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ON A POSSIBILITY OF CONTINUOUS TRANSITION FROM WAVE OPTICS TO GEOMETRICAL ONE

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1. A continuous transition from the wave optics to geometrical one can be achieved with the help of a wave packet the dimensions of which decrease with wave length. This approach gives no difficulties for the case of electromagnetic waves but is not comletely satisfactory in the case of massive particles as it is impossible to construct the nonspreading wave packet within the framework of canonical quantum mechanics /1/. In this connection many attempts are undertaken to construct the packet with the aid of different tricks going out of the canonical formalism /2/. The present note is dealing with one of such attempts.

2. For a free particle moving with velocity $\vec{k}$ (in what follows time t is supposed to be linked with the real time τ through the relation $t = h\tau/m$, where m is the particle mass) we postulate a wave function in the form:

$$\psi(\vec{r},t) = c\,\frac{e^{-q|\vec{r}-\vec{k}t|}}{|\vec{r}-\vec{k}t|}\,e^{i\vec{k}\vec{r}-i\omega t}, \qquad (1)$$

where $c = \sqrt{q/2\pi}$ is the normalization constant, $\omega = \frac{1}{2}(k^2-q^2)$. We will here consider only the nonrelativistic case. After the Schrödinger operator $(i\frac{\partial}{\partial t} + \frac{1}{2}\Delta)$ operates on the wave function (1) we get

$$(i\frac{\partial}{\partial t} + \frac{1}{2}\Delta)\psi(\vec{r},t) = -2\pi c\,\delta(\vec{r}-\vec{k}t)\psi_0, \qquad (2)$$

where $\psi_0 = e^{ikr - i\omega t}$. Therefore the function $\psi(\vec{r},t)$ can be regarded as a solution of equation (2). When external fields are present the function $\psi(r,t)$ obeys the following equation

$$[i\frac{\partial}{\partial t}+\frac{1}{2}\Delta-\frac{1}{2}u(\vec{r})]\psi(\vec{r},t)=f(\vec{r},t), \qquad (3)$$

where $\frac{1}{2}u(\vec{r})$ is the potential of external field multiplied by the quantity m/h^2, and the function $f(\vec{r},t)$ is defined by the same equation (3) and by the asymptotical condition requiring that for $t\to-\infty$ the function $\psi(\vec{r},t)$ describe the free particle moving along a definite straight-line path with velocity $\vec{k}$, i.e., that it coincides with (1). Thus, on the function $f(\vec{r},t)$ there is imposed the following condition:

$$\lim_{t\to-\infty} f(\vec{r},t)=-2\pi c\delta(\vec{r}-\vec{k}t)e^{i\vec{k}\vec{r}-i\omega t}. \qquad (4)$$

It is stated that eqs. (3) and (4) uniquely define the function $\psi(\vec{r},t)$ with the asymptotic behaviour (1) for $t\to-\infty$.

It can be shown that eq. (2) allows one to introduce the usual quantum-mechanical expressions for current and density. This has not been proved in general, but in the below examined case the probability conservation law is fulfilled. Note also that the condition (4) makes equation (3) dependent on initial conditions. And equations dependent upon initial conditions are employed in physics /3/.

3. Consider next the reflection of a scalar neutron by the plane $z=0$ restricting the half-space $z>0$ with the potential $u=u_0=4\pi Nb$, where b is the amplitude of scattering on one nucleus, N is the number of nuclei per unit volume.

Let us expand the function (1) in the Fourier integral

$$\psi(\vec{r},t)=\frac{4\pi c}{(2\pi)^3}\int\frac{d^3p}{p^2+q^2}e^{i(\vec{p}+\vec{k})\vec{r}-i(\omega+\vec{p}\vec{k})t}. \qquad (5)$$

The axis $\vec{x}$ is taken to be perpendicular to $\vec{k}$ and z-axis. Then we integrate (5) over p_x denoting $p_2=(p_y,p_z)$ and obtain

$$\psi(\vec{r},t)=\frac{c}{2\pi}\int\frac{d^2p_2}{p_x}e^{-p_x|x|-i(\omega+\vec{p}_2\vec{k})t}\eta_0(\vec{r},t,\vec{p}_2), \qquad (6)$$

where $p_x = \sqrt{q^2 + p_2^2}$ and $\eta_0(\vec{r}, t, \vec{p}_2) = e^{i(\vec{p}_2 + \vec{k})\vec{r}}$. When a potential is present the integrand will contain the function $\eta(\vec{r}, t, \vec{p})$ of the general form, for which the function η_0 plays the role of an incident wave. Substituting (6) into (3) shows that $\eta(\vec{r}, \vec{p})$ obeys the equation

$$[\frac{1}{2}\Delta - \frac{1}{2} u_0 \theta(z) + (\omega + \vec{p}_2 \vec{k}) + \frac{1}{2} p_x^2 - p_x \epsilon(x) \frac{\partial}{\partial x}]\,\eta(\vec{r}, \vec{p}) = 0, \quad (7)$$

where $\theta(x)$ and $\epsilon(x)$ are step functions, such that for $x > 0$ $\theta(x) = \epsilon(x) = 1$ and for $x < 0$ $\theta(x) = 0, \epsilon(x) = -1$. Since the potential does not depend on x, and so does η_0, equation (7) is readily solved and we find

$$\eta(\vec{r}, \vec{p}) = [e^{i\vec{Q}\vec{r}} + R(Q_z) e^{i\vec{Q}'\vec{r}}]\,\theta(-z) + T(Q_z) e^{i\vec{Q}''\vec{r}}\,\theta(z), \quad (8)$$

where $Q = (Q_y, Q_z) = p_2 + k,\ Q' = (Q_y, -Q_z),\ Q'' = (Q_y, Q_z'')$.

$$Q_z'' = \mathrm{sign}(Q_z)\sqrt{Q_z^2 - u_0},\ R(Q_z) = (Q_z - Q_z'')/(Q_z + Q_z''),$$

$$T(Q_z) = 2Q_z/(Q_z + Q_z'').$$

Substituting (8) into (6) and (6) into (3) gives for $q^2 \ll u_0 \ll k_z^2$

$$f(\vec{r}, t) = -2\pi c\ \{[e^{i\vec{k}\vec{r}}\,\delta(\vec{r} - \vec{k}t) + R(k_z) e^{i\vec{k}'\vec{r}}\,\delta(\vec{r} - \vec{k}'t)]\,\theta(-z) +$$
$$+ T(k_z) e^{i\vec{k}''\vec{r}}\,\delta(\vec{r} - \vec{k}''t) \cdot \theta(z)\}\, e^{-i\omega t}, \quad (9)$$

where k, k', k'' are connected with each other in the same way as Q, Q', Q''.

4. When $k_z^2 < u_0$, and as above $q < k_z$, from (8) and (6) we obtain the following: 1) the conventional Goose-Hanchen displacement /4/ which exhibits itself in that the path of reflected particle starts under the reflection surface at the distance $z_1 = 2/k_z$; 2) an effect which occurs only within the proposed here scheme. The effect is as follows: Despite that $k_z^2 < u_0$, there is a flux inside the medium. This flux is proportional to q. In what follows we shall assume that $q \sim k$, i.e., $q = \alpha k$. Then the probability

to come into the medium turns out to be proportional to α. This effect in principle can explain the small and universal storage time of ultracold neutrons (UCN) in traps [5]. The quantity α defined by experiments [5] appears to be $\sim 10^{-3}$. It means that the dimensions of packet (1), i.e., the coherence length for thermal neutrons is $\sim$ 1000 A. With increasing k the packet dimensions are decreasing and at $k \sim 10^3$ A^{-1} or at an energy E ~ 10 keV the packet dimensions prove to be smaller than the interatomic distances. If one takes a large, strongly absorbing in the range of 10 keV monocrystal it can serve as an ideal collimator, since neutrons are able to escape across it only through crystalline planes.

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