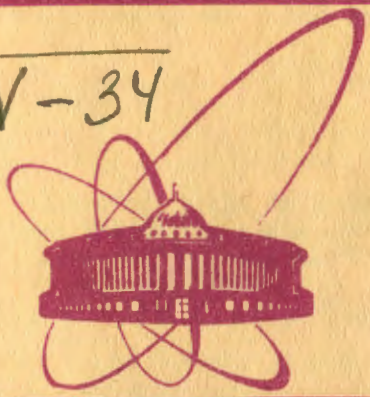

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ON THE FINE STRUCTURE OF THE RMS RADIUS

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According to the measurements, the mass number dependence of the RMS charge radii can be approximately calculated by the empirical expression [1]:

$$r_{st} = \sqrt{3/5}\,(1.15 + 1.80A^{-2/3} - 1.2A^{-4/3})A^{1/3}\ , \qquad (1)$$

which holds for nuclei with the number of nucleons A near the valley of stability.

For the isotopes of a given element, however, the increase of r_c (r_c^2 is the experimentally determined RMS radius of the proton distribution) with the neutron number N is systematically less than expected from average trend (1) [2-4]. In addition, the presence of shell effects in isotope shift data has been also suggested [5] and a more pronounced shell effect has been shown in work [6].

A comprehensive study of experimental RMS charge radii has been carried out by Angeli and Csatlós [7,8]. These systematics show that the deviations from the average A-dependence given in formula (1) follow simple trends. For an isotopic sequence of an element, the normalized RMS charge radii ν_c formed as

$$\nu_c = r_c / r_{st} \qquad (2)$$

and plotted as a function of N lie on or close to straight lines. The slopes of these lines denoted by a_z vary systematically as a function of atomic number Z. The products $\bar{A}_z a_z$, where $\bar{A}_z$ is the average mass number for the respective element, show a characteristic sawtooth structure [7] (see Fig.2). There are discontinuities in the values of the slopes for elements that contain N =20, 28, 50, 82, 88, 90 and 126 neutrons. The slopes for the normalized RMS charge radii for isotonic sequences show similar behaviour and the discontinuities can be found at sequences that contain Z =20, 28, 50, 82 protons [8] (see Fig.3).

The Hartree-Fock-BCS calculations using the energy density formalism and including a BCS treatment of pairing correlations have reproduced only the ~20% of the observed shell effects in isotopic and isotonic sequences [9]. The calculations have been performed by assuming spherical shapes. The calculated shell effect is attributed to the electromagnetic spin-orbit effect [10], the contribution to r_c^2 of the moving magnetic moments.

On the other hand the calculations have reproduced the average trend of RMS-radius to <1% over the entire periodic table. They reproduce the average behaviour of $\bar{A}_z a_z$ having an approximate $A^{1/5}$ dependence for r_c as estimated in [4]. On the grounds of the calculated Hartree-Fock charge densities, a simple parametrization of the two parameter Fermi function

$$\rho_q(r) = \rho_0 (1 + \exp(r - R_q)/a_q)^{-1} \qquad (3)$$

has been suggested where the parameters R_q and a_q have the following form:

$$R_q(N, Z) = -0.5401 + 1.249A^{1/3} - 0.9532(N-Z)/A, \qquad (4)$$

$$a_q(N, Z) = 0.4899 - 0.1236(N-Z)/A. \qquad (5)$$

In this paper we present the results of some simple phenomological model which investigates one of the possible interpretations of the observed trends of the RMS charge radii. We accept the assumption of spherical nuclear shape and the form of density function (3). In this case the shortcoming of the above-mentioned Hartree-Fock calculation lies in its inability to account for the change of stiffness of nuclei with the occupation of the valence shell. We assume that this change of stiffness can be attributed to the change of the nuclear surface and can be described by a suitable change of the parameter a_q in the density function (3).

For the change of the parameter a_q in an isotopic sequence we assume the following very simple scheme. A nucleus containing magic number of neutrons has a stiffer surface than other nuclei. For a nucleus which contains N neutron and $N_{md} \le N \le N_{mu}$, where N_{mu} and N_{md} are the closest to N magic neutron numbers, we assume a change of a_q, which depends on the ratio

$$R_N = \frac{N - N_{md}}{N_{mu} - N_{md}} \qquad (6)$$

in the following form. At the values $R_N = 0$ and $R_N = 1$, as mentioned above, a_q has minimal values and at $R_N = 1/2$, in the middle of the valence shell, a_q has maximal values. The change of a_q is supposed to be smooth in the interval $0 \le R_N \le 1$. These assumptions correspond to the picture that the valence neutrons change step by step the diffusity of surface of the nucleon distribution.

For the sake of simplicity we choose sinusoidal form for the assumed surface change and substitute the a_q parameter of the density function (3) by a_q^*:

$$a_q^* = a_q(N, Z) + s \sin R_N \pi. \qquad (7)$$

For the s parameter we suppose a N_{mu}, N_{md} dependence. As s has length dimension we have chosen it for isotopic sequence with atomic number Z as

$$s = s_0 [R_n(N_{mu}, Z) - R_n(N_{md}, Z)], \qquad (8)$$

where s_0 is a free parameter and $R_n(N, Z)$ is the radius of the neutron distribution, see ref. /9/

Neglecting the electromagnetic spin-orbit term, giving only 20% of the observed shell effects, and other small terms /11/ giving very small corrections to r_c^2, the calculated ratio ν_c^{th} has the following form:

$$\nu_c^{th} = \left(\int_0^\infty \rho_q(r)\, r^4 dr \right)^{1/4} / r_{st}, \qquad (9)$$

where $\rho_q(r)$ is normalized to unity.

If the s quantity is small compared to a_q then the ν_c^{th} ratios for isotopic sequences for a limited range of nuclei lie approximately on straight lines according to the experiments. For the first shells which contain only a few particles there exist data only for two, maximum three neighbouring isotopes so the deviations from the straight line cannot be discussed. The only exception is the sequence of Ca isotopes from N=40 to N=48, but here a systematic deviation from the straight line can be observed experimentally. The calculated values of ν_c^{th} show similar behaviour, see Fig.1.

Choosing the value of $s_0 = -0.07$ for $Z < 20$ and $s_0 = 0.07$ for $Z \geq 20$ we can reproduce the sawtooth structure over the whole region of Z, see Fig.2. In the range of the strongly deformed nuclei we can only reproduce the average decrease of the $\bar{A}_z a_z$ values. Small discontinuities in the calculated values can be explained by the investigated ranges of the given isotopes.

It is interesting that the shell correction parameter s is quite small, its value is about 5-15% of a_q over the whole Z-range.

For the verification of the above model the study of isotonic sequences is a very effective method. If such a surface effect exists then we have to reproduce the similar trends in the isotonic sequences. If N=const then like in formula (7) we can write

$$a_q^* = a_q(N, Z) + s \sin R_z \pi \qquad (10)$$

with

$$R_z = \frac{Z - Z_{md}}{Z_{mu} - Z_{md}} \qquad (11)$$

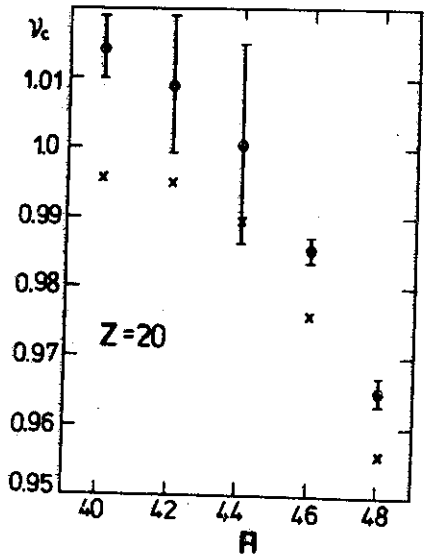
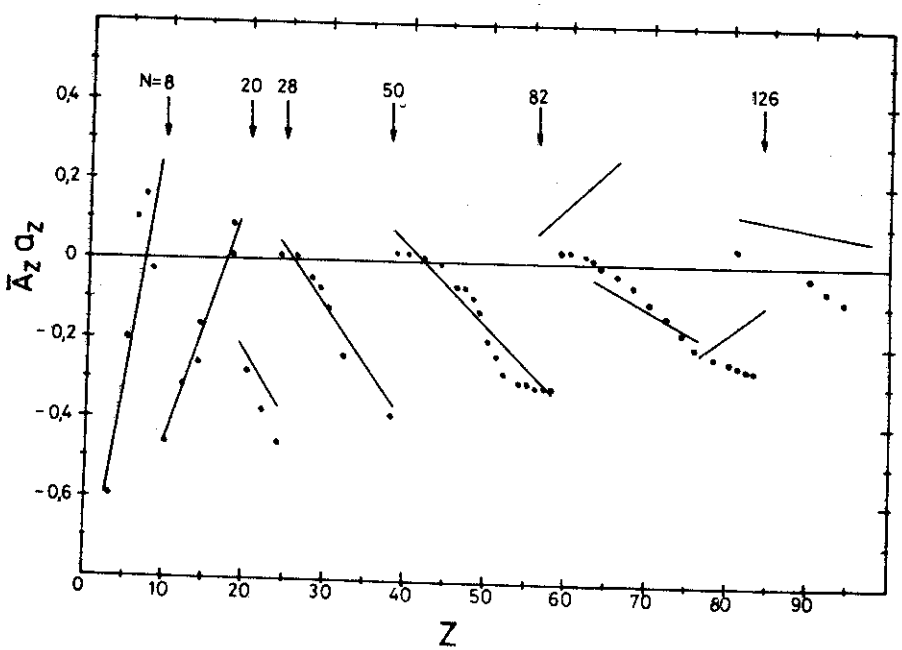


Fig.1. The RMS charge radii divided by formula (1) for Ca isotopes. The experimental data are taken from ref.[8], the calculated one's are denoted by x.

Fig.2. The a_z slopes of normalized RMS radii for isotopic sequences multiplied by the average mass numbers $\bar{A}_z$ as a function of atomic numbers. The solid lines show the result of a weighted least-squares fit to $\bar{A}_z a_z$ values derived from experimental data[7]. Circles denote the calculated values.



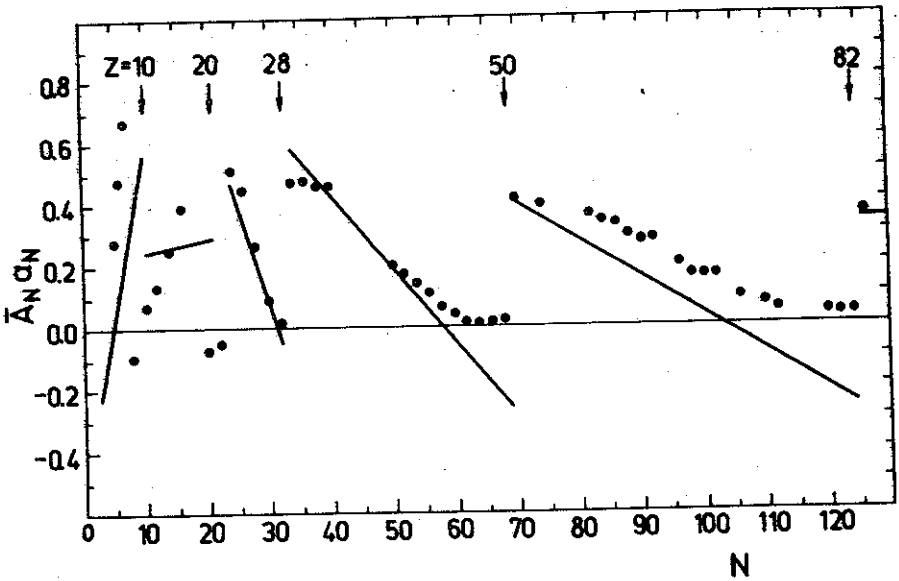


Fig.3. The a_N slopes of normalized RMS radii for isotonic sequences multipied by the average mass numbers $\bar{A}_N$ as a function of neutron numbers. The solid lines show the result of a weighted least-squares fit to $\bar{A}_N a_N$ values derived from experimental data /8/. Circles denote the calculated values.

and

$$s = s_0[R_q(N, Z_{mu}) - R_q(N, Z_{md})], \qquad (12)$$

where Z_{mu} and Z_{md} are the closest Z magic numbers, $Z_{mu} \geq Z \geq Z_{md}$. Having the same values of parameter s_0 as above that is $s_0 = -0.07$ for $N \leq 20$ and $s_0 = 0.07$ for $N > 20$, we have calculated the $\bar{A}_N a_N$ values, where a_N is the slope, $\bar{A}_N$ is the average mass number for the isotonic sequence of neutron number N. The results are presented in Fig.3. As the figure shows there is a satisfactory agreement between the calculated values and the experimental trend.

The picture given by the model seems to be acceptable. The magic nuclei have a stiffer surface. With the occupation of the valence shell the nuclear surface becomes a little more diffuse to the half-occupation and then the surface width decreases with the shell closure.

Light nuclei ($A < 40$) present a special problem. It is easy to imagine that at the opening of the valence shell the surface

becomes stiffer by adding an extra nucleon which plays the role of glue, see the Li isotopes. But the growth of the width of the surface region with the shell closure would be a surprising result.

This type of interpretation, which accepts the spherical shape of nuclei (except in the region of strongly deformed nuclei), calls for the inclusion of some kind of polarizing interaction between the valence nucleons and the proton distribution in the theoretical calculations.

We can say that this very simple model based on the small changes of surface width parameter a_q given by formula (7-8) provides a good parametrization for explanation of the shell effects of RMS-radii. But we have to note that a similar model based on small changes of the R_q radius or on deformation effects depending on the valence shell occupation might give similar good results. Anyway the present model suggests that for the explanation of the shell effects merely a quite small modification of parametrization (3-5) is required. In this case the parametrization (3-5) can provide in a first approximation a suitable tool for the investigation of nuclear effects depending on the surface behaviour of the proton and neutron distributions.

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