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PHONON OPERATORS IN DEFORMED NUCLEI

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1. INTRODUCTION

In the deformed nuclei the collective vibrational states are described by using the phonon operators. The nuclear many-body problem is solved in the RPA. The introduced in ref. [1] phonons independent of the sign of the angular momentum projection into the symmetry axis are used. These phonons are extensively used in the nuclear theory. A fairly good description of the low-lying collective vibrational states [1-5] and of the states of giant resonance-type [6] in deformed nuclei is obtained in the one-phonon approximation.

Within the quasiparticle-phonon nuclear model developed recently [7,8], the calculations are made with the wave functions containing two-phonon or quasiparticle plus one and two phonons components. In these calculations one should take Pauli principle into account correctly. The effect of the Pauli principle in the two-phonon components of the wave functions has been studied in refs. [9,10]. It was shown that the two-phonon collective states are strongly influenced by the Pauli principle. The use of the phonon operators independent of the sign of the angular momentum projection may lead to incorrect results in the calculations with the wave functions containing two-phonon or quasiparticle plus two phonons components and when the Pauli principle is consistently taken into account.

In order to correctly describe the two, three and more phonons components and the quasiparticle plus one, two and more phonons components of the nuclear wave functions, one should introduce the phonons depending on the sign of the angular momentum projection into the symmetry axis. Just this problem is solved in the present paper. Moreover, the formulae have been obtained with the new phonons, in which the Pauli principle is consistently taken into account in the two-phonon components of the excited state wave functions in deformed nuclei.

2. MULTIPOLE-MULTIPOLE INTERACTIONS

The Hamiltonian of the quasiparticle-phonon nuclear model includes the Saxon-Woods potential which describes the average field of the nucleus and interactions leading to the superconducting pairing correlations. It contains also the multipole-multipole and spin-multipole - spin-multipole isoscalar and isovector forces. For simplicity we shall use only the multipole-multipole isoscalar forces. It is not difficult to generalize the obtained equations to the case when the separable spin-multipole and isovector forces are introduced into the Hamiltonian

The multipole-multipole interaction can be written as

$$H_Q = - \sum_{\lambda\mu\sigma} \varkappa_1^{(\lambda\mu)} Q_{\lambda\sigma\mu} Q_{\lambda-\sigma\mu} , \qquad (1)$$

where $\varkappa_1^{(\lambda\mu)}$ are the constants of the isoscalar multipole forces. The operator of the multipole moment λ with projection μ has the form

$$Q_{\lambda\sigma\mu} = \sum_{\substack{q_1 q_2 \\ \sigma_1 \sigma_2}} < q_1 \sigma_1 | f^{\lambda\mu} | q_2 \sigma_2 > \delta_{\sigma_1 K_1 - \sigma_2 K_2, \sigma\mu} a^+_{q_1\sigma_1} a_{q_2\sigma_2} , \qquad (2)$$

here $<q_1\sigma_1|f^{\lambda\mu}|q_2\sigma_2>$ is the single-particle matrix element of

$$f^{\lambda\mu} = R_\lambda(r)\ Y_{\lambda\mu}(\theta\varphi)\ ,$$

the radial dependence is usually $R_\lambda(r)=r^\lambda$ or $R_\lambda = \frac{\partial V(r)}{\partial r}$, where $V(r)$ is the central part of the Saxon-Woods potential. The single-particle states are specified by the set of quantum numbers $q\sigma$ which involve the angular momentum projection into the symmetry axis denoted by K. Everywhere $K>0$ and $\mu>0$. The states differing by $\sigma(\sigma=\pm 1)$ are conjugated with respect to the time reflection. The operator (2) is rewritten as

$$Q_{\lambda\sigma\mu} = \sum_{q_1 q_2 \sigma'} \Big\{ f^{\lambda\mu}(q_1 q_2)\,\delta_{\sigma'(K_1-K_2),\sigma\mu}\ a^+_{q_1\sigma'}\ a_{q_2\sigma'} + \tag{2'}$$

$$+\bar{f}^{\lambda\mu}(q_1 q_2)\,\delta_{\sigma'(K_1+K_2),\sigma\mu}\ \sigma' a^+_{q_1\sigma'}\,a_{q_2-\sigma'}\Big\}\ ,$$

where

$$<q_1\sigma'|f^{\lambda\mu}|q_2\sigma'> = <q_1-\sigma'|f^{\lambda\mu}|q_2-\sigma'> = f^{\lambda\mu}(q_1 q_2)\ ,$$
$$<q_1\sigma'|f^{\lambda\mu}|q_2-\sigma'> = -<q_1-\sigma'|f^{\lambda\mu}|q_2\sigma'> = \sigma'\bar{f}^{\lambda\mu}(q_1 q_2)\ .$$

Now we perform the canonical Bogolubov transformation

$$a_{q\sigma} = U_q\,\alpha_{q\sigma} + \sigma\, v_q\, \alpha^+_{q-\sigma} \tag{3}$$

and introduce the dependent on σ operators

$$A(q_1 q_2;\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma\mu}\ \sigma'\alpha_{q_2-\sigma'}\,\alpha_{q_1\sigma'} \tag{4}$$

$$\bar{A}(q_1 q_2;\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1+K_2),\sigma\mu}\ \alpha_{q_1\sigma'}\,\alpha_{q_2\sigma'} \tag{4'}$$

and the corresponding Hermitian-conjugate operators as well as

$$B(q_1 q_2;\mu\sigma)=\sum_{\sigma'}\delta_{\sigma'(K_1-K_2),\sigma\mu}\,\alpha^+_{q_1\sigma'}\,\alpha_{q_2\sigma'}\,, \tag{5}$$

$$\bar B(q_1 q_2;\mu\sigma)=\sum_{\sigma'}\delta_{\sigma'(K_1+K_2),\sigma\mu}\,\sigma'\alpha^+_{q_1\sigma'}\,\alpha_{q_2-\sigma'}\,. \tag{5'}$$

Using them, we can rewrite the operator (2') in the form

$$Q_{\lambda\sigma\mu}=\frac{1}{2}\sum_{q_1 q_2}\Big\{f^{\lambda\mu}(q_1 q_2)\,U^{(+)}_{q_1 q_2}(A^+(q_1 q_2;\mu\sigma)+A(q_1 q_2;\mu-\sigma))+$$

$$+\bar f^{\lambda\mu}(q_1 q_2)\,U^{(+)}_{q_1 q_2}(\bar A^+(q_1 q_2;\mu\sigma)+\bar A(q_1 q_2;\mu-\sigma))+ \tag{6}$$

$$+2\,\mathcal{V}^{(-)}_{q_1 q_2}(f^{\lambda\mu}(q_1 q_2)B(q_1 q_2;\mu\sigma)+\bar f^{\lambda\mu}(q_1 q_2)\bar B(q_1 q_2;\mu\sigma))\Big\}\,,$$

where $U^{(+)}_{q_1 q_2}=U_{q_1}\mathcal{V}_{q_2}+U_{q_2}\mathcal{V}_{q_1}$, $\mathcal{V}^{(-)}_{q_1 q_2}=U_{q_1}U_{q_2}-\mathcal{V}_{q_1}\mathcal{V}_{q_2}$. The multipole moment operator independent of σ has been used in refs.[1,2]. It has been written in the same way through the operators

$$A(q_1 q_2)=\frac{1}{\sqrt2}\sum_\sigma \sigma\alpha_{q_2\sigma}\,\alpha_{q_1-\sigma}\,,\quad \bar A=\frac{1}{\sqrt2}\sum_\sigma\alpha_{q_1\sigma}\,\alpha_{q_2\sigma}\,,$$

$$B(q_1 q_2)=\sum_\sigma\alpha^+_{q_1\sigma}\,\alpha_{q_2\sigma}\,,\qquad \bar B(q_1 q_2)=\sum_\sigma\sigma\alpha^+_{q_1-\sigma}\,\alpha_{q_2\sigma}\,.$$

After the transformations the multipole-multipole interaction is

$$H_Q=H_1+H_2+H_3\,, \tag{7}$$

$$H_1=-\sum_{\lambda\mu\sigma}\frac{\varkappa_0^{(\lambda\mu)}}{4}\sum_{\substack{q_1 q_2\\ q_1' q_2'}}U^{(+)}_{q_1 q_2}U^{(+)}_{q_1' q_2'}\Big\{f^{\lambda\mu}(q_1' q_2')(A^+(q_1' q_2';\mu\sigma)+A(q_1' q_2';\mu-\sigma))+$$

$$+\bar{f}^{\lambda\mu}(q_1'q_2')(\bar{A}^+(q_1'q_2';\mu\sigma)+\bar{A}(q_1'q_2';\mu-\sigma))\}\{f^{\lambda\mu}(q_1q_2)(A^+(q_1q_2;\mu-\sigma)+$$

$$+\bar{A}(q_1q_2;\mu\sigma))+\bar{f}^{\lambda\mu}(q_1q_2)(\bar{A}^+(q_1q_2;\mu-\sigma)+\bar{A}(q_1q_2;\mu\sigma))\}\ , \tag{8}$$

$$H_2=-\sum_{\lambda\mu\sigma}\frac{\varkappa_0^{(\lambda\mu)}}{2}\sum_{\substack{q_1q_2\\q_1'q_2'}}u^{(+)}_{q_1'q_2'}v^{(-)}_{q_1q_2}\{[f^{\lambda\mu}(q_1'q_2')(A^+(q_1'q_2';\mu\sigma)+A(q_1'q_2';\mu-\sigma))+$$

$$+\bar{f}^{\lambda\mu}(q_1'q_2')(\bar{A}^+(q_1'q_2';\mu\sigma)+\bar{A}(q_1'q_2';\mu-\sigma))][f^{\lambda\mu}(q_1q_2)B(q_1q_2;\mu-\sigma)+ \tag{9}$$

$$+\bar{f}^{\lambda\mu}(q_1q_2)\bar{B}(q_1q_2;\mu-\sigma)]+e.c.\}\ ,$$

the remaining terms being included in H_3.

3. PHONON OPERATORS DEPENDING ON THE SIGN OF THE ANGULAR MOMENTUM PROJECTION

Now we introduce the phonon absorption operator depending explicitly on σ.

$$Q_{\lambda\sigma\mu i}=\frac{1}{2}\sum_{q_1q_2}\{\psi^{\lambda\mu i}_{q_1q_2}A(q_1q_2;\mu\sigma)-\varphi^{\lambda\mu i}_{q_1q_2}A^+(q_1q_2;\mu-\sigma)+$$
$$+\bar{\psi}^{\lambda\mu i}_{q_1q_2}\bar{A}(q_1q_2;\mu\sigma)-\bar{\varphi}^{\lambda\mu i}_{q_1q_2}\bar{A}^+(q_1q_2;\mu-\sigma)\}\ . \tag{10}$$

Its form is similar to that of the operator independent of σ (see refs. [1,2]). The wave functions of the one-phonon states are

$$Q^+_{\lambda\sigma\mu i}\Psi_0\ , \tag{11}$$

where Ψ_0 is the phonon vacuum, which is also the ground state wave function of a doubly even nucleus. Here i is the number of the one-phonon state, $i = 1, 2, 3, \ldots$; in what follows we use the notation $g \equiv \lambda\mu i$.

The commutation relations are

$$
\begin{aligned}
[Q_{g'\sigma'}, Q^{+}_{g\sigma}] = {} & \delta_{\lambda\lambda'}\,\delta_{\mu\mu'}\,\delta_{\sigma\sigma'}\,\frac{1}{2}\sum_{q_1 q_2}\Big(\psi^{g'}_{q_1 q_2}\psi^{g}_{q_1 q_2} - \varphi^{g'}_{q_1 q_2}\varphi^{g}_{q_1 q_2} + {} \\
& + \bar{\psi}^{g'}_{q_1 q_2}\bar{\psi}^{g}_{q_1 q_2} - \bar{\varphi}^{g'}_{q_1 q_2}\bar{\varphi}^{g}_{q_1 q_2}\Big) - \\
& - \sum_{q_1 q_2 q_3 \sigma_3}\Big\{\Big[\psi^{g'}_{q_1 q_3}\psi^{g}_{q_1 q_2}\,\delta_{\sigma_3(K_3-K_1),\sigma'\mu'}\,\delta_{\sigma_3(K_2-K_1),\sigma\mu} - \varphi^{g'}_{q_1 q_2}\varphi^{g}_{q_1 q_3}\,\delta_{\sigma_3(K_1-K_2),\sigma'\mu'}\,\delta_{\sigma_3(K_1-K_3),\sigma\mu} + {} \\
& + \bar{\psi}^{g'}_{q_1 q_3}\bar{\psi}^{g}_{q_1 q_2}\,\delta_{\sigma,\sigma'}\,\delta_{\sigma_3,\sigma} - \bar{\varphi}^{g'}_{q_1 q_2}\bar{\varphi}^{g}_{q_1 q_3}\,\delta_{\sigma,\sigma'}\,\delta_{\sigma_3,-\sigma}\Big]\alpha^{+}_{q_2\sigma_3}\,\alpha_{q_3\sigma_3} - \\
& - \Big[\psi^{g'}_{q_1 q_3}\bar{\psi}^{g}_{q_1 q_2}\,\delta_{\sigma_3(K_3-K_1),\sigma'\mu'}\,\delta_{\sigma_3,\sigma} + \varphi^{g'}_{q_1 q_2}\bar{\varphi}^{g}_{q_1 q_3}\,\delta_{\sigma(K_2-K_1),\sigma'\mu'}\,\delta_{\sigma_3,\sigma} - \\
& - \bar{\psi}^{g'}_{q_1 q_3}\psi^{g}_{q_1 q_2}\,\delta_{\sigma'(K_2-K_1),\sigma\mu}\,\delta_{\sigma_3,-\sigma'} - \bar{\varphi}^{g'}_{q_1 q_2}\varphi^{g}_{q_1 q_3}\,\delta_{\sigma'(K_3-K_1),\sigma\mu}\,\delta_{\sigma_3,-\sigma'}\Big]\sigma_3\,\alpha^{+}_{q_2\sigma_3}\,\alpha_{q_3 -\sigma_3}\Big\}.
\end{aligned}
\tag{12}
$$

From the relation (12) and if quasiparticles in the ground state are neglected, i.e., if $<\alpha^{+}_{q_1\sigma_1}\,\alpha_{q_2\sigma_2}> = 0$, we have

$$
\begin{aligned}
<[Q_{g'\sigma'}, Q^{+}_{g\sigma}]> = {} & \delta_{gg'}\,\delta_{\sigma\sigma'} = \\
= {} & \delta_{\lambda\lambda'}\,\delta_{\mu\mu'}\,\delta_{\sigma\sigma'}\,\frac{1}{2}\sum_{q_1 q_2}\Big(\psi^{g'}_{q_1 q_2}\psi^{g}_{q_1 q_2} - \varphi^{g'}_{q_1 q_2}\varphi^{g}_{q_1 q_2} + \bar{\psi}^{g'}_{q_1 q_2}\bar{\psi}^{g}_{q_1 q_2} - \bar{\varphi}^{g'}_{q_1 q_2}\bar{\varphi}^{g}_{q_1 q_2}\Big).
\end{aligned}
\tag{13}
$$

Thus we have obtained the same orthonormalization condition as in refs. /1,2/.

The Hamiltonian of the quasiparticle-phonon nuclear model for the isoscalar multipole-multipole interaction is expressed through the operators of quasiparticles and phonons as follows:

$$H_M = H_\vartheta + H_{\vartheta q} \quad ,$$

$$H_\vartheta = \sum_{q\sigma} \varepsilon(q) \alpha^+_{q\sigma} \alpha_{q\sigma} - \sum_{\lambda\mu} \frac{\varkappa^{(\lambda\mu)}}{2} \sum_{ii'\sigma} \sum_{\substack{q_1 q_2 \\ q_1' q_2'}} U^{(+)}_{q_1' q_2'} U^{(+)}_{q_1 q_2} \cdot \tag{14}$$

$$\cdot \left\{ f^{\lambda\mu}(q_1' q_2')(\psi^{\lambda\mu i'}_{q_1' q_2'} + \varphi^{\lambda\mu i'}_{q_1' q_2'}) + \bar{f}^{\lambda\mu}(q_1' q_2')(\bar{\psi}^{\lambda\mu i'}_{q_1' q_2'} + \bar{\varphi}^{\lambda\mu i'}_{q_1' q_2'}) \right\} \cdot \tag{15}$$

$$\cdot \left\{ f^{\lambda\mu}(q_1 q_2)(\psi^{\lambda\mu i}_{q_1 q_2} + \varphi^{\lambda\mu i}_{q_1 q_2}) + \bar{f}^{\lambda\mu}(q_1 q_2)(\bar{\psi}^{\lambda\mu i}_{q_1 q_2} + \bar{\varphi}^{\lambda\mu i}_{q_1 q_2}) \right\} Q^+_{\lambda\sigma\mu i'} Q_{\lambda\sigma\mu i} \quad ,$$

where $\varepsilon(q)$ is the quasiparticle energy (see ref.[1]).

To find the energies $\omega_{\lambda\mu i}$ and wave functions of one-phonon states we calculate the average H_ϑ over the state (11). Based on the variational principle and taking into account the normalization condition (13), we get a secular equation for calculating the one-phonon energies. From the corresponding equations and condition (13), we find $\psi^{\lambda\mu i}_{q_1 q_2}$, $\bar{\psi}^{\lambda\mu i}_{q_1 q_2}$, $\varphi^{\lambda\mu i}_{q_1 q_2}$ and $\bar{\varphi}^{\lambda\mu i}_{q_1 q_2}$. The secular equation and wave functions coincide with those obtained in refs.[1,2] with the phonons independent of σ. Therefore, the description of the one-phonon states with the phonons dependent on or independent of σ is completely equivalent.

We take into account the secular equation for the one-phonon energies and transform the term $H_{\vartheta q}$ as

$$H_{\vartheta q} = \frac{-1}{2\sqrt{2}} \sum_{g\sigma} \sum_{qq'} \frac{\vartheta^{(-)}_{qq'}}{\sqrt{Y_g}} \left\{ (Q^+_{g\sigma} + Q_{g-\sigma})(f^{\lambda\mu}(qq') B(qq'; \mu-\sigma) + \right. \tag{16}$$

$$\left. + \bar{f}^{\lambda\mu}(qq') \bar{B}(qq'; \mu-\sigma)) + e.c. \right\} \quad ,$$

the function Y_g is given in ref.[2]. This term describes the quasiparticle-phonon interaction.

The nonrotational states in nuclei with an odd number of nucleons are described with the wave function

$$\Psi_n(\sigma_0 K_0)=\Big\{\sum_{q_0} C^n_{q_0}(K_0)\alpha^+_{q_0\sigma_0}+\sum_{\substack{gq_3\\ \sigma\sigma_3}} D^n_{gq_3}(K_0)\delta_{\sigma_3K_3+\sigma\mu,\sigma_0K_0}\alpha^+_{q_3\sigma_3}Q^+_{g\sigma}\Big\}\Psi_0. \quad (17)$$

The corresponding equations coincide with the equations in ref.[2], which have been obtained with the phonons independent of σ. The difference arises when the Pauli principle is taken into account in the quasiparticle plus phonon components and when the quasiparticle plus two phonons components are included in (17). The rotational bands should be calculated with the wave function (17) taking into account the Coriolis interaction.

4. EFFECT OF THE PAULI PRINCIPLE ON THE TWO-PHONON STATES

The effect of the Pauli principle on the two-phonon states in doubly even deformed nuclei has been investigated in refs. [9,10]. It was shown[10] that the inclusion of the Pauli principle causes the energy shift of the collective two-phonon states towards larger energies. Since at the energies larger than 3 MeV the two-phonon states are strongly fragmented, the two-phonon collective states should not appear in the deformed nuclei, as it was stated in ref.[10]. In view of the importance of this statement, it is necessary to make the calculations with the phonons dependent on σ.

The excited state wave function of a doubly even deformed nucleus can be written as a superposition of the one-or two-phonon components

$$\Psi_n(\sigma_0 K_0)=\Big\{\sum_i R_i^n(K_0)\,Q^+_{g_0\sigma_0}+\frac{1}{2}\sum_{\substack{g_1 g_2\\ \sigma_1\sigma_2}}\sqrt{1+\delta_{g_1 g_2}}\;\delta_{\sigma_1 K_1+\sigma_2 K_2,\,\sigma_0 K_0}\;\cdot \tag{18}$$

$$\cdot\, P^n_{g_1 g_2}(K_0)\,Q^+_{g_1\sigma_1}\,Q^+_{g_2\sigma_2}\Big\}\,\Psi_0 ,$$

with $\mu_0 \equiv K_0$. While calculating with the Hamiltonian (14) the energies and coefficients $R_i^n(K_0)$ and $P^n_{g_1 g_2}(K_0)$, we deal with expressions of the type

$$\sum_{\substack{\sigma_1\sigma_2\\ \sigma_1'\sigma_2'}}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\,\sigma_0 K_0}\,\delta_{\sigma_1'\mu_1'+\sigma_2'\mu_2',\,\sigma_0 K_0}<Q_{g_2'\sigma_2'}\,Q_{g_1'\sigma_1'}\,Q^+_{g_1\sigma_1}\,Q^+_{g_2\sigma_2}>= \tag{19}$$

$$=(\delta_{\mu_1+\mu_2,K_0}+\delta_{|\mu_1-\mu_2|,K_0})(\delta_{g_1,g_1'}\,\delta_{g_2,g_2'}+\delta_{g_1,g_2'}\,\delta_{g_2,g_1'})+\mathcal{K}^{K_0}(g_2' g_1'/g_1 g_2).$$

The function $\mathcal{K}^{K_0}(g_2' g'/g g_2)$ calculated taking into account (12) is

$$\mathcal{K}^{K_0}(g_2' g'/g g_2)=-\sum_{\sigma\sigma_2\sigma'\sigma_2'}\delta_{\sigma\mu+\sigma_2\mu_2,\,\sigma_0 K_0}\cdot\delta_{\sigma'\mu'+\sigma_2'\mu_2',\,\sigma_0 K_0}\;\cdot$$

$$\cdot\sum_{q_1 q_2 q_3 q_4}\sum_{\sigma_3}\Big\{\Big[\psi^{g'}_{q_1 q_3}\,\psi^{g}_{q_1 q_2}\,\delta_{\sigma_3(K_1-K_3),\,\sigma'\mu'}\cdot\delta_{\sigma_3(K_1-K_2),\,\sigma\mu}\;-$$

$$-\varphi^{g'}_{q_1 q_2}\,\varphi^{g}_{q_1 q_3}\,\delta_{\sigma_3(K_2-K_1),\,\sigma'\mu'}\cdot\delta_{\sigma_3(K_3-K_1),\,\sigma\mu}+\bar{\psi}^{g'}_{q_1 q_3}\,\bar{\psi}^{g}_{q_1 q_2}\,\delta_{\sigma,\sigma'}\cdot\delta_{\sigma_3,-\sigma}-\bar{\varphi}^{g'}_{q_1 q_2}\,\bar{\varphi}^{g}_{q_1 q_3}\,\delta_{\sigma,\sigma'}\,\delta_{\sigma_3,\sigma}\Big]\cdot$$

$$\cdot\Big[\psi^{g_2'}_{q_4 q_2}\,\psi^{g_2}_{q_4 q_3}\,\delta_{\sigma_3(K_4-K_3),\,\sigma_2\mu_2}\,\delta_{\sigma_3(K_4-K_2),\,\sigma_2'\mu_2'}+\varphi^{g_2'}_{q_4 q_3}\,\varphi^{g_2}_{q_4 q_2}\,\delta_{\sigma_3(K_2-K_4),\,\sigma_2\mu_2}\,\delta_{\sigma_3(K_3-K_4),\,\sigma_2'\mu_2'}+$$

$$+\bar{\psi}^{g_2'}_{q_4 q_2}\,\bar{\psi}^{g_2}_{q_4 q_3}\,\delta_{\sigma_2,\sigma_2'}\,\delta_{\sigma_2,-\sigma_3}+\bar{\varphi}^{g_2'}_{q_4 q_3}\,\bar{\varphi}^{g_2}_{q_4 q_2}\,\delta_{\sigma_2,\sigma_2'}\,\delta_{\sigma_2,\sigma_3}\Big]+$$

$$+[\bar\psi^{g'}_{q_1q_3}\psi^{g}_{q_1q_2}\delta_{\sigma',\sigma_3}\delta_{\sigma'(K_1-K_2),\sigma\mu}+\bar\varphi^{g'}_{q_1q_2}\varphi^{g}_{q_1q_3}\delta_{\sigma',\sigma_3}\delta_{\sigma'(K_1-K_3),\sigma\mu}-$$

$$-\psi^{g'}_{q_1q_3}\bar\psi^{g}_{q_1q_2}\delta_{\sigma,-\sigma_3}\delta_{\sigma(K_1-K_3),\sigma'\mu'}-\varphi^{g'}_{q_1q_2}\bar\varphi_{q_1q_3}\delta_{\sigma,-\sigma_3}\delta_{\sigma(K_1-K_2),\sigma'\mu'}]\cdot$$

$$\cdot[\bar\psi^{g_2}_{q_4q_3}\psi^{g_2'}_{q_4q_2}\delta_{\sigma_2,\sigma_3}\delta_{\sigma_3(K_4-K_2),\sigma_2'\mu_2'}-\bar\varphi^{g_2}_{q_4q_2}\varphi^{g_2'}_{q_4q_3}\delta_{\sigma_2,\sigma_3}\delta_{\sigma_3(K_4-K_3),\sigma_2'\mu_2'}-$$

$$-\psi^{g_2}_{q_4q_3}\bar\psi^{g_2'}_{q_4q_2}\delta_{\sigma_2',-\sigma_3}\delta_{\sigma_3(K_3-K_4),\sigma_2\mu_2}+\varphi^{g_2}_{q_4q_2}\bar\varphi^{g_2'}_{q_4q_3}\delta_{\sigma_2',-\sigma_3}\delta_{\sigma_3(K_2-K_4),\sigma_2\mu_2}]\}. \tag{20}$$

The functions $\psi^{g}_{q_1q_2}$ and $\varphi^{g}_{q_1q_2}$ differ from zero at $\mu=K_1-K_2$ or $\mu=K_2-K_1$. If one of these possibilities is realized, then there are the factors $\delta_{K_1-K_2,\mu}$ or $\delta_{K_2-K_1,\mu}$. The functions $\bar\psi^{g}_{q_1q_2}$ and $\bar\varphi^{g}_{q_1q_2}$ differ from zero at $\mu=K_1+K_2$, i.e., there is one possibility and the factor $\delta_{K_1+K_2,\mu}$ is omitted. The normalization condition of the wave function (18) is

$$\sum_i (R_i^n(K_0))^2+\frac{1}{2}\sum_{g_1g_2}(1+\delta_{g_1g_2})(\delta_{\mu_1+\mu_2,K_0}+\delta_{|\mu_1-\mu_2|,K_0})(P^n_{g_1g_2}(K_0))^2+$$

$$+\frac{1}{4}\sum_{\substack{g_1g_2\\g_1'g_2'}}\sqrt{1+\delta_{g_1g_2}}\sqrt{1+\delta_{g_1'g_2'}}\,P^n_{g_1g_2}(K_0)P^n_{g_1'g_2'}(K_0)\mathcal{K}^{K_0}(g_2',g_1'/g_1,g_2)=1. \tag{21}$$

The last term is due to the inclusion of the Pauli principle in the two-phonon components of the wave function (18).

The function $\mathcal{K}^{K_0}(g_2'g_1'/g_1g_2)$ differs from $\mathcal{K}(g_2'g_1'/g_1g_2)$ calculated in ref. /9/ with the phonons independent of σ. The function $\mathcal{K}^{K_0}(g_2'g_1'/g_1g_2)$ takes different values at $K_0=\mu_1+\mu_2$ and $K_0=|\mu_1-\mu_2|$. It is responsible for the shift of the two-phonon poles in the secular equation given in refs.[9,10]. The energies and wave functions of the excited states with new phonons are calculated in the same way as in refs.[9,10]. The numerical calculations should show how large are the differences in the calculations with previous and new phonons and in what cases they occur.

Thus, at the present stage of the study of the structure of deformed nuclei, the mathematical apparatus turned out to be insufficient. The new phonons introduced in this paper will serve as a basis for many calculations of the properties of deformed nuclei.

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