two-phonon and many-phonon states over nuclear levels. Some examples of fragmentation of the strength of subshells due to deformation and fragmentation of two-quasiparticle states caused by multipole-multipole phonon-generating interactions are given in ref.<sup>10</sup>.

The development of a unified description of low, intermediate and highly excited states and the study of the process of structure complication with increasing excitation energy are performed by us along two lines: 1) a general semi-microscopic description on the basis of the operator form of the wave function<sup>10-13</sup>; and 2) calculations on the basis of a model taking into account the interactions of quasiparticles with phonons<sup>10,14-17</sup>.

3. Let us consider a model for the description of fragmentation in the case of an odd deformed nucleus. Originally in ref.<sup>14</sup> fragmentation was studied by the method applied for the calculation of the structure of low-lying states. It was also shown that this method requires some modifications. In ref.<sup>15</sup> a model for describing the nonrotational state structure at intermediate and high energies and for studying fragmentation was formulated which was then generalized in ref.<sup>16</sup>.

The model Hamiltonian is taken in the form (I) without the term  $H'$ . The wave function of a nonrotational state with a given  $K^\pi$  is written as follows

$$\begin{aligned} \psi_i(K^\pi) = & \frac{C_p^i}{\sqrt{2}} \sum_{\sigma} \{ \mathcal{L}_{p\sigma}^+ + \sum_{g,\nu} D_{p\sigma\nu}^{g_i} \mathcal{L}_{\nu\sigma}^+ Q_{g\nu}^+ + \\ & + \sum_{g_2,\nu_2} \sum_{\nu} F_{\sigma\nu\nu_2}^{g_2 g_i} \mathcal{L}_{\nu\sigma}^+ Q_{g_2\nu_2}^+ \} \psi_0, \end{aligned} \quad (2)$$

where  $\psi_0$  is the ground state wave function for a doubly even nucleus, 1 is the number of the state. The normalization condition (2) is of the form

$$\begin{aligned} (\psi_i^*(K^\pi) \psi_i(K^\pi)) = & 1 = (C_p^i)^2 \{ 1 + \sum_g \sum_{\nu} (D_{p\nu}^{g_i})^2 + \\ & + 2 \sum_{g_2,\nu_2} \sum_{\nu} (F_{\nu\nu_2}^{g_2 g_i})^2 \}, \end{aligned} \quad (3)$$

since by introducing the phase factors  $\delta_1$  and  $\delta_2$  it is possible to get rid of the  $\sigma$  dependence. We use the following notation:

$Q_g$ ,  $\omega_g$  and  $\gamma_g$  are the operator, the energy and the phonon characteristic; by  $g$  we mean  $\lambda, \mu_j$ ,  $f^g(\nu, \nu')$  is the matrix element of the multipole operator  $q = \lambda, \mu$ ,  $j$  is the number of the root of the secular equation for a phonon;  $\mathcal{L}_{\nu\sigma}^+$  and  $\mathcal{E}(\nu)$  are the creation operator and the quasiparticle energy, respectively;  $\Gamma^g(\nu, \nu') = U_{\nu\nu'} f^g(\nu, \nu') / 2\sqrt{\gamma_g}$ ,  $U_{\nu\nu'} = U_\nu U_{\nu'} - V_\nu V_{\nu'}$ ,  $(\nu\sigma)$  and  $G = \pi/2$  is the set of quantum numbers of a single-particle level,  $(\rho\sigma)$  the same for levels with fixed  $K^\pi$ .

The nonrotational state energies  $\zeta_i$  and the functions  $C_p^i$ ,  $D_{p\nu}^{g_i}$  and  $F_{p\nu\nu_2}^{g_2 g_i}$  are defined by the variational principle

$$\delta \{ (\psi_i^*(K^\pi) H_M \psi_i(K^\pi)) - \zeta_i [ (\psi_i^*(K^\pi) \psi_i(K^\pi)) - 1 ] \} = 0. \quad (4)$$

After some transformations the system of basic equations is written in the form

$$\mathcal{E}(\nu) - \zeta_i - \sum_g \sum_{\nu'} \Gamma^g(\rho, \nu) D_{\rho\nu'}^{g_i} = 0, \quad (5)$$

$$\begin{aligned} (\mathcal{E}(\nu) + \omega_g - \zeta_i) D_{p\nu}^{g_i} - \sum_{g_2} \sum_{\nu_2} \frac{1}{p(\nu_2, g_2) - \zeta_i} \left[ \delta_1 \Gamma^{g_2}(\nu, \nu_2) \Gamma^{g_2}(\nu_2, \nu_2) D_{p\nu_2}^{g_2 i} + \right. \\ \left. + \delta_2' \Gamma^{g_2}(\nu, \nu_2) \Gamma^g(\nu_2, \nu_2) D_{p\nu_2}^{g_2 i} \right] = \Gamma^g(\rho, \nu) \end{aligned} \quad (6)$$

$$F_{p\nu\nu_2}^{g_2 g_i} = \frac{1}{p(\nu_2, \nu_2) - \zeta_i} \sum_{\nu_2} \left[ \delta_2 \Gamma^{g_2}(\nu_2, \nu) D_{p\nu_2}^{g_2 i} + \delta_2' \Gamma^g(\nu_2, \nu) D_{p\nu}^{g_2 i} \right]. \quad (7)$$

The poles like  $p(\nu, g_2) = \mathcal{E}(\nu) + \omega_g + \omega_{g_2}$  are called fundamental poles. To find the energies  $\zeta_i$  it is necessary to solve eq. (6) with respect to  $D_{p\nu}^{g_i}$  and insert the obtained expression in eq. (5). After solving eq. (5), that is after finding the roots  $\zeta_i$ , and using eqs. (3) and (7) it is not difficult to derive  $F_{p\nu\nu_2}^{g_2 g_i}$  and  $C_p^i$ . The solution for eq. (6) can be represented in the form

$$D_{p\nu}^{g_i} = \frac{A_p^g}{\Delta}, \quad (8)$$

where the denominator  $\Delta$  is the determinant of the system, and the numerator  $A_p^g$  is the determinant which is obtained from  $\Delta$  by replacement of the coefficients for the unknown quantity by the appropriate free terms. In some real cases, when a large number of single-particle states and phonons are taken into account, eq. (8) comprises determinants of the order of  $10^4$  and higher.

An approximate method of solving eqs. (5) and (6) was develop-

ped in ref.<sup>17</sup>. According to this method, all the coherent terms and pole noncoherent terms are taken into account. The obtained approximate equations which differ from each other by the pole terms, are solved analytically. The expressions for  $D_{p,i}^{j'}$  are inserted in eq. (5) which is then solved numerically. In each case the solution corresponding to the fundamental pole is sought. An effective method of numerical solution of the secular equations (5) was suggested there, and the secular equation derived by this method contain no superfluous solutions.

The accuracy of the approximate solution is investigated in ref.<sup>17</sup> by the example of a restricted basis. The comparison of the wave function components for the exact and approximate solutions shows that their large components are close to one another, while very small components may strongly differ from one another.

The approximate method of solving the model equations is used for the study of fragmentation of the one-quasiparticle, quasiparticle plus phonon and quasiparticle plus two-phonons states over many nuclear levels, as well as for the study of the structure of the nonrotational states of intermediate excitation energy in odd deformed nuclei.

The study of the model shows that there is a direct and rather simple method of calculating the level density at different excitation energies and different spins. It is based on the calculation of the number of fundamental poles with fixed  $K^\pi$  or  $I^\pi$  in a given energy interval. This method was used in refs.<sup>18,19</sup> to calculate the density of a large number of spherical and deformed nuclei. The account of vibrational and rotational motions has resulted in a good description of the level density near  $B_n$ .

At intermediate excitation energies, the energy and spin dependence of the level density calculated by our method may strongly differ from the statistical model calculations. For example, in <sup>157</sup>Fe at  $\epsilon = B_n \pm 0.5$  MeV the density of negative parity states is larger than that of positive parity states by a factor of 1.5 - 2.0. On the contrary, in <sup>58</sup>Fe the density of the positive parity states is several times larger than that of negative parity states. In both the nuclei the  $\rho(I)$  dependence is close to that obtained by the statistical model. At high excitation energies the results of semi-microscopic calculations come nearer to the statistical ones. At high energies troubles with direct calculations become much more serious while the accuracy becomes worse due to insufficiently correct account of the Pauli principle because of the neglect of degeneration of certain states in spherical nuclei and ambiguity in the expression of the many-

-quasiparticle configuration in terms of noncollective phonons. Therefore one should use the direct method for  $\epsilon \leq B_n$  and the statistical one for  $\epsilon > B_n$ .

The correct description of the level density provides also evidence for the fact that this model may serve as a basis for describing the highly excited state structure since the configurational space of the model is large enough to cover the complexity of highly excited states.

4. We give a part of the results on fragmentation obtained in collaboration with Dr. Malov by means of an approximate solving of eqs. (5) and (6). The calculations are made for the states with  $K^\pi = 1/2^+$  in <sup>239</sup>U. The Saxon-Woods potential parameters, the interaction constants and some particular features of the calculations are given in ref.<sup>15</sup>. In studying fragmentation, similarly as in calculating the level density, we use the parameters which were fixed in the study of low-lying states. Fifteen multipolarities  $\lambda\mu$ , which are given in Table I, and 10 - 70 solutions of the secular equations for phonons are utilized in the calculations. The wave function (2) is seen to have a large number of different components and therefore it can describe the complex structure of states.

Table I  
The  $g = \lambda\mu$  values and the number of terms of the wave function (2).

$\lambda\mu$								Number of terms of the type	
20	22	30	31	32	33	41	43	$L_{\nu\sigma}^+ Q_g^+$	$L_{\nu\sigma}^+ Q_g^+ Q_{g_1}^+$
44	54	55	65	66	76	77			
j = 1, 2, ... 10								870	$9 \cdot 10^4$
j = 1, 2, ... 35								$1 \cdot 10^4$	$1 \cdot 10^6$
j = 1, 2, ... 70								$5 \cdot 10^4$	$5 \cdot 10^6$

Finding of the solutions for eqs. (5) and (6) and calculation of the wave function components require much electronic computer time. Up to the present time one has obtained a total of about 100 solutions for these equations and for the appropriate wave functions. As an example, in table 2 we give a typical state structure. The component values are calculated from the normalization condition for the wave function and are expressed in percent. This state corresponds to the  $622^+ + Q_1(31) + Q_1(31)$  fundamental pole of an energy 2.07 MeV. For the phonon we use the

Table 2

Squared components ( in percent) of the wave function (2) for the 1.9 MeV  $1/2^+$  state in  $^{239}\text{U}$

Configuration	Components %
620†	6.10 <sup>-4</sup>
501† + Q <sub>1</sub> (31)	28.98
761† + Q <sub>1</sub> (31)	1.47
752† + Q <sub>1</sub> (31)	0.15
725† + Q <sub>1</sub> (55)	0.04
624† + Q <sub>2</sub> (44)	0.01
622† + Q <sub>1</sub> (31) + Q <sub>1</sub> (31)	52.42
770† + Q <sub>1</sub> (22) + Q <sub>1</sub> (31)	6.66
631† + Q <sub>1</sub> (30) + Q <sub>1</sub> (31)	2.14
606† + Q <sub>1</sub> (31) + Q <sub>1</sub> (55)	1.99
501† + Q <sub>1</sub> (20) + Q <sub>1</sub> (31)	1.30
743† + Q <sub>1</sub> (22) + Q <sub>1</sub> (31)	0.88
503† + Q <sub>1</sub> (22) + Q <sub>1</sub> (31)	0.69
624† + Q <sub>1</sub> (31) + Q <sub>1</sub> (55)	0.44
606† + Q <sub>1</sub> (31) + Q <sub>1</sub> (55)	0.30
503† + Q <sub>1</sub> (31) + Q <sub>1</sub> (44)	0.25
633† + Q <sub>1</sub> (31) + Q <sub>1</sub> (31)	0.22

of the strength from level to level are due to the fact that, firstly, account is taken of only the multipole collective states, other collective vibrations, e.g., giant resonances, are disregarded. Secondly, the wave function (2) has no terms containing three and more phonons, thirdly, eqs. (5) and (6) are solved approximately. To get better description the approximate solutions for eqs. (5) and (6) corresponding to the one-quasiparticle and quasiparticle plus phonon fundamental poles should be improved. It is also necessary to perform in the wave function (2) a summation over one-quasiparticle states  $\rho$ .

Let us consider the fragmentation of the one-quasiparticle states and look how it changes depending on the position of the single-particle level with respect to the Fermi level. We take four one-quasiparticle states with  $K^\pi = 1/2^+$ , two of them are near the Fermi level, one state is a high particle state and the other is a deep hole state. To study the fragmentation of these states

notation  $Q_j(\lambda\mu)$  and for the single-particle state the Nilsson model notation  $Nn_z \lambda \dagger$  and  $Nn_z \lambda \dagger$ . The root of eq. (5) is lowered by 0.17 MeV respectively to the fundamental pole. With increasing excitation energy the roots drop down still further with respect to the fundamental poles quasiparticle plus phonon. It is seen from table 2 that 52.4% of the state strength is concentrated about the component corresponding to the fundamental pole. Next, the phonon-containing components entering the fundamental pole are predominant. This appears to be due to the approximate method of solving eqs. (5) and (6).

In some states the value of the component corresponding to the fundamental pole reaches 90%. Insufficient fragmentation of certain states quasiparticle plus phonon and quasiparticle plus two phonons and great fluctuations

it is necessary to find several thousands of solutions for eqs. (5) and (6) and their wave functions. The investigations showed that the energies and wave functions for the solutions corresponding to the fundamental poles quasiparticle plus phonon strongly differ in the two-phonon and one-phonon approximations, i.e., in the case when the terms quasiparticle plus phonon are taken into account or disregarded. In the one-phonon approximation the fragmentation of states quasiparticle plus phonon is badly described, and for many solutions the component corresponding to the fundamental pole exceeds 99%. However, the distribution of the single-particle strength is nearly the same in the one- and two-phonon approximations. For example, for the 631† state at the first level in the one-phonon approximation the strength concentration is 90.1%, in the two-phonon one 85.6%. The total 631† strength at all the levels of an energy up to 1.9 MeV in the one-phonon approximation is 93%, in the two-phonon one 92.5%. For the 620† state without the solution corresponding to the one-quasiparticle fundamental pole the total strength of the 620 state at all the levels up to an energy 1.9 MeV is 16% in the one-phonon and 14% in the two-phonon approximations. Therefore, to study fragmentation of one-quasiparticle states we use the one-phonon approximation.

The calculation results are given in Fig. 1. The  $(C_p^i)^2$  values are calculated from the normalization condition (3), represented as a sum over the states lying in the 0.2 MeV energy interval, denoted as  $(C_p)^2 = \sum_i (C_p^i)^2$  and given in percent. The excitation energy is plotted on the abscissa axis. The  $(C_{631\dagger})^2$  quantity is plotted on the left ordinate axis. The  $(C_p)^2$  values for the 620†, 600† and 640† states are plotted on the right ordinate axis. The compact presentation of the data has given no possibility of following firmly the scale for some  $(C_p)^2$  values, which are therefore marked by appropriate numbers.

In the lower part of the figure we give the fragmentation of the 631† single-particle state placed below the 622† Fermi level by 0.2 MeV, for it  $\epsilon(631\dagger) = 0.8$  MeV. At the first level the strength concentration is 90.1%, at the levels in the 1-3 MeV interval we have another 5.9% of the strength. A total of 96.6% of the 631† strength is exhausted up to an excitation energy 3.8 MeV the remaining 3.4% relate to higher levels. Such a situation with the main component is typical of the low-lying states of odd deformed nuclei, ref. 8. The particle 620† state is by 2 MeV higher than the Fermi level, for it  $\epsilon(620\dagger) = 2.1$  MeV. At the lowest 0.8 MeV level the strength concentration is 60%, at the levels of 1-2 MeV another 16.5% of the strength is concentrated. Further the

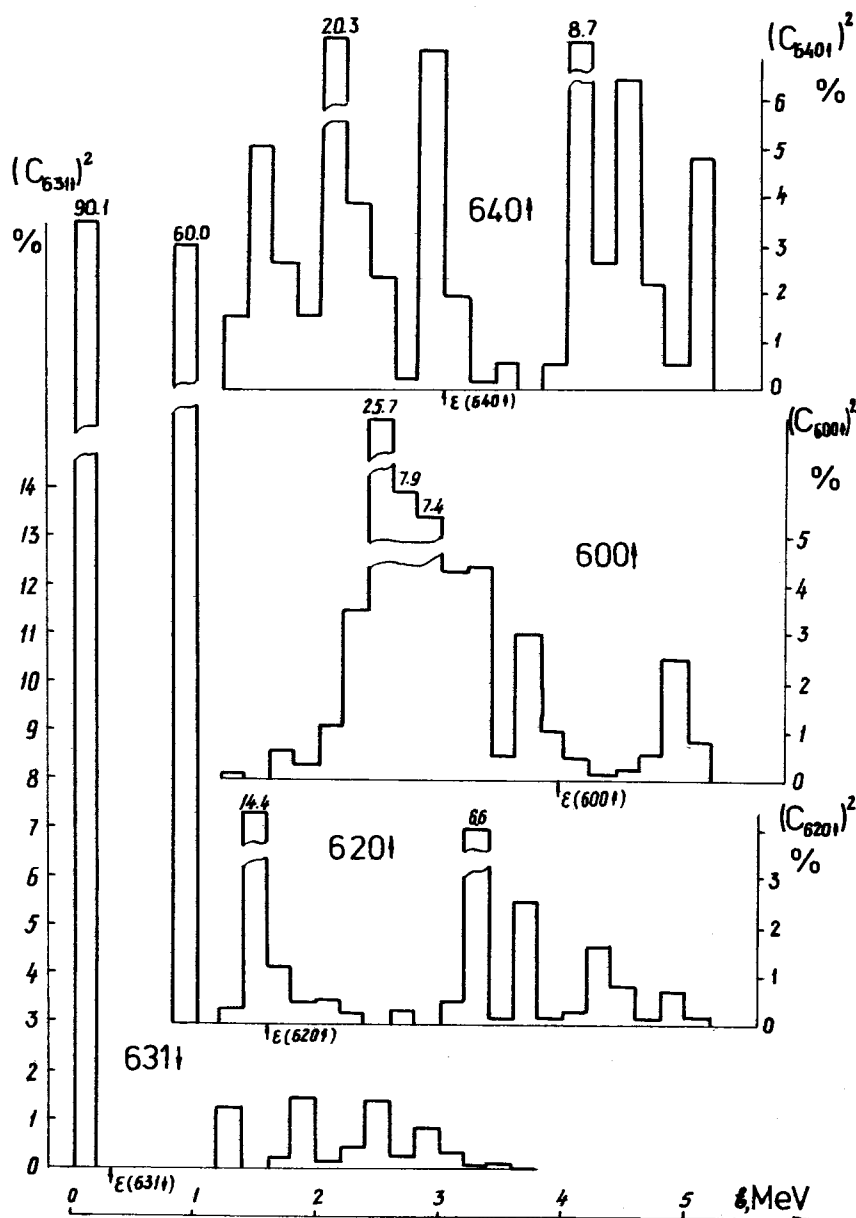


Fig.1  
Strength distribution for single-particle states with  $K^{\pi} = \frac{1}{2}^{+}$   
in  $^{239}\text{U}$

strength function has a minimum, after 3 MeV a rise is observed. Up to 5.2 MeV excitation energy 90.5% of the 620<sup>+</sup> state strength is exhausted. The study of the fragmentation of the 631<sup>+</sup> and 620<sup>+</sup> state near the Fermi surface has shown that their strength distribution has a long tail. By means of this tail it appears to be possible to explain the values of the s- and p-wave neutron strength functions in the range of their minima.

The particle 600<sup>+</sup> state is by 4.3 MeV higher than the Fermi level,  $\epsilon(660^{+}) = 4.36$  MeV. At the levels below 2 MeV there is less than 1 percent of the strength, in the interval 2.4–3.4 MeV there is 50% of the 600<sup>+</sup> state strength. Up to 5.2 MeV energy 64.5% of the strength of this state is exhausted. It is unclear how the remaining 35.5% is distributed. It should be noted, that, in spite of the common opinion, the particle state strength is distributed in a nonsymmetric way with respect to the single-particle energy. So, 75% of the 620<sup>+</sup> state strength is exhausted up to 2 MeV, 60% of the 600<sup>+</sup> state strength up to 4.3 MeV and 50% of its strength is at the levels up to 3.2 MeV. Thus, for the particle states the distribution maximum is biased in favour of low energies.

The 640<sup>+</sup> hole state is by 3.04 MeV lower than the Fermi level,  $\epsilon(640^{+}) = 3.44$  MeV. At the  $1/2^{+}$  levels of 1–3 MeV energy there is 45% of the 640<sup>+</sup> state strength, between 3 and 4 MeV the distribution has a deep minimum, then a noticeable increase is observed. Up to 5.2 MeV excitation energy 72.7% of the 640<sup>+</sup> state strength is exhausted. The fragmentation of this state is characterized by strong fluctuations of the strength not only from one 0.2 MeV zone to another but especially from one level to another. So, at excitation energies exceeding 4 MeV there may be levels which have 640<sup>+</sup> components equal to 3–5%. It is unclear whether these large fluctuations are due to specific features of the 640<sup>+</sup> state or to the hole-state fragmentation.

The account of the terms quasiparticle plus two-phonons which is taken in the study of the distribution of the one-quasiparticle state strength results, firstly, in an increase of the state density due to the appearance of the solutions corresponding to the fundamental poles quasiparticle plus two phonons and due to decreasing energy of the states corresponding to the poles quasiparticle plus one phonon, secondly, in an increase of the fragmentation of one quasiparticle components over the levels and in a decrease of the strength fluctuation in the transition from one level to another.

The study of the fragmentation of the states quasiparticle plus phonon and quasiparticle plus two phonons continues. As an example, table 3 contains the fragmentation of six states quasiparticle plus



phonon over eight levels in  $^{239}\text{U}$ . We give only those components which are larger than 0.01%. For the time being it is still difficult to formulate the particular features of the fragmentation of the states quasiparticle plus two phonons.

5. The model considered above is rather simple. It permits describing the energy and the structure of nonrotational states with a lower excitation energy than that at which there appear fundamental quasiparticle plus three phonons. At excitation energies, at which the number of the fundamental poles quasiparticle plus three phonons is large enough, the calculations with the wave function (2) yield the strength functions rather the structure of individual resonances.

The approximate method developed in ref. <sup>17</sup> is found to be very effective. Using it one has succeeded in obtaining approximate solutions for more complicated cases when the wave function (2) is supplemented with the terms quasiparticle plus three phonons and when the wave function consists of one-, two-, three- and four-phonon components. Thus, there is a possibility of continuing the study of the state structure at high excitation energies and the fragmentation of components with large number of phonons.

Of great interest is the study of the widths and the fine structure of giant resonances and the coupling between these collective states with different multipolarities. The model in question and its modifications are found to be suitable for solving these problems. For example, collective states, such as giant resonances in odd deformed nuclei, can be described by

$$\psi_i(K^\pi) = \sum_{j, \nu\sigma} \left\{ D_{\nu\sigma}^{j_1} L_{\nu\sigma}^+ Q_{j_1}^+ + \sum_{j_2} F_{\nu\sigma}^{j_2} L_{\nu\sigma}^+ Q_{j_2}^+ Q_{j_2}^+ + \sum_{j_2, j_3} R_{\nu\sigma}^{j_2 j_3} L_{\nu\sigma}^+ Q_{j_2}^+ Q_{j_3}^+ Q_{j_3}^+ \right\} \psi_0 \quad (9)$$

In this wave function we have neglected the one-quasiparticle terms which little contribute to the wave function normalization. The probabilities of  $\gamma$ -ray excitation for these states are proportional to  $|D_{\nu\sigma}^{j_1}|^2$ , where  $\nu_0$  - corresponds to the ground one-quasiparticle state. The summation over  $j_1, j_2$  and  $j_3$  is performed not only over collective states of the type of giant resonances with different multipolarities, but also over low-lying collective states. The second and third terms in eq. (9) are responsible for the width and the fine structure of the giant resonance of appropriate multipolarity as well as for their mutual influence. The model considered in ref. <sup>16</sup> can be used for the study of the structure of giant resonances in doubly even deformed

nuclei.

It is interesting to clarify how the giant resonances affect the process of fragmentation and thereby the structure of excited states of energies lower than the energy of giant resonances. It may be hoped that our model will be effective for answering this question.

The suggested model cannot cover the whole complexity of highly excited states. Structure complications are, to a large extent, due to the interaction of single particle motion with various types of collective and weakly collective motions. Therefore it is necessary to account for other collective states, such as spin-multipole and Gamow-Teller states, giant resonances, etc. Searches for other, not so clear-cut, collective branches at intermediate and high excitation energies should be intensified.

Table 3

Fragmentation of states quasiparticle plus two phonons over eight levels of  $^{239}\text{U}$

State Configuration	fund. pole, MeV	Components in percent, Energy, MeV							
		1.52	2.02	2.09	2.20	2.32	2.52	2.53	2.54
622 <sup>+</sup> +Q(30)+Q(32)	2.49	12.1	-	75.9	-	-	-	-	-
631 <sup>+</sup> +Q(31)+Q(31)	2.57	-	61.5	-	0.92	-	-	-	0.16
622 <sup>+</sup> +Q(31)+Q(32)	2.74	-	1.0	-	58.6	0.06	-	0.80	-
622 <sup>+</sup> +Q(32)+Q(55)	2.90	-	-	-	-	-	79.2	0.44	20.3
624 <sup>+</sup> +Q(31)+Q(32)	2.94	-	-	-	-	0.38	5.75	0.97	55.1
631 <sup>+</sup> +Q(32)+Q(32)	2.96	0.38	-	0.06	0.15	0.64	0.01	40.4	6.16

6. We formulate the foundations of the general semi-microscopic approach based on the operator form of the highly excited state wave function and estimate the contribution of some simple configurations in the wave functions of neutron resonances.

Within the framework of the semi-microscopic approach of the superfluid nuclear model we construct <sup>11,12,15</sup> the wave function of a highly excited state which is presented in the form of an expansion in the number of quasiparticles. The state structure complication with increasing excitation energy is seen from the fact that in the wave function components with over-increasing number of quasiparticles become more and more important. In constructing such a wave function we start from the interaction Hamiltonian (1) supplemented with other residual interactions and employ the single-particle states of the average field potential and the mathema-

tical apparatus of the superconducting pairing correlation theory.

The wave function of the highly excited state of an odd-A-spherical nucleus is of the form

$$\begin{aligned} \psi_{\lambda}^{\dagger}(I^{\pi}M) &= b_{\lambda}^{\dagger} \mathcal{A}_{I\lambda}^{\dagger} \psi_0^{\dagger} + \\ &+ \sum_{\substack{j_1, j_2, j_3 \\ m_1, m_2, m_3}} b_{\lambda}^{\dagger}(j_1, m_1, j_2, m_2, j_3, m_3) \mathcal{A}_{j_1, m_1}^{\dagger} \mathcal{A}_{j_2, m_2}^{\dagger} \mathcal{A}_{j_3, m_3}^{\dagger} \psi_0^{\dagger} + \\ &+ \sum_{\substack{j_1, j_2, j_3, j_4, j_5 \\ m_1, m_2, m_3, m_4, m_5}} b_{\lambda}^{\dagger}(j_1, m_1, j_2, m_2, j_3, m_3, j_4, m_4, j_5, m_5) \mathcal{A}_{j_1, m_1}^{\dagger} \mathcal{A}_{j_2, m_2}^{\dagger} \mathcal{A}_{j_3, m_3}^{\dagger} \mathcal{A}_{j_4, m_4}^{\dagger} \mathcal{A}_{j_5, m_5}^{\dagger} \psi_0^{\dagger} + \dots \end{aligned} \quad (10)$$

This expression should be supplemented with the terms having the operators of pairing vibrational phonons which replace the operators  $(\mathcal{A}_{j, m}^{\dagger}, \mathcal{A}_{j, -m}^{\dagger})_{\mathbf{I}=\mathbf{0}}$ . In addition, we can introduce explicitly in (10) the operators of any phonons.

When constructing the wave function (10) we assume that the density matrix is diagonal for the ground state of the nucleus. In this representation the wave function of a highly excited state must contain thousands of different components. The use of this representation is physically justified. In the majority of cases the formation of a highly excited state occurs due to capture of a slow neutron or a high-energy  $\gamma$ -ray by a target nucleus in the ground zero-quasiparticle or one quasiparticle state. Therefore the expansion (10) seems to be performed in the basis functions of the target-nucleus.

The operator form of the wave function is used in refs. 11,12 to express the reduced neutron, radiational and alpha widths in neutron resonances in terms of the coefficients  $b_{\lambda}^{\dagger}$ . In this way one formulates the problem of experimental determination of the coefficients  $b_{\lambda}^{\dagger}$ . The coefficients  $b_{\lambda}^{\dagger}$  can be found from the spectroscopic factors of the reactions of the type (dp) and (dt), the  $\beta$  decay probabilities, gamma transition probabilities between excited states, etc.

Consider, for example, the one-quasiparticle components. The strength of each single-particle state is distributed over several levels. At low energies this fragmentation is manifested in (dp) and (dt) reactions when some levels are excited. With increasing excitation energy the level density grows, and it is difficult to resolve them experimentally. Therefore, in ref. 12 it is proposed to determine experimentally the strength functions in (dp) reaction with momentum transfer  $\ell=1$  and compare them with the strength functions for p-wave neutrons. In this way it is possible to study

the energy dependence of the strength function. The (dt) reaction cross sections can yield information about strength functions associated with the hole state fragmentation. It is of particular concern to clarify how the fragmentation of the hole state strength differs from that of the particle state strength. There has appeared first theoretical and experimental investigations<sup>20,21</sup> on the study of (dp) reactions on unbound states. In ref. 22 an attempt has been made to obtain information on neutron strength functions in deformed nuclei. In that paper one has not succeeded in obtaining the values of the strength functions for fixed momentum transfers.

Let us consider, for example a reduced neutron width which can approximately be written in the form

$$\Gamma_{n\lambda}^{\circ} = \Gamma_{sp}^{\circ} |b^{\lambda}(j) U_j|^2, \quad (11)$$

where the function  $U_j$  indicates that the state  $j$  must be a  $j$  particle state. Knowing the experimental values of  $\langle \Gamma_{n\lambda}^{\circ} \rangle$  averaged over a number of resonances it is not difficult to find  $|b^{\lambda}|^2$ . For  $\mathcal{E} \approx B_n$  they are found to be the following: for nuclei Ca-Ni  $|b^{\lambda}|^2 \sim 10^{-3}$  for Zn-Ba and certain isotopes of Au and Hg  $|b^{\lambda}|^2 \sim 10^{-2} - 10^{-5}$  for Pb isotopes  $|b^{\lambda}|^2 \sim 10^{-3} - 10^{-4}$ , for deformed nuclei  $|b^{\lambda}|^2 \sim 10^{-6} - 10^{-7}$ . Note that the values obtained in refs. 12,13 are found to be underestimated by one-two orders because of an incorrect interpretation of the experimental data.

We now investigate in which cases the valence neutron model is valid. The matrix element of the E1 transition from the highly excited state described by the wave function (10) to the one-quasiparticle state  $\mathcal{A}_{j_f, m_f}^{\dagger} \psi_0^{\dagger}$  is of the form

$$\begin{aligned} M(E1, I^{\pi}\lambda \rightarrow j_f, m_f) &= \frac{(-1)^{j_f - M + 1}}{\sqrt{3}} (j_f, m_f, I - M ||) b_{\lambda}^{\dagger} U_{j_f} \langle j_f || \Gamma(E1) || j_f \rangle - \\ &- \frac{1}{\sqrt{3}} \sum_{j_1, j_2} (-1)^{j_1 + j_2 - 1} (j_1, m_1, j_2, m_2 || 1 ||) b_{\lambda}^{\dagger}(j_1, m_1, j_2, m_2) U_{j_1, j_2} \langle j_2 || \Gamma(E1) || j_f \rangle + \dots, \end{aligned} \quad (12)$$

where  $\langle j_f || \Gamma(E1) || j_f \rangle$  - is the single-particle matrix element of the E1 transition. The first term in (12) describes the E1 transition in the valence neutron model, the remaining terms describe the E1 transitions from the three-quasiparticle components of the neutron resonance wave function.

Consider the E1 transitions from p-wave neutron resonances to the low-lying states in <sup>93</sup>Mo which illustrate the valence neutron model<sup>23</sup>. Fig.2 gives the <sup>93</sup>Mo low-lying states with the indication of the one-quasiparticle component contribution and the three-quasiparticle states lying, according to the calculations, in the

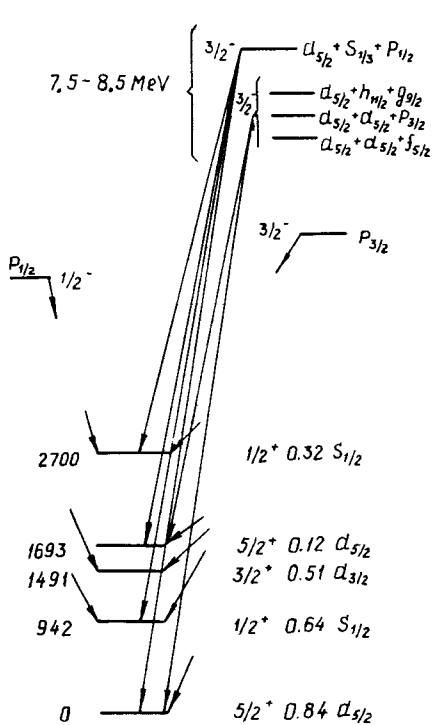


Fig.2 . EI-transitions from one- and three-quasiparticle components of p-wave neutron resonances to low-lying states in  $^{93}\text{Mo}$

case of  $^{99}\text{Mo}$  near  $B_n$  there are no three-quasiparticle states from which the EI transitions occur to the subshells  $3S_{1/2}$ ,  $2d_{5/2}$  and  $2d_{3/2}$ . Therefore in this case the valence neutron model is expected to show more clearly its validity.

According to ref.<sup>24</sup> this model is quite valid for the EI transitions in ( $\gamma n$ ) reaction on 36  $3/2^-$  resonances in the (5-225) KeV interval higher than  $B_n = 7.22$  MeV in  $^{91}\text{Zr}$ .

A strong correlation ( $\rho = 0.59$ ) between neutron and radiational widths is obtained. According to our calculations in  $^{91}\text{Zr}$ , the  $2p_{3/2}$  subshell has an energy of about 7 MeV, and in the 6-8 MeV interval there is not a single three-quasiparticle state to which the EI transition from the ground state could occur. Therefore in this case the valence neutron model is expected to work

7.5-8.5 MeV interval from which the EI transitions can occur to one-quasiparticle states. As the calculations show, the subshells  $3P_{1/2}$  and  $3P_{3/2}$  are relative to 5.06 MeV and 6.81 MeV energies. They are distributed over a large number of levels in a wide energy interval. Due to this fragmentation the wave functions of p-wave neutron resonances near  $B_n = 8.067$  MeV contain noticeable one-quasiparticle components  $3P_{1/2}$  or  $3P_{3/2}$ , which are responsible for the EI transitions in the valence neutron model. In addition, the EI transitions proceed from three-quasiparticle components which are due to fragmentation of three-quasiparticle states (Fig.2). These components lead to the violation of the valence neutron model in  $^{93}\text{Mo}$  for the  $I^\pi 3/2^-$  resonances which is confirmed by experimental data. It should be noted that the violation is also due to three-quasiparticle component admixtures in low-lying states. The calculations show that in the

well. However, in the 81-8.4 MeV interval we have the following three-quasiparticle states:

$2d_{5/2} + 2d_{5/2} + 1f_{5/2}$ ,  $2d_{5/2} + 2d_{5/2} + 2p_{3/2}$ ,  $2d_{5/2} + 3S_{1/2} + 2p_{1/2}$ ,  $2d_{5/2} + 1h_{11/2} + 1g_{9/2}$ , to which the EI transitions from the  $2d_{5/2}$  state can proceed. Therefore with increasing excitation energy the valence neutron model in ( $\gamma n$ ) reaction in  $^{91}\text{Zr}$  is expected to be less profitable.

In some cases experimental data on radiational widths permit obtaining information about certain components of the wave function (10). Of most convenience are the reactions of the type ( $\gamma n$ ), since the known structure of the initial state allows one to get individual components in the excited state. In some cases it is possible to obtain information about three-quasiparticle and five-quasiparticle components of the highly excited state wave functions. In this respect the most favourable is the study of the EI transitions in  $^{177}\text{Lu}$  from neutron resonances with  $I^\pi = 13/2^-$  and  $15/2^-$  to three-quasiparticle states.

Some simple configurations were estimated in ref.<sup>25</sup> from the experimental data on ( $n\gamma$ ) reactions by calculating the EI transitions from capture states to the low-lying ones. In these papers the low-lying states in  $^{57}\text{Fe}$ ,  $^{59,63}\text{Ni}$  and in nuclei with  $N=28$  and  $82$  are described taking into account quadrupole phonon admixtures. It is shown that the components quasiparticle plus phonon in the wave functions of the capture states are very important in the EI transitions to the low-lying states.

It is very interesting to clarify the problem about rotation and the nuclear shape in highly excited states. On the basis of a large amount of experimental information (strength function behaviour for s- and p-wave neutrons, probabilities for  $\Delta$  and  $J^\pi$  transitions from highly excited states, splitting of the EI giant resonance and others) it is possible to conclude that the shape of spherical and deformed nuclei for the majority of states does not change essentially with increasing excitation energy. The problem of the shape of the excited states of transition nuclei is more complicated. Undoubtedly of importance are direct measurements of the nuclear shape in highly excited states. In this connection, the suggestion of Ostanevich<sup>26</sup> to measure isomer shifts of neutron resonances is of great interest.

To clarify the particular features of nuclear rotation in highly excited states in ref.<sup>27</sup> it is proposed to investigate whether the  $K$  forbiddenness in the  $J^\pi$  transitions from neutron resonances to low-lying states takes place or not. The study<sup>28</sup> of the EI transitions in  $^{177}\text{Lu}$  from  $I^\pi = 13/2^-$  and  $15/2^-$  resonances produced as a result of S-wave neutron capture of  $^{176}\text{Lu}$  has not

shown any large enhancement of the EI transitions to low-lying levels with large  $K$  compared with the transitions to levels with small  $K$ . Hence it may be concluded that  $K$  is not a good quantum number in highly excited states. It should be noted that consideration of rotational motion in ref.<sup>19</sup> has led to an increase of the high-spin level density at  $\epsilon = B_n$  by several times. This points out that rotational motion is important in highly excited states of deformed nuclei. However, it seems to be impossible to distinguish it explicitly among other forms of nuclear motion.

7. At excitation energies close to the neutron binding energy  $B_n$  and higher the wave function (10) contains thousands of various components. In many quasiparticle components of the wave function the quasiparticles are distributed over bound single-particle average field levels, and not a single nuclear of this configuration is able to leave the nucleus. The few-quasiparticle components are responsible for the nucleon emission. The many-quasiparticle components of the wave function (10) correspond, to some extent, to the quasi-bound state discussed in ref.<sup>4</sup>.

Such wave functions possess the properties of the compound states introduced by N.Bohr. Our treatment of the highly excited state differs from the Bohr conception of compound state. We are based on the introduction of the average field and residual interactions and use the representation in which the wave function of a highly excited state is a many-component one. In this way it is possible to understand all the effects which are interpreted by means of the compound state. However, with such an approach the question does not arise as to how such a complicated state is formed dynamically from a simple state due to nucleon or  $\gamma$ -ray capture.

In ref.<sup>29</sup> one puts the question as to whether all the components of the highly excited state wave function are small or among them there are relatively large ones. The ways of experimental discovery of large many-quasiparticle components of the wave functions of neutron resonances are discussed in refs.<sup>10,13</sup>. At present

the most available way of clarifying the role of the many-quasiparticle components is the study of E1, M1 and E2-transitions from neutron resonances to states of an energy by (1.0-1.5) MeV lower than their energy. Possibly the probabilities for these transitions can be estimated from the study of the subsequent alpha-decay of the excited state (e.g., in ref.<sup>30</sup>), fission or neutron emission. Observation of the  $\gamma$ -transition cascades, the reduced probabilities of which are close to the single-particle ones, provides evidence for the existence of large many-quasiparticle components in the wave functions of neutron resonances and in states of inter-

mediate excitation energy. Information about the magnitude of individual four- and six-quasiparticle components can be extracted from the study of  $\gamma$ -transitions from neutron resonances to states of intermediate excitation energy. For example, in ref.<sup>31</sup>  $\gamma$ -transitions in <sup>58</sup>Fe, obtained after capture of a thermal neutron, to states of an energy higher than 5 MeV are studied.

It should be noted that the presentation of the wave function as simple and more complicated parts is widely used in treating excitation reactions of highly excited states. However, the mathematical method of a gradual introduction of simple and more complicated components should not be understood literally. It is impossible to assign to this method the physical meaning of transitions from simple to complicated configurations.

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