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THRESHOLD π^0-MESON PHOTORODUCTION ON THE LIGHTEST NUCLEI

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1. INTRODUCTION

In the last years due to a great effort of the French group [1] a large amount of the experimental data on π°-meson photoproduction near threshold were stored. There are three motivations for the experimental and theoretical research of the threshold photoproduction of π°-mesons.

The first is the desire to get a new information concerning the elementary $\gamma N \to \pi N$ amplitude.

The second is the possibility to extract from experiment the information on a nuclear matrix element of the type $\langle\Psi|\frac{1}{r}|\Psi\rangle$. It is possible because of specific feature of the mechanism of photoproduction shown in ref. [2].

And finally the interpretation of results on the low energy photoproduction of π-mesons on few-body systems is almost model-independent as compared to those on heavy nuclei*.

Below we consider the threshold π°-meson photoproduction on nuclei d, ^3He, ^3H and ^6Li. The ^6Li nucleus will be described as a three-body system, $\alpha + n + p$. For the dynamical part of the problem that means for the final state wave function of the system $\pi + A$ we use equations of the evolution with respect to coupling constant (ECC) method [3].

These equations are rather suitable and effective for the description of π-nucleus interaction at low energies [4,5]. The probability of the process:

$$\gamma + A \to \pi^\circ + A \tag{1.1}$$

can be defined by the matrix element

$$M_{\gamma A \to \pi^\circ A} = (\pi A|\hat{H}_e|\gamma A)\,. \tag{1.2}$$

Here $(\pi A|$ is an eigenvector for the πA-system, $\hat{H}_e$ is the Hamiltonian of electromagnetic interaction.

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* Everywhere in this work we suppose the πN and NN-interaction potential type. Also, we omit the discussion of the role of exchange currents in π-meson photoproduction.

For the matrix element (1.2) we have

$$(\pi A|M|\gamma A) = \int \frac{d\vec{k}_1}{(2\pi)^3} \prod_{i=1}^{A-1} \frac{d\vec{Q}_i}{(2\pi)^3} \Psi^*_{\vec{k}} (\vec{k}_1, \vec{Q}_1, \ldots, \vec{Q}_{A-1}) \times$$
$$\times <\pi A; \vec{k}_1, \vec{Q}_1, \ldots, \vec{Q}_{A-1}|M|\gamma A; \vec{q}>, \qquad (1.3)$$

where $\Psi_{\vec{k}}$ is the wave function of the π+A system; $\vec{k}$, the momentum of the relative motion of π-meson and nucleus; $\vec{q}$, the momentum of the photon.

The wave function of the π+A system is usually obtained by using the optical model with the first order optical potential in multiplicity of πN-collisions. However, the optical model has some defects, one of which is unacceptable while describing the photoproduction of pions. Really, the "optical" wave function takes into account the rescattering of a π-meson on all nucleons in a target including the nucleon producing the π-meson. But in calculations we use the experimental value of the elementary amplitude, which by definition takes into account the rescattering on this nucleon. So, the calculation of photoproduction of π-mesons within the optical model contains the double counting of rescattering effects. We show in the next section large contributions from the double counting.

2. THRESHOLD π°-MESON PHOTOPRODUCTION ON DEUTERON

As we pointed out in the Introduction, the wave function of the π+A system will be found via dynamical equations of the ECC-method. The total Hamiltonian H of the π+A system has the form

$$H = H_0 + h + gV_\pi , \qquad (2.1)$$

here H_0 is the kinetic energy operator for the relative motion of a pion and nucleus; h the target Hamiltonian; $V_\pi = \sum_{i=1}^{A} V_{\pi N_i}$, the potential of interaction of a pion and nucleons of the nucleus; g is the dynamical variable of the ECC-method; $0 \le g \le 1$.

The equation for the eigenvector $|\pi A)$ of the Hamiltonian H in ECC-method is [3]

$$\frac{\partial}{\partial g}|\pi A) = \sum_\sigma |\sigma) \frac{(\sigma|V_\pi|\pi A)}{E - E_\sigma + i\epsilon} . \qquad (2.2)$$

For the matrix element in the r.h.s. we have the representation

$$(\sigma|V_\pi|\pi d;\vec{k}) = \int(\sigma|\vec{k}_3\vec{Q}_3><\vec{k}_3\vec{Q}_3|V_\pi|\pi d;\vec{k}>d\vec{k}_3\,d\vec{Q}_3 = \\ = \frac{1}{g}\int\frac{d\vec{k}_3\,d\vec{Q}_3}{(2\pi)^6}\Psi_\sigma(\vec{k}_3,\vec{Q}_3)<\vec{k}_3\vec{Q}_3|T|\phi_d;\vec{k}>, \tag{2.3}$$

where $\Psi_\sigma(\vec{k}_3,\vec{Q}_3)$ is the wave function of the $2\pi N$-system with quantum numbers σ, T is the 3-body transition operator. Using (2.3), we immediately obtain the equation for the wave function of the $\pi+d$ system:

$$\frac{\partial}{\partial g}\Psi_{\vec{k}}(\vec{k}_2,\vec{Q}_2) = \frac{1}{g}\sum_\sigma\int\frac{d\vec{k}_3 d\vec{Q}_3}{(2\pi)^6}\,\frac{\Psi_\sigma(\vec{k}_2,\vec{Q}_2)\Psi_\sigma(\vec{k}_3,\vec{Q}_3)}{k^2/2\mu+E_d-E_\sigma+i\epsilon}<\vec{k}_3\vec{Q}_3|T|\phi_d;\vec{k}>. \tag{2.4}$$

This equation will be solved by using two approximations, which hold at low energies of photons. The first approximation consist in the restriction in the r.h.s. of (2.4) only to term linear in the function $t_{\pi N}(g)$. Reliability of this approximation was confirmed by the authors in calculations of the πA-scattering length /5/. This restriction immediately leads to the following form of $\Psi_\sigma(\vec{k}_2,\vec{Q}_2)$ in the r.h.s. of eq. (2.4):

$$\Psi_\sigma(\vec{k}_2,\vec{Q}_2) = \begin{cases} (2\pi)^3\delta(\vec{k}_1-\vec{k}_2)\,\phi_d(\vec{Q}_2), & \sigma = |\pi d) \\ (2\pi)^3\delta(\vec{k}_1-\vec{k}_2)\,\Psi_{\vec{Q}_1}(\vec{Q}_2), & \sigma = |\pi 2N). \end{cases}$$

The second approximation is the approximation of the static nucleons, i.e., $\mu_\pi/m_N \ll 1$. Due to this approximation terms linear in $t_{\pi N}$ and containing NN-interaction can be omitted (see fig.1). The boundary condition in g for eq. (2.4) has, obviously, the form

$$\Psi_{\vec{k}}(\vec{k}_2,\vec{Q}_2)|_{g=0} = (2\pi)^3\delta(\vec{k}_2-\vec{k})\,\phi_d(\vec{Q}_2). \tag{2.5}$$

So, for the wave function in the threshold energy region we have

$$\Psi_{k=0}(\vec{k}_2,\vec{Q}_2) = (2\pi)^3\delta(\vec{k}-\vec{k}_2)\,\phi_d(\vec{Q}_2) + \frac{2\pi}{\mu_{\pi N}}\,\frac{1}{\frac{k_2^2}{2\mu}-\frac{k^2}{2\mu}-i\epsilon}\times \\ \times[\bar{a}_{\pi N_1}\phi_d(\vec{Q}_2+\frac{\vec{k}_2}{2})+\bar{a}_{\pi N_2}\phi_d(\vec{Q}_2-\frac{\vec{k}_2}{2})], \tag{2.6}$$

where

$$\bar{a}_{\pi N_i} = \int_0^1 \frac{dg}{g} a_{\pi N_i}(g) \tag{2.7}$$

and $a_{\pi N_i}(g)$ is a solution of the corresponding two-particle problem.[1]

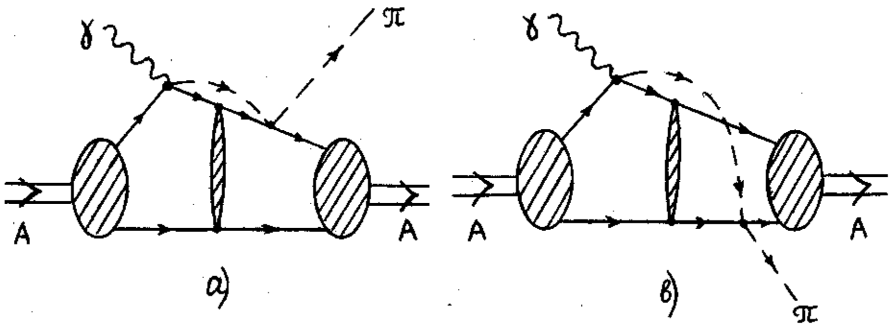


Fig. 1. Diagrams including NN-interaction and disappearing in the static limit.

Taking into account the isotopic structure of the πN-amplitude for $a_{\pi N}(g)$ we have:

$$a^{\alpha\beta}_{\pi N} = a^{(+)}(g)\,\delta_{\alpha\beta} + \frac{1}{2} a^{(-)}(g)\,[\tau_\beta, \tau_\alpha], \tag{2.8}$$

$[\tau_\beta, \tau_\alpha]$ is the commutator of isospin Pauli matrices.

If we restrict ourselves only to the S-wave πN-interaction

$$V_{\pi N} = \Lambda v(k)\, v(k'), \qquad v(k) = \frac{1}{k^2 + \gamma^2}$$

for the function $a^{(\pm)}(g)$ we can get

$$a^{(\pm)}(g) = \frac{g a^{(\pm)}_{exp}}{1 - \frac{\gamma}{2}(g-1)\, a^{(\pm)}_{exp}},$$

where $a^{(\pm)}_{exp}$ are experimental value of πN-scattering lengths. The matrix element $\langle \pi 2N; \vec{k}_1 \vec{Q}_1 | M | \gamma d; \vec{q} \rangle$ from (1.3) has the form

$$\langle \pi 2N; \vec{k}_1 \vec{Q}_1 | M | \gamma d; \vec{q} \rangle = \phi_d\left(\vec{Q}_1 + \frac{\vec{k}_1}{2} - \frac{\vec{q}}{2}\right) M^{\alpha}_{N_1} + \phi_d\left(-\vec{Q}_1 + \frac{\vec{k}_1}{2} - \frac{\vec{q}}{2}\right) M^{\alpha}_{N_2}. \tag{2.9}$$

Here $M^{\alpha}_{N_j}$ is the π-meson photoproduction amplitude on a nucleon N_j. For this amplitude near threshold we have

$M^{\alpha}_{N_j} = i(\vec{\sigma}\vec{\epsilon})E^{\alpha}_{N_j}$,

where $\vec{\epsilon}$ is the vector of the photon polarization, $\vec{\sigma}$ is the spin Pauli matrix of the nucleon; $E^{\alpha}_{N_j}$ is the multipole amplitude.

$E^{\alpha}_{N_j}$ has the isotopic structure

$$E^{\alpha}_{N_j} = E^{(+)}\delta_{\alpha 3} + \frac{1}{2}E^{(-)}[\tau^{(j)}_{\alpha}, \tau^{(j)}_{3}] + E^{(0)}\tau^{(j)}_{\alpha} . \qquad (2.10)$$

Substituting (2.6) and (2.9) into (1.2) and taking into account the isotopic structure of all quantities, we obtain the following expression for the π°-photoproduction amplitude:

$$(\pi^{\circ}d|\tilde{M}|\gamma d> = 2E^{(+)}F_d(q^2) - 2<\phi_d|\frac{e^{i\vec{q}\vec{r}/2}}{r}|\phi_d>\times \qquad (2.11)$$
$$\times[2\bar{a}^{(-)}E^{(-)} - \bar{a}^{(+)}E^{(+)}],$$

where $F_d(q^2)$ is the deuteron form-factor and

$$<\phi_d|\frac{e^{i\vec{q}\vec{r}/2}}{r}|\phi_d> = \int d\vec{r}\,\frac{e^{i\vec{q}\vec{r}/2}}{r}|\phi_d(r)|^2 ,$$

$\tilde{M}$ stands for the following summation

$$\tilde{M} \sim \sum_{i \neq j} a^{\beta\alpha}_{\pi N_i} E^{\alpha}_{N_j}$$

that means the rescattering of π-mesons on the nucleon producing pion was excluded. If we sum over all nucleons, then we have the additional contribution

$$\Delta M = 2[\bar{a}^{(+)}E^{(+)} + 2\bar{a}^{(-)}E^{(-)}]\gamma F_d(q^2) , \qquad (2.12)$$

which is five times as large as the amplitude (2.11).

If the expression (2.11) is compared to the diagrams (see fig.2), then the Feynman rules should be slightly modified, namely, to the πN-vertices we compare not the amplitudes $\frac{a_{\pi N}}{\bar{a}_{\pi N}}$ but the unitarized, that is integrated over g, quantity.

The calculation was done with different sets of the πN-interaction parameter. The pion-photoproduction amplitude near threshold in contradiction to the elastic π-nuclear scattering amplitude appeared to be low-sensitive both to the unitarization procedure, that is to the change $a_{\pi N} \to \int \frac{dg}{g} a_{\pi N}(g)$

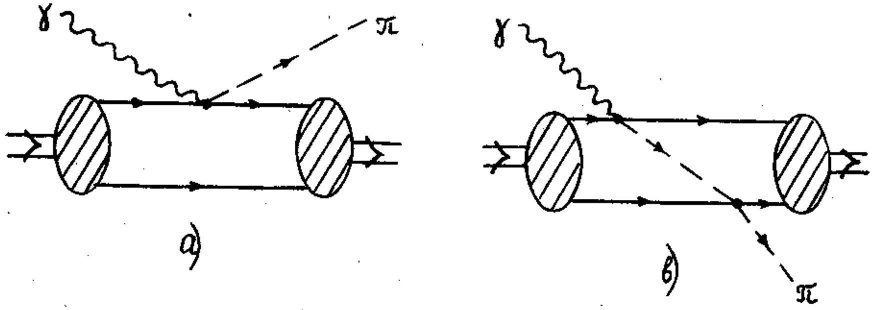


Fig.2. Diagrams illustrating expression (2.11): a) without pion rescattering; b) including rescattering.

and to the elementary πN -data. Both these facts correspond to that the expression $a^{(-)}E^{(-)}$ gives the main contribution to the pion photoproduction amplitude. The isovector part of πN-scattering amplitude $a^{(-)}$ in contradiction to the isoscalar part $a^{(+)}$ depends weakly on the πN-parameter sets and almost do not change after the integration over g.

We take isotopic combinations of multipole amplitudes in the Born approximation with the pseudovector coupling /6/. The values of $F_d(q^2)$ and $\langle \frac{e^{i\vec{q}\vec{r}/2}}{r} \rangle$ were taken from ref. /7/, there they had been computed by using the McGee wave function for the deuteron (at threshold q = 0.66 fm). The amplitude (2.11) equals*

$$M_{\gamma d\to\pi^\circ d} = (-8.2 \pm 0.2)\cdot 10^{-3} \text{ fm}.$$

The experimental value of this amplitude is /8/

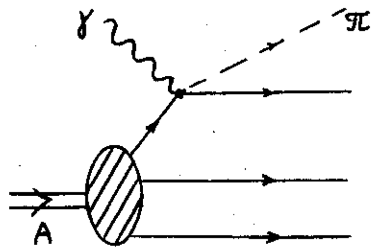
$$M^{exp}_{\gamma d\to\pi^\circ d} = (-8.90 \pm 0.45)\cdot 10^{-3} \text{ fm}.$$

3. THRESHOLD π°- MESON PHOTOPRODUCTION ON NUCLEI ^{3}He AND ^{3}He

The matrix element $\langle \pi A|M|\gamma A\rangle$ given by (1.3) and the wave function of the πA -system should be known to obtain the amplitude of the process (1.1). In the framework of the approximations discussed above the π -nuclear wave function may be

* With the experimental values of the πN -scattering lengths from ref. /9/.

Fig.3. Direct photopion production diagram which corresponds to the case of noninteracting final particles.



found from equation (2.2) with the boundary condition in g in the form:

$$\Psi_{\vec{k}}(\vec{k}_2, \vec{P}, \vec{Q})|_{g=0} = (2\pi)^3 \delta(\vec{k}-\vec{k}_2)\Psi_A(\vec{P}, \vec{Q}) . \quad (3.1)$$

Here $\Psi_A(\vec{P}, \vec{Q})$ is the nuclear wave function of ^3He (^3H), $\vec{P}, \vec{Q}$ are Jacobi variables.

In the first order in the constant of electromagnetic interaction the matrix element $\langle \pi A|M|\gamma A\rangle$ is a sum of diagrams one of which is shown in fig.3.

After some calculations we get the following expression:

$$M_{\gamma A \to \pi^0 A} = (E^{(+)} \mp E^{(0)}) F_1(q) - $$
$$- [4\bar{a}^{(-)}E^{(-)} - 2\bar{a}^{(+)}(E^{(+)} \mp E^{(0)})]F_2(q) , \quad (3.2)$$

where

$$F_1(q) = \int \frac{d\vec{P}\, d\vec{Q}}{(2\pi)^6} \Psi^*(\vec{P} + \frac{\vec{q}}{2}, \vec{Q} - \frac{\vec{q}}{3})\Psi(\vec{P}, \vec{Q})$$

$$F_2(q) = \int \frac{d\vec{k}_1\, d\vec{P}\, d\vec{Q}}{(2\pi)^9} \Psi^*(\vec{P} + \frac{\vec{q}}{2} - \vec{k}_1, \vec{Q} - \frac{\vec{q}}{3}) \frac{1}{k_1^2} \Psi(\vec{P}, \vec{Q}) ,$$

the upper sign corresponds to ^3He and the lower to ^3H.

The wave function of nucleus ^3He (^3H) has the simple form /10/

$$\Psi_A(\vec{P}, \vec{Q}) = \frac{1}{\sqrt{2}}(\chi_1 \eta_2 - \chi_2 \eta_1) u(\vec{P}, \vec{Q}) ,$$
$$u(P, Q) = (\frac{3\alpha^2}{\pi})^{3/2} \exp[-\alpha^2 (P^2 + \frac{3}{4} Q^2)] . \quad (3.3)$$

The parameter α was taken from the fit of the form-factor ^3He (^3H) at small q^2.

Using function (3.3) we find at $q = 0.70$ fm $F_1(q) = 0.83$, $F_2(q) = 0.47\ fm^{-1}$. For these F_1 and F_2 the numerical value of the π^0 photoproduction amplitude equals $-5.85 \cdot 10^{-3}$ fm. This value is very close to the experimental one [8]:

$$M^{exp}_{\gamma ^3He \to \pi^o ^3He} = (-5.80 \pm 0.60) \cdot 10^{-3} \text{ fm}.$$

As the amplitude (3.2) is very sensitive to the function $F_2(q)$ * we are left with the equation on the stability of $F_2(q)$ with respect to variations of the wave function form of ^{3}He nucleus.

A partial calculation of nonstatic corrections or binding corrections has been made in ref. [11]. It gives a value of the amplitude $M_{\gamma ^3He \to \pi^o ^3He}$ by about 20% smaller than the experimental amplitude. Our numerical value of the amplitude for ^{3}H given by expression (3.2) is $10.1 \cdot 10^{-3}$ fm. This value is two times as large as the value of the amplitude for ^{3}He what qualitatively agrees with the one-particle process picture.

4. THRESHOLD π^o- MESON PHOTOPRODUCTION ON ^{6}Li

The nucleus ^{6}Li is treated as a three body system consisting of a proton, a neutron and an α-particle. In the description of π^o photoproduction on ^{6}Li an α-particle can be considered as an inert core. Actually, since the α-particle has zero spin, the expansion of the photoproduction amplitude over multipoles does not contain the dipole electric amplitude E_{0+} nonvanishing at threshold. The rescattering process of pions on the α-particle takes place only via π^o-mesons and therefore is not important.

Considering that nucleons of an α-particle core in ^{6}Li are in the triplet spin state (S=1) and in the singlet isospin state (T=0), we get the following expression for the process

$$M_{\gamma ^6Li \to \pi^o ^6Li} = 2E^{(+)} F_1(q) - 2[2\bar{a}^{(-)} E^{(-)} - \bar{a}^{(+)} E^{(+)}] \cdot F_2(q), \quad (4.1)$$

where

$$F_1(q) = \int \frac{d\vec{P}\, d\vec{Q}}{(2\pi)^6} \Psi^*(\vec{P} + \frac{\vec{q}}{2}, \vec{Q} - \frac{2}{3}\vec{q})\, \Psi^*(\vec{P}, \vec{Q}),$$

$$F_2(q) = \int \frac{d\vec{k}_1\, d\vec{P}\, d\vec{Q}}{(2\pi)^9} \Psi^*(\vec{P} - \vec{k}_1 + \frac{\vec{q}}{2}, \vec{Q} - \frac{2}{3}\vec{q})\, \frac{1}{k_1^2} \Psi(\vec{P}, \vec{Q}),$$

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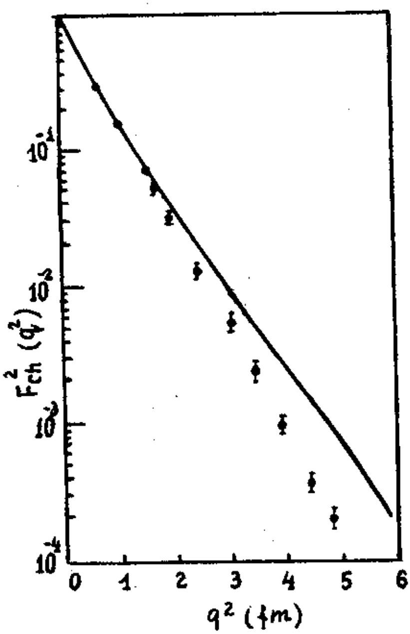
* The reason is that the double scattering via charge exchange becomes important.



while obtaining the wave function of ^{6}Li, we have used α–N and NN-potentials in a separable form as given in ref. [12]. Using this wave function we calculated the charge form-factor of ^{6}Li and obtained good agreement with the experiment at small q^2 (see fig.4). This fact makes us hope that the calculated values $F_1(q)$ and $F_2(q)$ are close to the true ones. Our numerical results are

$F_1(q) = 0.50$, $F_2(q) = 0.32\ \mathrm{fm}^{-1}$

at $q = 0.70$ fm.

Fig.4. The charge form factor $F_{ch}(q^2)$ calculated with the three body ^{6}Li (g.s.) wave function and experimental data from ref. [13].

Finally, we get

$$F_{\gamma\,^6Li \to \pi^0\,^6Li} \approx -6.3\cdot 10^{-3}\ \mathrm{fm}.$$

There is no experimental value to be compared with, there is only the theoretical calculation (see ref. [14]), which gives for the amplitude $M_{\gamma\,^6Li \to \pi^0\,^6Li}$ at threshold the value approximately one and one half times as small as our value. In ref. [14] there are used elementary photoproduction amplitudes which somewhat differ from ours and the oscillator wave function of ^{6}Li.

5. CONCLUDING REMARKS

In this work we apply the ECC-method to describe the π^0 - photoproduction near threshold on nuclei with nonzero spins (d, ^{3}He, ^{3}H, ^{6}Li). Using this method we can make unitarization of the contribution of πN -rescattering.

It is necessary to emphasize a specific situation, first noted in [2] arising in the description of process (1.1). Processes of the intermediate production of charged pions are important in π^0 photoproduction because of the big value for the amplitude. The photoproduction amplitude is dominated by the combination $a^{(-)}E^{(-)}$ related to the charged mesons, which is low sensitive to the unitarization sets of πN-data. Note the opposite situation for the elastic πA-scattering described within the ECC-method: the unitarization was substantial and there was a strong dependence on sets of the πN- interaction parameters (see ref. [5]). The reason for this difference is that in the elastic πA-scattering the dominant (especially for even-even nuclei) contribution to the π-nuclear scattering length comes from the isoscalar part of the πN-amplitude $a^{(+)}$ which is strongly renormalized by unitarization and is very sensitive to sets of πN-data.

With the same input data, the description of process (1.1) within the ECC-method is similar to the description of that process within the multiple scattering theory in the double scattering approximation (photoproduction and a subsequent scattering of the pion on one of a nuclear nucleon).

As the ECC-equations are dynamical equations equivalent to the Schrödinger equation, then the proximity of the results obtained by that method on the π^0-meson photoproduction at threshold to the results of the multiple-scattering theory should be considered as a justification of the applicability of approximations of the multiple scattering theory to this process.

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