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SPECTRAL FUNCTIONS AND HOLE NUCLEAR STATES

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1. INTRODUCTION

Systematic experimental and theoretical investigations of knock-out, stripping and pick-up reactions (e.g., refs. [1-11]) give information about the possibility of using a simple shell model, in general, about the quasiparticle approach[12-14] to the description of the single-particle nuclear states.

The study of important single-particle characteristics, such as occupation numbers, deep hole energies and widths is closely related to this problem.

The comparison between single-particle strength functions for the deep hole nuclear states and those for the states with energies near the Fermi energy is of peculiar interest.

A great deal of theoretical investigations, especially those, based on Brueckner's nuclear matter theory of spectral functions and widths of the hole nuclear states, take into account the mass-operator deviations from the Hartree-Fock values.

An object of interest is, however, how to use the theoretical results obtained about infinite nuclear matter for description of the finite nuclei[6,15]. There are several approaches to this problem. Köhler[4] calculates the 1s-single-particle width in a finite nucleus from the approximation

$$\Gamma = 2W(\langle k\rangle,\ E(\langle k\rangle)), \qquad (1)$$

where the imaginary part W of the mass-operator on-shell value is calculated in nuclear matter at density equal to the mean density $\langle\rho\rangle$ felt by the hole in the finite nucleus, and $\langle k\rangle$ is the average momentum of the hole. The quantities $\langle\rho\rangle$ and $\langle k\rangle$ are calculated by means of harmonic oscillator wave functions. It is pointed out in ref.[11], however, that the good agreement between theoretical and empirical values is worsened when one takes into account renormalization corrections and the widths are calculated more stringently from the spectral functions[8,9].

In Ref.[11], the difference $E_F - E(k)$ between the Fermi energy and the quasiparticle energy is adopted as a more relevant variable [10] for adaptation of the nuclear matter results to finite nuclei. This difference is less model dependent than $\langle k\rangle$ or $E(k)$ and determines the phase space available for the decay of the quasiparticle state.

The spectral functions and the widths for finite nuclei, obtained in ref.[11], however, are strongly dependent on the value of the infinite nuclear matter density ρ (or the Fermi momentum k_F) at which the mass-operator is calculated.

A new approach for calculation of spectral functions, widths and energies of single-particle states in finite nuclei is proposed in the present paper. This approach is based on the possibility offered in the coherent fluctuation model (CFM)[16,17] of direct use of nuclear matter theory results in calculations of finite nuclei quantities.

Here the single-particle widths do not depend as in ref.[4] on introduced characteristic densities felt by the particle in a considered state but are functionals of the nuclear ground state density distribution.

In Sec. 2 the spectral functions in the framework of CFM are introduced. Calculations based on spectral function are carried out in Sec. 3.

2. SPECTRAL FUNCTIONS IN TERMS OF CFM

In CFM the mixed density matrix $\rho(\vec{r},\vec{r}')$ is [17]:

$$\rho(\vec{r},\vec{r}') = \int_0^\infty |f(x)|^2 \rho_x(\vec{r},\vec{r}')\,dx\,, \tag{2}$$

where

$$\rho_x(r,r') = 3\rho_0(x)\,\frac{j_1(k_F(x)|\vec{r}'-\vec{r}|)}{(k_F(x)|\vec{r}'-\vec{r}|)}\cdot\Theta\left(x-\frac{|\vec{r}+\vec{r}'|}{2}\right), \tag{3}$$

$$\Theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0. \end{cases}$$

In (3) $\rho_x(\vec{r},\vec{r}')$ is the density matrix for uniform matter with fixed mean density

$$\rho_0(x) = \frac{3A}{4\pi x^3}. \tag{4}$$

The Fermi momentum is defined by

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\alpha}{x}\,, \tag{5}$$

where

$$\alpha \equiv \left(\frac{9}{8}\pi A\right)^{1/3}.$$

The representation (2) corresponds to the general statement of the CFM that the density distribution of the nuclear matter fluctuates nearly the average distribution, keeping spherical symmetry and uniformity.

The function $|f(x)|^2$ is a weight function for the different uniform distributions in the average density distribution. It is expressed in terms of the nuclear density distribution $\rho(r)$ [17]

$$|f(x)|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr}\Big|_{r=x} \tag{6}$$

and can be experimentally determined, for example, from the elastic electron-nucleus scattering data.

The corresponding to the mixed density matrix $\rho(\vec{r},\vec{r}')$ Wigner distribution function $n(\vec{k},\vec{r})$ is:

$$n(\vec{k},\vec{r}) = \int_{|\vec{r}|}^{\infty} dx\,|f(x)|^2\,\Theta(k_F(x) - |\vec{k}|). \tag{7}$$

The function $n(\vec{k},\vec{r})$ determines the particle density in phase space in the vicinity of the point $(\vec{r},\vec{k})$.

As far as the interior of the nucleus can be considered as having properties closed to infinite nuclear matter ones, we can estimate $n(\vec{r},\vec{k})$ at point $\vec{r}=0$.
Then

$$n(\vec{k}) = \int_0^{\infty} dx\,|f(x)|^2\,\Theta(k_F(x) - |\vec{k}|) \tag{8}$$

may be considered as a function, which determines occupation numbers of states with a given momentum $\vec{k}$. The occupation numbers $n_x(k)$ of single-particle states in nuclear matter with a Fermi momentum $k_F(x)$ are given by:

$$n_x(k) = \Theta(k_F(x) - |\vec{k}|). \tag{9}$$

Keeping in mind the relation between occupation numbers $n_x(k)$ and the spectral function for uniform system $S_x(k,\omega)$:

$$n_x(k) = \int_{-\infty}^{\epsilon_F(x)} \frac{d\omega}{2\pi} S_x(k,\omega) \tag{10}$$

equation (8) can be written in the form:

$$n(k) = \int_0^{\infty} dx\,|f(x)|^2 \int_0^{h^2 k_F^2(x)/2m} d\omega\,\delta\Big(\omega - \frac{h^2 k^2}{2m}\Big). \tag{11}$$

The following approximation is used to obtain (11):

$$S_x(k,\omega) = 2\pi\delta\Big[\omega - \frac{h^2 k^2}{2m}\Big], \qquad \text{for } k < k_F(x). \tag{12}$$

From (11) follows immediately:

$$n(k) = \int_{\mu+E_F}^{E_F} \frac{d\omega'}{2\pi} \int_0^\infty dx|f(x)|^2 2\pi\delta[\omega' - E_F - \mu \frac{k^2}{k_F^2(x)}], \qquad (13)$$

if one uses the substitution

$$\omega' = \frac{\mu}{h^2 k_F^2(x)/2m} \cdot \omega + E_F . \qquad (14)$$

The energy E_F in (13,14) is considered as Fermi energy of the nucleus and is identified with the separation energy. The parameter μ can be interpreted as the energy of the deepest level, while $\mu + E_F$ in (3) – the depth of the potential well.

It is evident from (13) that

$$S(k, \omega \le E_F) \equiv 2\pi \int_0^\infty dx|f(x)|^2 \delta[\omega - E_F - \mu \frac{k^2}{k_F^2(x)}] \cdot \Theta(k_F(x) - k) \qquad (15)$$

plays the role of a spectral function for nuclear hole states.

Finally, after integrating on x in (15), the explicit expression of the spectral function follows:

$$S(k, \omega \le E_F) = \frac{\pi a}{k\sqrt{\mu(\omega - E_F)}} |f(\frac{a}{k}\sqrt{\frac{\omega - E_F}{\mu}})|^2 . \qquad (16)$$

3. SINGLE-PARTICLE WIDTHS, CENTROID ENERGIES, EFFECTIVE MASS

Now we use equation (16) to calculate the spectral function and the quantities related to it for the hole nuclear states. The spectral functions for $^{58}Ni, ^{40}Ca, ^{28}Si$ obtained by means of (16) are given in Figs. 1,2 and 3 respectively. The corresponding empirical strength functions from /2/ are included there for comparison.

The function $|f(x)|^2$ is calculated from (6) by the use of the well-known from electron-nuclei scattering experiments Fermi-type density distribution:

$$\rho(r) = \rho_0 \frac{1}{1+e^{(r-R)/b}}; \quad \rho_0 = \frac{3A}{4\pi R^3[1+(\pi b/R)^2]} . \qquad (17)$$

The values of parameters R and b are taken from /18/: ^{58}Ni (R=4,153 fm; b=0,566 fm), ^{40}Ca (R=3,556 fm; b=0,578 fm), ^{28}Si (R=3,085 fm; b=0,563 fm). The value of parameter μ is chosen to be –50 MeV.

Since the data do not directly yield the normalized spectral function, we attach to the theoretical curves one normalization coefficient, chosen in such a way that the height of the 1p-peak coincides with the experimental value.

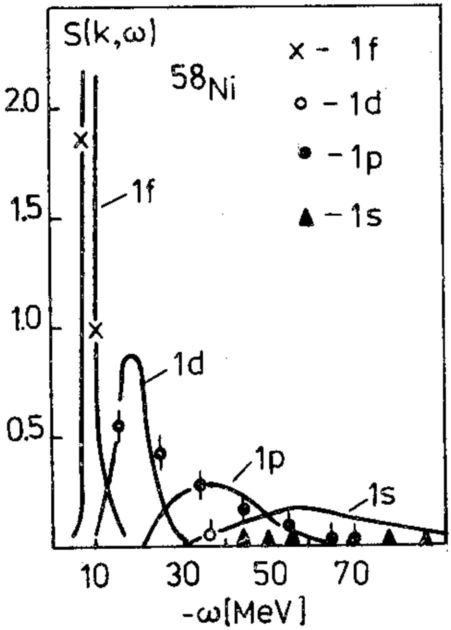

Fig.1. Comparison between empirical strength functions (Ref. [2]) and spectral functions calculated from Eq. (16) for ^{58}Ni. The ordinate scale is in arbitrary units.

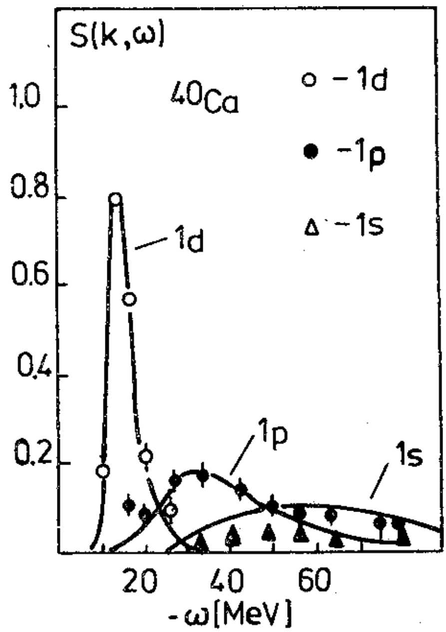

Fig.2. Same as Fig.1 for ^{40}Ca.

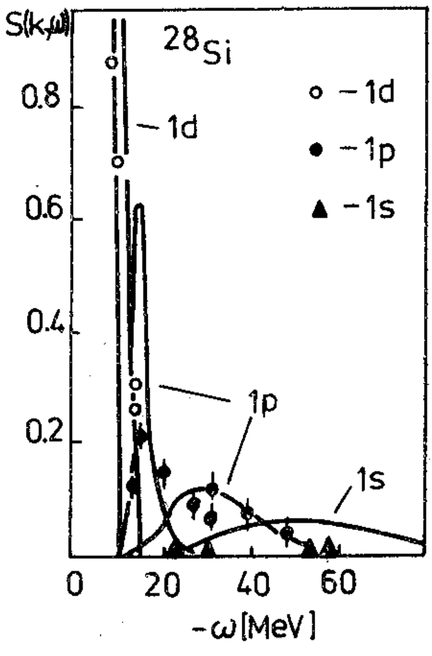

Fig.3. Same as Fig.1 for ^{28}Si.

The spectral functions (16) are calculated at such values of the momentum k (or $k\sqrt{\mu}$), for which the positions of the peak of $S(k,\omega)$ are in keeping with the empirical ones. Here we have to notice that this procedure is close to the one in ref.[8]. The spectral function $S(k, \omega)$ in ref.[8], however, is proposed to be calculated for each value of ρ, while the hole state width for a finite nucleus has to be determined by interpolation of widths obtained from $S(k, \omega)$ at $\rho = \langle\rho\rangle$ [4].

From (15) follows that our approach distinguishes essentially from the proposed in ref.[8] and others.

In our case the spectral function for a hole nuclear state is a superposition of spectral functions for nuclear matters with different densities $\rho_0(x)$ multiplied by corresponding weight factor $|f(x)|^2$. Moreover, one can see that (15) gives a possibility to determine the interval Δx (or interval of densities $\Delta\rho$) which takes part in the formation of the peak of $S(k, \omega)$ in the interval $\Delta\omega$, in the vicinity of $\omega = \omega_{max}$ (at a fixed momentum k).

We can note the good agreement between theoretical and experimental strength distributions for 1s, 1p, 1d, 1f states.

We emphasize that the asymmetry of the curves in Figs.1-3. and the increasing widths for deeply bound states are characteristic features and such behaviour of the spectral functions is in accordance with the experimental regularities.

The theoretical results give a good description of the observed in the experimental data large spread of the 1s-hole strength, more than 40 MeV [2].

In the framework of the proposed method, the centroid energy can be calculated:

$$W(k) = \frac{\int_{-\infty}^{E_F} \frac{d\omega}{2\pi}\, \omega S(k, \omega)}{\int_{-\infty}^{E_F} \frac{d\omega}{2\pi} S(k, \omega)} . \qquad (18)$$

Using now Eq. (16) in (18), one finds:

$$E(k) = \frac{h^2 k^2}{2m}\mu \frac{\int_0^{a/k} dx f^2(x) \Big/ \frac{h^2 k_F^2(x)}{2m}}{\int_0^{a/k} dx f^2(x)} + E_F . \qquad (19)$$

At small values of k

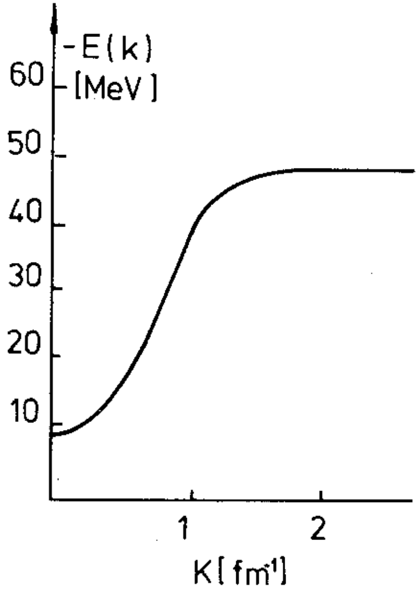

Fig.4. Centroid energy E(k) calculated from Eq. (19) for ^{58}Ni.

$$E(k) \approx \frac{h^2k^2}{2m^*}, \qquad (20)$$

where the quasiparticle effective mass m*is expressed by:

$$m^* = m[\,|\mu| \int_0^\infty dx\,|f(x)|^2 / \frac{h^2k_F^2(x)}{2m}]^{-1} . \qquad (21)$$

In Fig.4 the function E(k) for the nucleus ^{58}Ni is represented graphically. In this case $m^* \approx 0,6$ m.

4. SUMMARY

In the present paper an attempt is made to apply the coherent fluctuation model to the description of the single-particle nuclear properties. An explicit expression is given for the spectral function of hole nuclear states. Some single-particle nuclear characteristics are analyzed on this basis.

In the framework of CFM, these characteristics are functionals of the ground state nucleon density distribution. The calculated values of the single-particle quantities are in agreement with the experimental data. We emphasize that if the weight function $|f(x)|^2$ is determined from the data on elastic electron-nuclei scattering, the analysis is made without fitted parameters.

We note that there are approaches in which nuclear matter density ρ is specified for calculation of each single-particle state spectral function. The success of such approaches becomes clear from CFM point of view. The density interval giving basic contribution to the formation of the corresponding peak of the spectral function can be found in the CFM.

Although the spectral functions $S_x(k,\omega)$ for different densities $\rho_0(x)$ are chosen in a maximum simplified form (see Eq. (12)), it can be pointed out that the decisive factor for the appearance of realistic properties of the finite nuclei spectral function $S(k,\omega)$ is the right account of the $|f(x)|^2$

spectral function $S(k, \omega)$ is the right account of the weight $|f(x)|^2$ for each density $\rho_0(x)$ participating in the formation of the ground state nucleon density distribution $\rho(r)$.

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