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FERMI LIQUID TREATMENT OF ALPHA DECAY IN THE LEAD REGION

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1. INTRODUCTION

An analysis[1] of different actual models of the α-decay process shows that: 1°) the α-transition operator was not correctly chosen and 2°) the Pauli principle was not correctly considered.

In the R-matrix approach[2] the α-transition operator depends on the channel radius which is to a great extent artificially introduced and the internal region nuclear states are not correctly defined at the matching point.

In the Feshbach like models[3-7] an operator obtained in the first order perturbation theory was used for the α-transition operator, i.e., a Fermi gas treatment was considered.

All these models lead to constantly lower values of the theoretical α-widths with respect to the experimental data.

In the above-mentioned models the Pauli principle in the α-channel wave function was not correctly taken into account. In this paper we show that the correct consideration of the Pauli principle leads to a decrease of the theoretical α-widths unlike the results obtained by some authors[8-10].

In the previous paper[11] we have proposed, in the framework of Migdal's[12] Fermi liquid theory, a new model for the α-transition operator with the following properties: a) the operator is able to transfer large enough momenta to the four nucleons that participate in the α-clusterization process; b) the operator has a universal character, determined by the properties of the nuclear matter (through the density dependence), i.e., by the s.p. states deep inside the Fermi sea; c) the operator reflects the fact that the clusterization is a surface phenomenon; d) the model for the α-transition operator is practically determined by the irreducible amplitude of the α-particle formation in the four particle channel[11]:

from which the contribution from s.p. states around the Fermi sea is excluded; e) the operator contains a universal constant having a unique value for all the α-transitions.

Our Fermi liquid model for the α-transition operator explains the experimental data within less than an order of magnitude (less than 100%).

In Sec. 2, the correct consideration of the Pauli principle in the α-channel wave function is discussed. In Sec. 3, new results within our Fermi liquid model of α-decay are reported, concerning the α-transitions in the lead region.

2. ANTISYMMETRIZATION IN THE α-CHANNEL STATE

The α-decay width is defined as elsewhere

$$\Gamma_\alpha = 2\pi \sum_c |\langle \phi_{\epsilon c} | T_{4\to\alpha} | \Phi_{A+4} \rangle|^2 , \tag{1}$$

where $\phi_{\epsilon c}$ is the many body α-channel wave function, $T_{4\to\alpha}$ is the α-transition operator describing the clusterization of two protons and two neutrons into the α-particle and Φ_{A+4} is the many body wave function describing the initial nucleus state.

The many body α-channel wave function ϕ_ϵ (here and in the following we drop the index c for simplicity) is a solution of the scattering Schrödinger equation

$$(\epsilon - \mathcal{H})\phi_\epsilon = 0 \qquad \phi_\epsilon = \mathcal{A}(u_\epsilon \phi_\alpha \phi_A) = \mathcal{A}(Q u_\epsilon \phi_\alpha \Phi_A) \tag{2}$$

with the normalization condition

$$\langle \phi_\epsilon | \phi_{\epsilon'} \rangle = \delta(\epsilon - \epsilon') = \langle u_\epsilon | 1 - K | u_{\epsilon'} \rangle . \tag{3}$$

Here ϕ_α and Φ_A are bound states, w.f., totally antisymmetrized and normalized to unity, describing the internal motion of the free α-particle and residual nucleus respectively. $Q = Q^2 = Q^+$ projects onto four s.p. orbitals which do not occur in Φ_A (for ground state Φ_A, the mentioned four orbitals must be chosen among orbitals above the Fermi sea, when Φ_A is a Slater determinant). In eq. (2) we have made the standard assumption that $\mathcal{H}$ does not depend on energy ϵ.

The relative motion w.f. u_ϵ is normalized according to the eq. (3) and is the solution of the equation

$$(\epsilon - \epsilon K - h) u_\epsilon = 0, \tag{4}$$

where

$$\langle \vec{R} | h | \vec{R}' \rangle = \langle \mathcal{A}(\delta(\vec{R} - \vec{R}_\alpha)\phi_\alpha \Phi_A) | \mathcal{H} | \mathcal{A}(\delta(\vec{R}' - \vec{R}_\alpha)\phi_\alpha \Phi_A) \rangle \tag{5}$$

and

$$\begin{aligned}\langle \vec{R} | 1 - K | \vec{R}' \rangle &= \langle \mathcal{A}(\delta(\vec{R} - \vec{R}_\alpha)\phi_\alpha \Phi_A) | \mathcal{A}(\delta(\vec{R}' - \vec{R}_\alpha)\phi_\alpha \Phi_A) \rangle \simeq \\ &\simeq \langle \delta(\vec{R} - \vec{R}_\alpha)\phi_\alpha | Q | \delta(\vec{R}' - \vec{R}_\alpha)\phi_\alpha \rangle .\end{aligned} \tag{6}$$

The correct consideration of the Pauli principle in the α-channel w.f. means the inclusion of the exchange kernel K in the eqs. (1) and (3), i.e.,

$$\langle\phi_\epsilon|T_{4\to\alpha}|\Phi_{A+4}\rangle = \langle Q u_\epsilon \phi_\alpha \Phi_A|T_{4\to\alpha}|\Phi_{A+4}\rangle \left[\binom{Z+2}{2}\binom{N+2}{2}\right]^{1/2}$$

$$\cong \langle(1-K)u_\epsilon \phi_\alpha\phi_A|T_{4\to\alpha}|\Phi_{A+4}\rangle \left[\binom{Z+2}{2}\binom{N+2}{2}\right]^{1/2}. \qquad (7)$$

At this point it should be mentioned that no theory include either the operator Q or 1-K in the α-transition matrix element [1-10].

Using for Φ_A a Slater determinant and a Gaussian w.f. [2] (with the strength β^2 =0.47 fm^{-2}) for the α-particle we can find an explicit expression for the exchange kernal:

$$K = 2K_p + 2K_n - K_{pp} - K_{nn} - - 4K_{pn} + 2K_{pnn} + 2K_{npp} - K_{ppnn} \qquad (8)$$

in which p and n stand for protons and neutrons and

$$K_n(\vec{R},\vec{R}') = \langle\delta(\vec{R}-\vec{R}_\alpha)\phi_\alpha|\prod_{i=1}^{n} P_i|\phi_\alpha\delta(\vec{R}_\alpha-\vec{R}')\rangle , \qquad (9)$$

where n stands for the number of the exchanged nucleons and P_i projects onto s.p. levels below the Fermi sea.

As far, no explicit expressions for the K-kernel were published and no qualitative analysis concerning its properties (magnitude, degree of nonlocality, relative importance of one-, two-, three- and four-nucleon exchange) has been done.

In the following we use some approximations to get an explicit expression of K-kernel.

First we used the Slater approximation for the nonlocal density

$$\langle x|P|x'\rangle = \rho(x,x') \cong \frac{1}{2}\rho(\vec{r}_+)\rho_{s\ell}(\vec{r}_-)\delta_{\hat{s},\hat{s}'} , \qquad (10)$$

where $\vec{r}_+ = \frac{1}{2}(\vec{r}+\vec{r}')$; $\vec{r}_- = \vec{r}-\vec{r}'$; $\vec{x} \equiv \{\vec{r},\hat{s}\}$ and [13]

$$\rho_{s\ell}(\vec{r}_-) = \frac{3}{K_F r_-} j_1(K_F r_-) \qquad (11)$$

in which K_F is the Fermi momentum and $j_1(z)$ is the first order spherical Bessel function.

For the density $\rho(\vec{r}_+)$ the step-function is used:

$$\rho(\vec{r}) \cong \frac{1}{2}\rho_0 \theta(R_0 - r). \tag{12}$$

To compute the integral (9) we use the generating function of the oscillator functions $<x|n\ell m>$:

$$\begin{aligned} f_\alpha(\vec{x},\vec{y};a) &= \sum_{n\ell m} <x|n\ell m> a^{2n+\ell} <n\ell m|\vec{y}> = \\ &= \left(\frac{\alpha^2}{\pi(1-a^2)}\right)^{3/2} \exp\left\{\frac{2a}{1-a^2}\alpha^2 \vec{x}\vec{y} - \frac{1+a^2}{1-a^2}\frac{\alpha^2}{2}(x^2+y^2)\right\} \end{aligned} \tag{13}$$

with $|a| < 1$ and with the following properties

$$\int_c d^3z\, f_\alpha(\vec{x},\vec{z};a)\, f_\alpha(\vec{z},\vec{y};b) = f_\alpha(\vec{x},\vec{y};ab), \tag{14}$$

$$\int_c d^3z\, f_\alpha(\vec{x},\vec{z};a)\, f_\alpha(\vec{z},y;\frac{1}{a}) = \delta(\vec{x}-\vec{y}), \tag{15}$$

where the integration contour is a line parallel to the imaginary axis, and:

$$\begin{aligned} \left(\frac{2\sqrt{\pi}\beta}{\beta^2+\alpha^2}\right)^{9/2} \prod_{i=1}^{3} f_\alpha(\vec{\xi}_i, 0; \sqrt{a}) &= \left(\frac{\beta}{\sqrt{\pi}}\right)^{9/2} e^{-\frac{\beta^2}{2}(\xi_1^2+\xi_2^2+\xi_3^2)} = \\ &= \phi_\alpha(\xi_\alpha) = \phi_\alpha \end{aligned} \tag{16}$$

with $a = \dfrac{\beta^2-\alpha^2}{\beta^2+\alpha^2}$ and

$$\begin{aligned} \delta(\vec{R}-\vec{R}_\alpha)\phi_\alpha &= 4^3\left(\frac{2\sqrt{\pi}\beta}{\beta^2+\alpha^2}\right)^{9/2} \int d^3z\, f_\alpha(2\vec{R},2\vec{z};\frac{1}{\sqrt{a}}) * \\ &* \prod_{s=1}^{4} f_\alpha(\vec{z},\vec{r}_s;\sqrt{a}). \end{aligned} \tag{17}$$

Thus

$$\begin{aligned} K_n(\vec{R},\vec{R}') &= \frac{1}{8} <\delta(\vec{R}-\vec{R}_\alpha)\phi_\alpha |\rho^n| \phi_\alpha \delta(\vec{R}_\alpha - \vec{R}')> = \\ &= \frac{1}{8}\left[4^3\left(\frac{2\sqrt{\pi}\beta}{\beta^2+\alpha^2}\right)^{9/2}\right]^2 \int_c d^3z\, d^3z'\, f_\alpha(2\vec{R},2\vec{z};\frac{1}{\sqrt{a}}) * \\ &* f_\rho^n(\vec{z},\vec{z}';a)\, f_\alpha^{4-n}(\vec{z},\vec{z}';a)\, f_\alpha(2\vec{z}', 2\vec{R}'; \frac{1}{\sqrt{a}}), \end{aligned} \tag{18}$$

where

$$f_\rho(\vec{z},\vec{z}';a) = \int d^3r\, d^3r'\, f_\alpha(\vec{z},\vec{r};\sqrt{a})\,\rho(\vec{r},\vec{r}')\,f_\alpha(\vec{n}',\vec{z}',\sqrt{a})\,. \qquad (19)$$

Performing the substitutions:

$$\vec{z}_+ = \frac{1}{2}(\vec{z}+\vec{z}')\,;\quad \vec{z}_- = \vec{z}-\vec{z}'\,;\qquad d^3z\, d^3z' = d^3z_+\, d^3z_-$$

and

$$i\vec{q} = \sqrt{\beta^4-\alpha^4}\,\vec{z}_+\,;\qquad -i\vec{k} = \frac{1}{2}\sqrt{\beta^4-\alpha^4}\,\vec{z}_-\,,$$

we obtain:

$$K_n(\vec{R},\vec{R}') \cong K_n^{(+)}(\vec{R}_+)\,K_n^{(-)}(\vec{R}_-) \qquad (20)$$

with

$$K_n^{(+)}(\vec{R}_+) = \frac{1}{(\beta\sqrt{\pi})^3}\int d^3q\, F^s(q)\exp(2\beta\vec{R}_+ + \frac{i\vec{q}}{\beta})^2\,, \qquad (21)$$

where

$$F(q) = \frac{1}{2}\operatorname{erf}(\beta R_0 - \frac{iq}{2\beta}) + \frac{1}{2}\operatorname{erf}(\beta R_0 + \frac{iq}{2\beta}) -$$
$$- \frac{2\beta}{q\sqrt{\pi}}\sin qR_0 \exp(-\beta^2R_0^2 + \frac{q^2}{4\beta^2}) \qquad (22)$$

and

$$K_n^{(-)}(\vec{R}_-) = (\frac{2}{\pi})^3 e^{\beta^2R_-^2}\int d^3k\, f_{k_f}^s(k)\exp(4i\vec{k}\vec{R}_-), \qquad (23)$$

where

$$f_{k_f}(k) = \frac{1}{2}\operatorname{erf}\frac{k_f+k}{\beta} + \frac{1}{2}\operatorname{erf}\frac{k_f-k}{\beta} -$$
$$- \frac{\beta}{k\sqrt{\pi}}\operatorname{sh}\frac{2k_fk}{\beta^2}\exp\{-\frac{1}{\beta^2}(k_f^2+k^2)\}\,. \qquad (24)$$

For n =1 (the one nucleon exchange term) the integrals can easily be performed:

$$K_1^{(+)}(R_+) = \frac{1}{2}\operatorname{erf}[\frac{2\beta}{\sqrt{3}}(R_0+R_+)] + \qquad (25)$$

$$+\frac{1}{2}\,\mathrm{erf}[\,\frac{2\beta}{\sqrt{3}}(R_0-R_+)] - \frac{1}{2\beta R_+}\sqrt{\frac{3}{\pi}}\,\mathrm{sh}\frac{8\beta^2 R_0 R_+}{3}\exp\{-\frac{4}{3}\beta^2(R_0^2+R_+^2)\}$$

and

$$K_1^{(-)}(R_-) = 16\rho_0\rho_{s\ell}(4k_f R_-)\exp(-\beta^2 R_-^2)\,. \tag{26}$$

$K_n^{(+)}$ has been chosen to be identically equal to unity for nuclear matter. For heavy nuclei $K_n^{(+)}$ will be equal to unity with high precision inside the nucleus. Thus the magnitude of the nucleon exchange kernels is contained in $K_n^{(-)}$.

The Slater (10) and the step (12) approximations are rather good inside the nucleus [13], where the fluctuations of the density are much smaller than the density itself. At the surface, however, these approximations do not seem to be so appropriate. Because of the approximative description of the density at the surface, errors will be induced in the exchange kernal K, and thus in the normalization of the channel function. But, since we are concerned with volume effects, the errors will be of the order of $A^{-1/3}$ (where A is the mass number of the residual nucleus), i.e., of about 20%, only, for heavy nuclei. The fact that the above-used approximations are rather good inside the nucleus is supported also by the high coincidence of our $K(\vec{R},\vec{R}')$ for $R=0$, $R'=0$ with those obtained elsewhere [14,15] (see, e.g., (A.1) of ref. [14]).

The calculated $K_n^{(\pm)}$ for $\alpha + {}^{208}Pb$ channel are shown in figs. 1,2. K_n decreases with n, but this decrease does not exceed an order of magnitude (fig. 2) unlike the results reported in ref.[10]. This result is explained by the fact that the volume occupied by the four nucleons (not "dressed" nucleons) in the α-particle or in the nucleus is almost the same.

The contribution of the ϵK term in the eq. (4) for small distances, much less than the nuclear radius is less than 2% of the optical potential with the depth of the order of 200 MeV. Indeed, bearing in mind that $K_n^{(+)}(R_+)\approx 1$ for such distances, we have:

$$\epsilon K u_\epsilon \cong \epsilon K^{(-)}(q_\epsilon)\,u_\epsilon\;, \tag{27}$$

where $K^{(-)}(q)$ is the Fourier transform of $K^{(-)}$ (see eq. 23).

The calculated $K^{(-)}(q)$ (see eq. 13) is given in fig. 3. From this figure we conclude also that the K-operator has a lot of eigenvalues equal to unity, that correspond to the spurious states, which must be eliminated from u_ϵ and the α-decay width, respectively. From fig. 3 we learn also that the depth of the optical potential should not be much less than 200 MeV, otherwise the characteristic momentum q_ϵ will

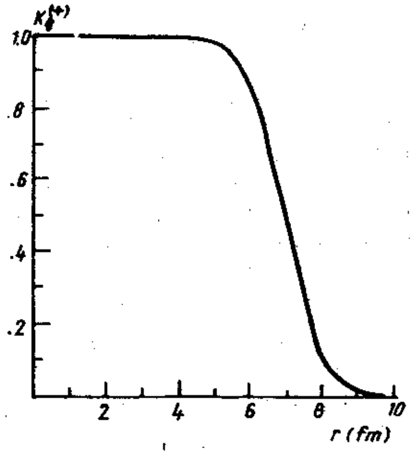

Fig. 1. The function $K_1^{(+)}$ from eq. (25) for α+^{208}Pb channel.

be in the region of spurious states. Approximately the mentioned spurious states are eliminated in the function $<\vec{R}|1-K|u_\epsilon>$ (see also eq.7). This function becomes equal to the $<\vec{R}|u_\epsilon>$-function at distances somewhat larger than R_0.

Thus, if one has in mind the application of the R-matrix theory to the α-decay in the barrier region at the matching point ($R_c = R_0 + R_\alpha \approx$ ≈ 9 fm for the lead region), the K-kernal already vanishes. Therefore at least in the R-matrix approach, one can neglect the effect of the K-operator on the α-decay width.

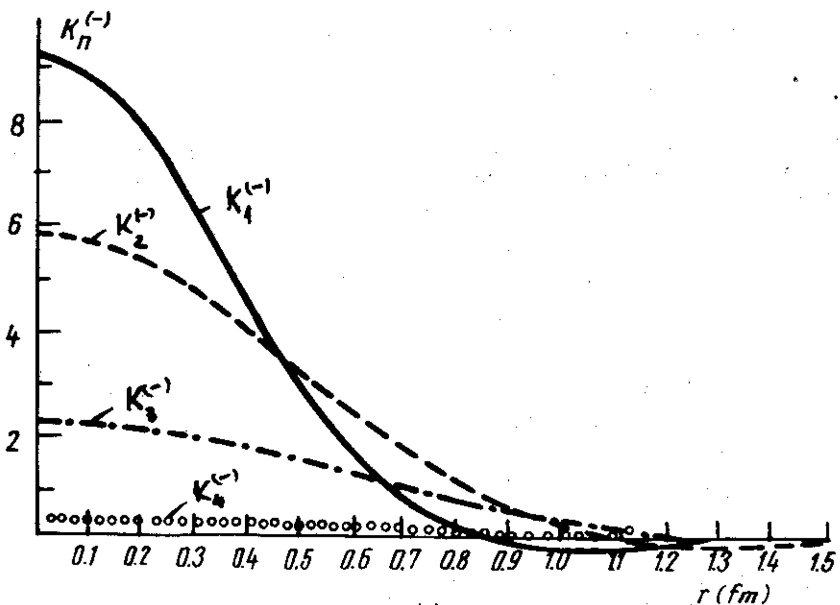

Fig. 2. The functions $K_n^{(-)}$ from eq. (23) for α + ^{208}Pb channel.

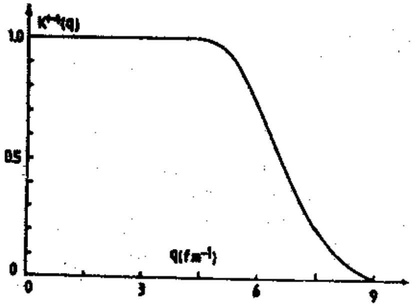

Fig.3. The Fourier transform of $K^{(-)}$ obtained from the eq. (8) with $K_n^{(+)} = 1$.

The large increase of the R-matrix α-decay width reported in [8] is obtained because, first, the operator Q (or $1-K$) in eq. (7) is omitted and, secondly, because some spurious states may occur in the operator $(1-K)^{-1/2}$ leading to a large increase.

In the integral theories [2-7] one may expect a decrease of the theoretical α-decay width (see eq. 7).

In the Fermi liquid model of α-decay [11] the situation is different, because we have the possibility, after including the Pauli principle correctly, to fix the universal constant κ, that stands for the form factor of the vertex 4-nucleons $\to \alpha$.

3. FERMI LIQUID MODEL OF α-DECAY

The α-decay width has the expression [11]

$$\Gamma_\alpha = 2\pi\kappa^2 \sum_c \left| \int_0^\infty dR\, u_{c\epsilon}(R) \frac{\partial\rho}{\partial R} g_c^{if}(R) \right|^2 , \tag{28}$$

where $u_{c\epsilon}$ is the radial part of the relative motion wave function given by the Folding Model potential [11,16], ρ is the density of the mother nucleus and

$$g_c^{if}(R) = \langle \phi_\alpha (Y_\ell \Phi_{I_f}^{\pi_f})_{I_i M_i} | \frac{\delta(R_\alpha - R)}{R} \delta(\vec{\xi}_1)\delta(\vec{\xi}_2)\delta(\vec{\xi}_3) | \Phi_{I_i M_i}^{\pi_i} \rangle \tag{29}$$

is the α-clusterization amplitude [11].

Using the structure of the initial and final states in the framework of the two-particles (holes) RPA model of refs. [17] the g_c^{if} (29) α-clusterization amplitudes become:

$$g_c^{if} = \frac{\sqrt{2}\,R}{(4\pi)^{3/2}} \left(\frac{\beta}{\sqrt{\pi}}\right)^{9/2} \frac{1}{\hat{I}_i} C_{0\,0\,0}^{I_i I_f \ell} F_{I_i}(p) F_{I_f}(n) \tag{30}$$

with

$$F_I = \sum_{s_1 s_2} \hat{j}_{s_1} \hat{j}_{s_2} (-1)^{j_{s_1}+\ell_{s_1}+1/2} C_{1/2\,-1/2\,0}^{j_{s_1} j_{s_2} I} \quad * \tag{31}$$

$$* \{ X^{I}_{s_1 s_2} + Y^{I}_{s_1 s_2} \} \mathcal{R}_{s_1}(R)\, \mathcal{R}_{s_2}(R)$$

for $^{210}Po \to {}^{206}Pb$,

$$g^{if}_{c}(R) = \frac{\sqrt{2}R}{(4\pi)^{3/2}} \left(\frac{\beta}{\sqrt{\pi}}\right)^{9/2} \frac{1}{\hat{I}_i} C^{I_i I_f \ell}_{0\,0\,0} \{ F^{(1)}_{I_i} F^{(1)}_{I_f} + \frac{I_i(I_i+1) + I_f(I_f+1) - \ell(\ell+1)}{2I_i\,(I_i+1)\,I_f(I_f+1)} F^{(-)}_{I_i} F^{(+)}_{I_f} \} \tag{32}$$

for α-transitions between the natural parity states $((-)^{\ell_{s1}+\ell_{s2}} = (-)^{I})$ of $^{210}Bi \to {}^{206}Tl$ and

$$g^{if}_{if}(R) = \frac{\sqrt{2}}{(4\pi)^{3/2}} \left(\frac{\beta}{\sqrt{\pi}}\right)^{9/2} \frac{(-1)}{\hat{I}_i} \frac{C^{I_f I_i \ell}_{1\,-1\,0} F^{(-)}_{I_i} F^{(+)}_{I_f}}{\sqrt{I_i(I_i+1)I_f(I_f+1)}} \tag{33}$$

for the α-transitions between a natural parity state and a non-natural parity state $((-)^{\ell_{s1}+\ell_{s2}} = (-)^{I+1})$ of $^{210}Bi \to {}^{206}Tl$, where

$$F^{(1)}_{I} = \sum_{s_1 s_2} \hat{j}_{s_1} \hat{j}_{s_2} (-)^{j_{s1}+\ell_{s1}+1/2} C^{j_{s1} j_{s2} I}_{1/2\,-1/2\,0} \times \{ X^{I}_{s_1 s_2} + Y^{I}_{s_1 s_2} \} \mathcal{R}_{s_1}(R)\, \mathcal{R}_{s_2}(R), \tag{34}$$

$$F^{(\pm)}_{I} = \sum_{s_1 s_2} \hat{j}_{s_1} \hat{j}_{s_2} (-)^{j_{s1}+\ell_{s2}+1/2} C^{j_{s1} j_{s2} I}_{1/2\,-1/2\,0} \times \{ X^{I}_{s_1 s_2} + Y^{I}_{s_1 s_2} \} \mathcal{R}_{s_1}(R)\, \mathcal{R}_{s_2}(R)\,(K_1 \pm K_2) \tag{35}$$

with

$$K_i = (2j_{s_i} + 1)(\ell_{s_i} - j_{s_i}) . \tag{36}$$

The quantities u_ϵ, $\partial\rho/\partial R$ and $g(R)$ are given in fig. 4 for some α-transitions. As a rule, the last minimum of u_ϵ coincides with the minimum of $\partial\rho/\partial R$ and with the maximum (minimum) of $g(R)$.

Table

The experimental α-kinetic energies (E_α) and α-decay widths together with the ratios calculated as in fig. 5 for $^{210}Bi \to {}^{206}Tl$ and $^{210}Po \to {}^{206}Pb$. The numbers in the brakets in the 5th column are the exponents of 10.

Nr	$I_i^{\pi_i}$	$\to (I_f^{\pi_f})_n$	E_α(MeV)	Γ_{exp}(MeV)	$A = \frac{\Gamma_{exp}}{\Gamma^{s.p.}_{mod}}$	$B = \frac{\Gamma_{exp}}{\Gamma^{Is}_{mod}}$	$C = \frac{\Gamma_{exp}}{\Gamma^{sk}_{th}}$	$D = \frac{\Gamma_{exp}}{\Gamma^{Is}_{th}}$
$^{210}Bi \to {}^{206}Tl$								
1.	1^-(g.s.)	1_1^-	4.649	7.90(-34)	269	158	1.55	1.1.
2.	1^-(g.s.)	2_1^-	4.686	5.26(-34)	164	51	1.26	0.20
3.	9^-	1_1^-	4.908	1.62(-36)	295	107	7.78	0.31
4.	9^-	2_1^-	4.946	2.27(-36)	42	21	0.19	0.026
5.	9^-	1_2^-	4.550	2.06(-38)	37	21	-	0.073
6.	9^-	2_2^-	4.568	1.98(-37)	620	415	-	2.39
7.	9^-	3_1^-	4.413	1.23(-38)	2016	92	-	0.4
8.	9^-	$2_3^-(2_4^-)$	4.224	4.13(-40)	-	-	-	17.57(1.6)
$^{210}Po \to {}^{206}Pb$								
1.	0^+(g.s.)	0^+(g.s.)	5.30451	3.8(-29)	6700[17]	134[17]	-	1.88
2.	0^+(g.s.)	2^+	4.525	4.56(-34)	24000[17]	114[17]	-	1.37

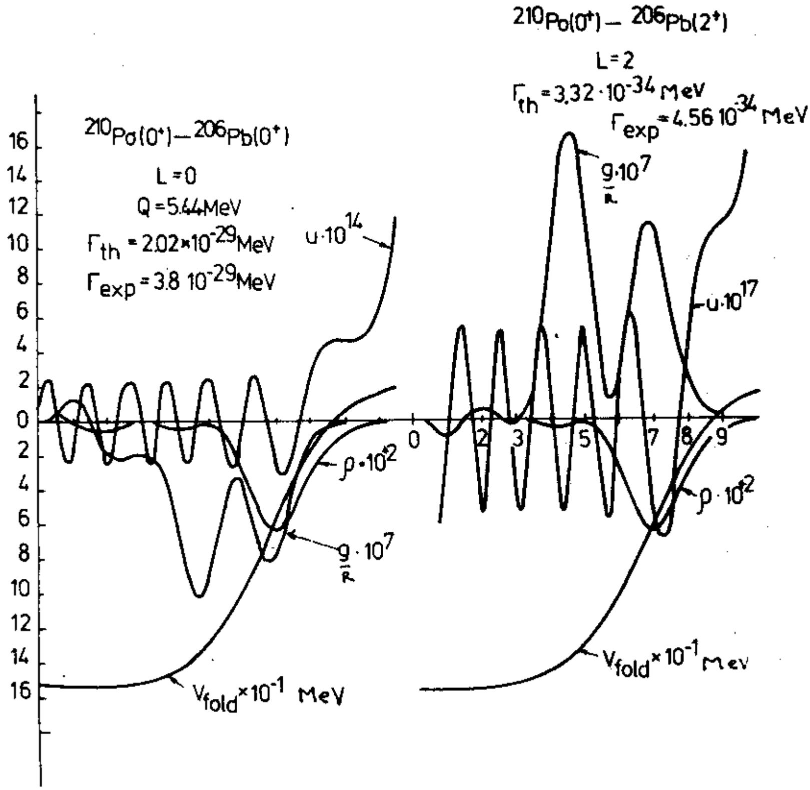

Fig.4. The u_ϵ, $\partial\rho/\partial R$ and g functions occuring in the integral (28) together with the folding potential for $^{210}Po(g.s) \to {}^{206}Pb(g.s)$ and $^{210}Po(g.s.) \to {}^{206}Pb(2^+_1)$.

The α-decay widths calculated in our model together with those calculated in the potential model of refs.[7,18] and with the experimental ones[19-21] are given in the table in which the universal constant κ was considered to be equal to $\kappa = 0.522 \cdot 10^7\ \mathrm{MeV \cdot fm^{13}}$. In the fig. 5 are shown the hindrance factors, where for favoured α-widths the Geiger-Nuttal expression was used.

One can see that our calculated α-decay widths for both favoured and unfavoured α-transitions are in agreement with the experimental data. The remained discrepancies are not large and they can be removed either by sophisticating the model

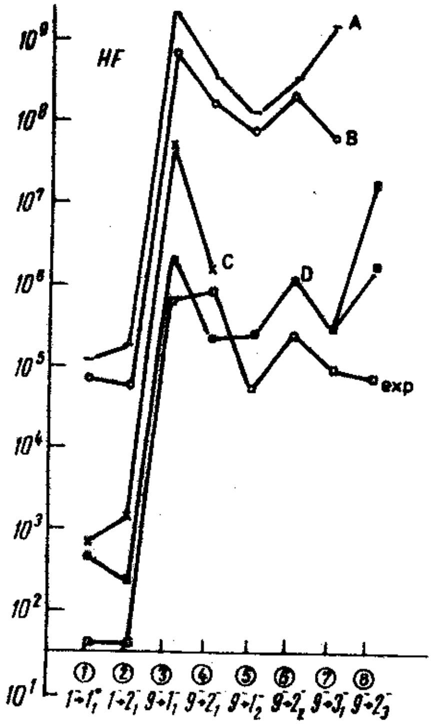

Fig.5. The experimental hindrance factors together with the theoretical ones calculated as follows: A) model of ref. /7/ and the s.p. structure of the nuclear states; B) model of refs. /7/ and the structure of ref. /17,18/; C) model and the structure of ref. /11/ and D) the present calculations with the structure of ref. /17/ The favoured α-widths have been calculated according to the Geiger-Nuttal low.

(e.g., by including the next perturbation term) or/and by improving the description of the nuclear structure.

4. CONCLUSIONS

From the detailed analysis of the Pauli kernel (see Sec. 2) we conclude that:

1°) The correct consideration of the Pauli principle does not change much the R-matrix decay width. Taking into account that K is a positive definite operator the correction ϵK from the eq. (3) will rather increase the barrier (if at such distances K does not vanishes), which lead to a decreases of the R-matrix α-decay width.

2) The K-kernel has many spurious states that has to be eliminated. From fig. 3 we learn that only the states with the momenta $q > q_\epsilon \approx 4K_F$ (where K_F is the Fermi momentum) contribute to the α-decay width, which lead to some selection among the optical model potentials (i.e., the accepted optical potentials must have the depth ≈ 200 MeV). The proposed /8-10/ transformation $\Omega_\epsilon = \sqrt{1-K}\,u_\epsilon$ may lead to uncontrolable errors, when calculating the renormalized amplitude of the reduced width $(1-K)^{-1/2}g$ because of the effect of the spurious states that

may still occur in the $(1-K)^{-1/2}$ operator and because of the nonuniqueness of $\sqrt{1-K}$.

3°) None of the known models [1-7] for α-transition operator can remove the discrepancy between the theoretical and the experimental α-decay widths except for our Fermi liquid model proposed in ref. [11].

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