объединенный
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исследований
дубна

E4-81-677

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DESCRIPTION OF THE NUCLEON, Λ-PARTICLE INTERACTION WITH THE LIGHTEST NUCLEI AT LOW ENERGIES

Submitted to ЯФ

1981

INTRODUCTION

One of the most interesting problems in the intermediate energy physics is the investigation of processes due to the exchange meson currents in nuclei. In this respect the radiative neutron capture by the few-nucleon systems is highly informative. The cross section of the n-p capture calculated by neglecting the meson currents, exceeds the experimental σ_{np} =334.2±0.5 mb by 10% /1/. The theory is consistent with experiment if the meson currents are taken into account /2/. The radiative capture cross sections n-d and n-^{3}He are still more sensitive to the meson current contribution.

Indeed, in these cases the single-particle matrix elements of the electromagnetic operators are very suppressed and proportional (due to selection rules) to the state weight of the mixed permutation symmetry, which equals 1-2%. In the case of the n-d capture σ_{nd} =650±50 μb /3/, and the contribution of meson currents is more than 50% /4/. The n-^{3}He capture cross section is less than σ_{n^3He} =27±9 μb /5/, 60±12 μb /6/.

At the same time the matrix elements of the meson current operators between the states n^{3}He and ^{4}He with large weights may be nonzero. Therefore, the process n+^{3}He→γ+^{4}He is unique for elucidating the role of the meson currents in nuclei. Towner and Khanna /7/ have evaluated the meson current contribution to the reaction n+^{3}He→γ+^{4}He cross section using the model functions n^{3}He and ^{4}He. They observed a strong dependence of the contribution on the wave functions n^{3}He and ^{4}He.

Thus, one should have the exactly calculated functions of ^{4}He- and n^{3}He -scattering. The known methods of solving the four-nucleon problem are very laborious. Within the Yakubovsky theory one should solve the systems of two-dimensional (even with the separable NN-potential) integral equations /8,9/, and the Coulomb interaction cannot still be taken into account. The methods of resonating groups /10/ and K-harmonics /11/ are not less laborious and their use results in the systems of integro-differential equations.

In our approach the amplitudes of n^{3}He-scattering satisfy the one-dimensional integral equations, that simplifies essentially the numerical calculations and allows determination of these amplitudes and wave functions under a few model assumptions.

The basic equations are obtained in solving the πd, $\pi {}^3He$ scattering problems[12]. The only approximation, consisting in substitution of the target Hamiltonian by the finite rank operator, turned out to be possible. Section 1 contains the derivation of equations for the amplitudes of elastic scattering by the nucleus for nonidentical particles and the calculation of the scattering lengths Λd, $\Lambda {}^3H$, $\Lambda {}^3He$ and binding energies. In section 2 the equations for the neutron scattering on the lightest nuclei are derived and the lengths of Nd, $N {}^3H$, $N {}^3He$-scattering are calculated.

1. EQUATIONS FOR THE SCATTERING AMPLITUDES BY THE NUCLEUS FOR NONIDENTICAL PARTICLES

Let us consider the method by the example of Λd-scattering; the $\Lambda + N \to \Sigma + N$ channel will not be taken into account.

Let $H = h_0 + V + h_c$ be the total three-particle Hamiltonian and $V = V_{\Lambda d} = V_{13} + V_{23}$ the potential of the particle-target interaction. $h_0 = -\frac{1}{2\mu}\Delta_\rho$ is the Hamiltonian of a free of Λ particle motion with respect to the center of mass of the target; h_c is the total Hamiltonian of the target. Let us determine $G_0(z) = (h_0 - z)^{-1}$ and $G_c(z) = (h_0 + h_c - z)^{-1}$. According to[12] we rewrite the three-particle Lippman-Shwinger equation for the transition operator T in the form

$$T(z) = T^{\bullet}(z) + T^{\bullet}(z)(G_0(z) - G_c(z))\, T(z), \qquad (1.1)$$

where $T^{\bullet}(z) = V - V G_0(z) T^{\bullet}(z)$.

We shall assume that particles 1,2,3 are spinless, and the Hamiltonian h_c has the bound state $|\chi_d\rangle$ and the scattering states $|\chi_{\vec p}\rangle$

$$h_c = \epsilon_d |\chi_d\rangle\langle\chi_d| + \int \frac{d\vec p}{(2\pi)^3} \frac{p^2}{m} |\chi_{\vec p}\rangle\langle\chi_{\vec p}| . \qquad (1.2)$$

The Λd-scattering amplitude is

$$f(\vec k', \vec k, z) = -\frac{\mu}{2\pi} \langle \chi_d \vec k' | T(z) | \chi_d \vec k \rangle,$$

where $\langle \vec r \vec\rho | \chi_d \vec k\rangle = \chi_d(\vec r) e^{i\vec k \vec\rho}$, $k' = k$, $z = \frac{k^2}{2\mu} + \epsilon_d + i0$.

Using (1.2) and (1.1), we get the exact equation

$$< \chi_d \vec{k}' | T | \chi_d \vec{k} > = < \chi_d \vec{k}' | T^\circ | \chi_d \vec{k} > +$$

$$+ \epsilon_d \int \frac{d\vec{k}''}{(2\pi)^3} \frac{< \chi_d \vec{k}' | T^\circ | \chi_d \vec{k}'' >}{(E'' - z)(E'' - z + \epsilon_d)} < \chi_d \vec{k}'' | T | \chi_d \vec{k} > + \tag{1.3}$$

$$+ \int \frac{d\vec{k}''}{(2\pi)^3} \frac{d\vec{p}}{(2\pi)^3} \frac{p^2}{m} \frac{< \chi_d \vec{k}' | T^\circ | \chi_{\vec{p}} \vec{k}'' >}{(E'' - z)(E'' - z + \frac{p^2}{m})} < \chi_{\vec{p}} \vec{k}'' | T | \chi_d \vec{k} > ,$$

where

$$E'' = \frac{k''^2}{2\mu} .$$

If the kinetic energy with respect to motion Λ–d satisfies inequality $E = \frac{k^2}{2\mu} << |\epsilon_d|$, then the decay amplitudes $< \chi_{\vec{p}} \vec{k}' | T | \chi_d \vec{k} >$ can be neglected in comparison with the elastic amplitude $< \chi_d \vec{k}' | T | \chi_d \vec{k} >$, and we can get from (1.3) the approximate equation

$$< \chi_d \vec{k}' | T | \chi_d \vec{k} > \approx < \chi_d \vec{k}' | T^\circ | \chi_d \vec{k} > + \tag{1.4}$$

$$+ \epsilon_d \int \frac{d\vec{k}''}{(2\pi)^3} \frac{< \chi_d \vec{k}' | T^\circ | \chi_d \vec{k}'' >}{(E'' - z)(E'' - z + \epsilon_d)} < \chi_d \vec{k}'' | T | \chi_d \vec{k} > .$$

Equation (1.4) is obtained from (1.3) by approximating the target Hamiltonian by the finite rank operator

$$h_c \cong {}^{(1)}h_c = \epsilon_d | \chi_d > < \chi_d | \tag{1.5}$$

that is the basic approximation.

In contrast with the method of strong coupling of channels, we approximate only a part of the total Hamiltonian H. Instead of $< \chi_d | V | \chi_d >$ there appears the operator $< \chi_d | T^\circ | \chi_d > = < \chi_d | \frac{V}{1 + VG_0} | \chi_d >$ which takes into account all rescattering in the system. To evaluate an accuracy of approximation (1.5), we have calculated the doublet Λd-scattering lengths ${}^2a_{\Lambda d}$ and the separation energies $B_\Lambda({}^3_\Lambda H)$; $z_\Lambda = -B_\Lambda({}^3_\Lambda H) + \epsilon_d + i0$ is the pole $< \chi_d \vec{k}' | T(z) | \chi_d \vec{k} >$. The nonlocal potentials

$$< \vec{k}' | V_{\Lambda N} | \vec{k} > = \lambda g(k') g(k), \quad g(k) = (k^2 + \beta^2)^{-1} \tag{1.6}$$

have been used, which act in the states ${}^3S_1$ and ${}^1S_0$. The potential parameters, corresponding to different lengths a_s, a_t and effective radii r_s, r_t of the ΛN-scattering in the singlet and triplet channels have been taken from ref. /13/, where ${}^2a^F_{\Lambda d}$ and $B^F_\Lambda({}^3_\Lambda H)$ were obtained from the solution of the Faddeev equations.

The deuteron wave function, as in ref.[13], has been taken in the form

$$\chi_d(r) = N\frac{1}{r}(e^{-br} - e^{-\beta r}),$$

where $b=\sqrt{m|\epsilon_d|}$, $\epsilon_d = -2.225$ MeV, $\beta = 1.4498\ fm^{-1}$.

It turned out that for all 11 sets of parameters $V_{\Lambda N}$ the following inequalities take place:

$$\left|\frac{{}^2a_{\Lambda d} - {}^2a^F_{\Lambda d}}{{}^2a^F_{\Lambda d}}\right| \le 0.25 \qquad \left|\frac{B_\Lambda({}^3_\Lambda H) - B^F_\Lambda({}^3_\Lambda H)}{B^F_\Lambda({}^3_\Lambda H)}\right| \le 0.3. \tag{1.6a}$$

The best agreement with the experimental values $B_{\Lambda exp}({}^3_\Lambda H)$ = =0.13±0.03 MeV [14] is obtained at

$$a_s = -1.80\ fm, \qquad a_t = -1.60\ fm,$$
$$r_s = 2.80\ fm, \qquad r_t = 3.30\ fm. \tag{1.7}$$

In this case

$${}^2a^F_{\Lambda d} = 12.2\ fm, \qquad B^F_\Lambda({}^3_\Lambda H) = 0.188\ MeV\ [13]$$

According to our calculations with (1.7)

$${}^2a_{\Lambda d} = 13.29\ fm, \qquad B_\Lambda({}^3_\Lambda H) = 0.214\ MeV.$$

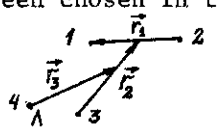
As is seen, for this set of parameters the deviation of our results from Faddeev's values is much less than in (1.6a). Equations (1.1), (1.4) can easily be generalized to the four-particle systems Λ^3H and Λ^3He. Assuming, as before, conservation of the total isotope-spin and its third projection, we approximate the target Hamiltonian by the first rank operators

$$h_c \cong {}^{(1)}h_{c\ {}^3He} = \epsilon_{{}^3He}|\chi_{{}^3He}\rangle\langle\chi_{{}^3He}|,$$
$$h_c \cong {}^{(1)}h_{c\ {}^3H} = \epsilon_{{}^3H}|\chi_{{}^3H}\rangle\langle\chi_{{}^3H}| \tag{1.8}$$

for Λ^3He- and Λ^3H scattering, respectively. For the potentials (1.6) and (1.7) we have calculated the triplet and singlet lengths A_t, A_s for the $\Lambda(3N)$-scattering and separation energy.

$A_s = 4.23$ fm, $A_t = 5.73$ fm, $B_{\Lambda s} = 2.82$ MeV for ${}^4_\Lambda He$,
$A_s = 4.53$ fm, $A_t = 4.45$ fm, $B_{\Lambda s} = 2.45$ MeV for ${}^4_\Lambda H$.

The experimental values of $B_\Lambda({}^4_\Lambda He) = 2.39 \pm 0.03$ MeV and of $B_\Lambda({}^4_\Lambda H) = 2.04 \pm 0.04$ MeV [14]. The wave functions ${}^3He$ and ${}^3H$ have been chosen in the form



$$\langle 1,2,3|\chi_{{}^3He}\rangle = N_{{}^3He}\exp(-\kappa_{{}^3He}\rho)\cdot\xi_{1/2}$$
$$\langle 1,2,3|\chi_{{}^3H}\rangle = N_{{}^3H}\exp(-\kappa_{{}^3H}\rho)\cdot\xi_{-1/2}, \tag{1.9}$$

where $\rho^2 = \frac{3}{2} r_1^2 + 2 r_2^2$, $\kappa_{^3He} = 0.5745\ fm^{-1}$, $\kappa_{^3H} = 0.635\ fm^{-1}$, that corresponds to the following values of the mean square radii

$$\sqrt{\overline{r^2_{^3He}}} = 1.88\ fm, \quad \sqrt{\overline{r^2_{^3H}}} = 1.70\ fm, \tag{1.10}$$

$$\epsilon_{^3He} = -7.718\ MeV, \quad \epsilon_{^3H} = -8.482\ MeV.$$

$\xi_{\pm 1/2}$ is a fully antisymmetric spin-isospin function of three nucleons in the state with the total spin, isospin 1/2 and third projection of the isotope-spin equal to $\pm 1/2$. Note, that in approximating (1.5) and (1.8), we retained only a negative definite part of the target Hamiltonian; therefore B_Λ, calculated from (1.4), will always be larger than the separation energies calculated exactly.

Now we proceed to the nucleon-nuclear scattering problem.

2. APPROXIMATE EQUATIONS FOR THE NUCLEON-NUCLEAR SCATTERING AMPLITUDES

Let us consider the low-energy nd-scattering. The total three-nucleon transition operator satisfies an equation analogous to (1.1)

$$T = T^\circ + T^\circ (G_0 - G_c) T, \tag{2.1}$$

where

$$T^\circ = V - V G_0 T^\circ, \quad V = V_{nd} = V_{13} + V_{23}, \quad \mu = \frac{2}{3} m .$$

In and out-states of the nd-scattering can be obtained by antisymmetrizing the functions $|\chi_d \vec{k}\rangle$

$$|\phi_{\vec{k}}\rangle_a = \sqrt{3} A |\chi_d \vec{k}\rangle, \quad A = \frac{1}{6}(1 - P_{12} - P_{13} - P_{23} + P_{12}P_{13} + P_{12}P_{23}),$$

where P_{ij} is the permutation operator of all the coordinates of particles i,j. The nd-scattering amplitude is /16/

$$f(\vec{k}', \vec{k}, z) = -\frac{\mu}{2\pi} {}_a\langle \phi_{\vec{k}'} | T | \phi_{\vec{k}} \rangle_a = -\frac{\mu}{2\pi} \langle \chi_d \vec{k}' | T(1 - 2P_{13}) | \chi_d \vec{k} \rangle .$$

Thus, one should calculate the matrix element of $\Gamma \equiv T(1 - 2P_{13})$ in the bracket of nonantisymmetrized states. From (2.1) we get

$$\Gamma = \Gamma^\circ + T^\circ (G_0 - G_c) \Gamma, \tag{2.2}$$

where $\Gamma^\circ = T^\circ (1 - 2P_{13})$.

Using approximation (1.5) we get from (2.2) approximate equations

$$\langle \chi_d \vec{k}' | \Gamma | \chi_d \vec{k} \rangle \approx \langle \chi_d \vec{k}' | \Gamma^\circ | \chi_d \vec{k} \rangle +$$

$$+\epsilon_d \int \frac{d\vec{k}''}{(2\pi)^3} \frac{\langle \chi_d \vec{k}' | T^\circ | \chi_d \vec{k}'' \rangle}{(E''-z)(E''-z+\epsilon)} \langle \chi_d \vec{k}'' | \Gamma | \chi_d \vec{k} \rangle . \qquad (2.3)$$

Using $V_{NN} = V^{10}P^{10} + V^{01}P^{01}$, (2.4)
where P^{st} is the projector onto the state of two nucleons with the total spin (isospin) s(t), and V^{10} and V^{01} in the form (1.6), we have calculated from (2.3) the quartet length of the nd-scattering ^4a =5.25 fm. The parameters V^{10} and V^{01} have been fixed by the low-energy data of the NN-scattering

ϵ_d =-2.225 MeV, a_s =-23.69 fm, a_t =2.378 fm,

r_t =2.7 fm.

The solution of the Faddeev equations with the same potentials yields

$^4a^F$ =6.28 fm /17/, $^4a_{exp}$ =6.35±0.02 fm /18/.

Now we proceed to the $n\,^3He$-scattering problem.

In the scattering of thermal neutrons in ^3He, the following processes are possible:

	$n + {}^3He$	$\sigma_{n+{}^3He \to n+{}^3He}$	=3.3±0.2 b	/19/
$n + {}^3He \to$	$p + {}^3H$	$\sigma_{n+{}^3He \to p+{}^3H}$	=5337±8 b	/19/
			=27±9 μb	/5/
	$\gamma + {}^4He$	$\sigma_{n+{}^3He \to \gamma+{}^4He}$	=60±12 μb	/6/.

An essential specific feature of the $n\,^4He$-scattering is the presence of the opean channel $n+{}^3He \to p+{}^3H$. This is the reason that the n^3He-scattering lengths are complex. The four particle transition operator satisfies the equation

$$T = T^\circ + T^\circ (G_0 - G_c) T , \qquad (2.5)$$

where $T^\circ = V - V G_0 T^\circ$, $V = \sum_{i=1,2,3} V_{i4}$.
The n^3He-scattering amplitude is

$$f(\vec{k}', \vec{k}, z) = -\frac{\mu}{2\pi} \langle \chi_{^3He} \vec{k}' | T(z)(1-3P_{34}) | \chi_{^3He} \vec{k} \rangle .$$

From (2.5) it follows

$$\Gamma = T(1-3P_{34}) = \Gamma^\circ + T^\circ (G_0 - G_c) \Gamma , \qquad (2.6)$$

where

$$\Gamma^\circ = T^\circ (1-3P_{34}) .$$

The three-nucleon Hamiltonian with two bound states is approximated as follows:

$$h_c = {}^{(2)}h_c = \epsilon_{^3He} | \chi_{^3He} \rangle \langle \chi_{^3He} | + \epsilon_{^3H} | \chi_{^3H} \rangle \langle \chi_{^3H} | . \qquad (2.7)$$

For the simplicity of calculation we have assumed that the spatial parts of the wave functions ^3He and ^3H are the same

$$\langle 1,2,3 | \chi_{^3He} \rangle = Ne^{-\kappa\rho} \xi_{1/2}, \langle 1,2,3 | \chi_{^3H} \rangle = Ne^{-\kappa\rho} \xi_{-1/2}\ \kappa = \kappa_{^3He} . \qquad (2.8)$$

Let S(T) be the total spin (isospin) of four nucleons, then

$$f^{s}_{n+^3He \to n+^3He} (\vec{k}', \vec{k}, z) = -\frac{\mu}{2\pi} \cdot \frac{1}{2} [\langle \chi \vec{k}' | \Gamma_{ST=0}(z) | \chi \vec{k} \rangle + \langle \chi \vec{k}' | \Gamma_{ST=1}(z) | \chi \vec{k} \rangle],$$

where S=0,1 , $\langle \vec{r}_1, \vec{r}_2 | \chi \rangle = Ne^{-\kappa\rho}$.

In approximating (2.7), (2.8) from (2.6), we get equations for the amplitudes Γ_{ST}

$$\langle \chi \vec{k}' | \Gamma_{ST} | \chi \vec{k} \rangle = \langle \chi \vec{k}' | \Gamma^{o}_{ST} | \chi \vec{k} \rangle + \qquad (2.9)$$
$$+ \int \frac{d\vec{k}''}{(2\pi)^3} \frac{R(k'',z)}{E''-z} \langle \chi \vec{k}' | T^{o}_{ST} | \chi \vec{k}'' \rangle \langle \chi \vec{k}'' | \Gamma_{ST} | \chi \vec{k} \rangle ,$$

where

$$R(k'',z) = \frac{1}{2} \epsilon_{^3He} (E'' - z + \epsilon_{^3He})^{-1} + \frac{1}{2} \epsilon_{^3H} (E'' - z + \epsilon_{^3H})^{-1} .$$

In the reaction $n + {}^3He \to n + {}^3He$ $R(k'',z)$ contain δ-functions even at zero energy of relative motion $n - {}^3He$, ($z = \epsilon_{^3He} + i0$) as $\epsilon_{^3H} < \epsilon_{^3He}$; therefore the $n\,^3He$-scattering lengths have imaginary parts.

Equations for the amplitudes $p + {}^3H \to p + {}^3H$ are the same as for (2.9). When calculating the p H-scattering lengths, one should assume that $z = \epsilon_{^3H} + i0$; in this case $R(k'',z)$ does not contain δ-function, the $p\,^3H$-scattering lengths are real.

Thus, within the approximation (2.7), (2.8), in the elastic $n\,^3He$-scattering, one is able to take into account the open channel $n + {}^3He \to p + {}^3H$ and in the elastic $p\,^3H$-scattering the channel $p + {}^3H \to n + {}^3He$, the threshold energy of which being $\epsilon_{^3He} - \epsilon_{^3H}$ =0.76 MeV.

The elastic $p\,^3He$, $n\,^3H$-scattering amplitudes are

$$f^{s}(\vec{k}', \vec{k}, z) = -\frac{\mu}{2\pi} \langle \chi \vec{k}' | \Gamma_{S1}(z) | \chi \vec{k} \rangle$$

and satisfy equations (2.9), in which

$$R(k'',z) = \epsilon_{^3He}(E'' - z + \epsilon_{^3He})^{-1}, \qquad z = E + \epsilon_{^3He} + i0$$

for $p + {}^3He \to p + {}^3He$,

$$R(k'',z) = \epsilon_{^3H}(E'' - z + \epsilon_{^3H})^{-1}, \qquad z = E + \epsilon_{^3H} + i0$$

for $n + {}^3H \to n + {}^3H$.

Using V_{NN}(2.4) we have calculated the N^3He, N^3H -scattering lengths. The results given below are compared with the calculations by the Yakubovsky equations /8/ with the same V_{NN} and with the calculations by MRG /10/. When solving Yakubovsky equations the wave functions of the states of ^3He and ^3H have the same energy and differ by the isospin-spin projection only; therefore, the $n\,^3He$-scattering lengths are real.

Channel lengths

$A_{ST}(n\,^3He)$ (fm)

ST	$h_c \approx {}^{(2)}h_c$	/8/
00	8.1- i1.5	12.34
10	2.9+ i0.1	3.03
01	4.0+ i0.06	3.77
11	5.6- i0.09	3.13

Lengths of $n\,^3He$-scattering

$A_S(n\,^3He) = \frac{1}{2}[A_{S0}(n\,^3He) + A_{S1}(n\,^3He)]$ (fm)

	$h_c \approx {}^{(2)}h_c$	/8/	experiment /20/
A_0	6.05- i0.72	8.05	$6.1 \pm 0.6 - i(4.4448 + 9 \cdot 10^{-4})$
A_1	4.25+ i0.005	3.08	$4.0 \pm 0.2 - i(1.7 \pm 0.8)10^{-8}$

Lengths of $n\,^3H$-scattering

$A_S(n\,^3H) = A_{S1}(n\,^3H)$ (fm)

	$h_c \approx {}^{(1)}h_c$	/8/	/10/	experiment /21/
A_0	3.8	3.77	3.38	3.91 ± 0.12
A_1	4.9	3.13	3.25	3.60 ± 0.10

Lengths of $p\,^3H$-scattering $A_0(p\,^3H) = 4.2$ fm, $A_1(p\,^3H) = 5.1$ fm,
lengths of $p\,^3He$-scattering $A_0(p\,^3He) = 4.1$ fm, $A_1(p\,^3He) = 5.2$ fm.

So, in the approximation (2.7), (2.8) the real parts of the lengths $A_S(n\,^3He)$ are close to the experimental ones. The lengths $A_S(n\,^3H)$ satisfy inequality $A_0(n\,^3H) < A_1(n\,^3H)$, though these lengths, calculated by other methods, satisfy an inverse relation.

The imaginary parts of the n^3He-scattering lengths turned out to be sensitive to the choice of V_{NN} and of the functions $\chi_{^3He}$ and $\chi_{^3H}$ in contrast with the real parts.

It should be expected that these calculations with more realistic $V_{NN}, \chi_{^3He}, \chi_{^3H}$ and more correct account Coulomb interation will provide a better agreement of the imaginary parts of the n^3He-scattering lengths with experiment.

CONCLUSION

The approximate equations are used to calculate the amplitudes of the elastic low-energy scattering of particles on the lightest nuclei and allow one to calculate the wave functions of the scattering of thermal neutrons on nuclei ^3He within the four-body theory taking into account the Coulomb splitting of energies of ^3He and ^3H.

The real parts of lengths $A_S(n^3He)$ are close to the experimental ones, i.e., the elastic n^3He-scattering is described adequately, and one holds out some hope of an adequate theoretical description of the process $n + {}^3He \to \gamma + {}^4He$. As an input information in our calculations we have used the interaction potential of an incident particle with the target nucleon and the target wave function. Note, that the model nature of the target wave function is used only to find the explicit form of the kernels and inhomogeneous terms of equations. The structure itself of approximate equations is independent of the concrete form of the target wave function. Here we investigate mostly n^3He-system. Therefore we shall compare $B_\Lambda({}^4_\Lambda He)$, $B_\Lambda({}^4_\Lambda H)$ with Gibson's, Lehman's results[22] in the next paper.

The main difficulty of these calculations is rather an accurate calculation of many-dimensional integrals. For this purpose we have used the functions of Haar and many-dimensional Π_τ-nets[23], that allowed the calculation of the scattering lengths with a relative accuracy not worse than 20% with rather a small quadrature nodes ($\sim 2^{13} - 2^{15}$).

In conclusion one of the authors (P.V.V.) is very indebted to V.G.Pupysheva for the calculation of Π_τ-lattices.

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Received by Publishing Department
on October 29 1981.