ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 8029

V.V.Burov, Yu.N.Eldyshev, V.K.Lukyanov,
Yu.S.Pol

ON ONE UNIVERSAL NUCLEAR CHARGE DENSITY DISTRIBUTION

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 8029

V.V.Burov,[1] Yu.N.Eldyshev,[2] V.K.Lukyanov,
Yu.S.Pol[3]

ON ONE UNIVERSAL NUCLEAR CHARGE DENSITY DISTRIBUTION

Submitted to
„Ядерные реакции при высоких энергиях"

---

[1] The Moscow State University, Moscow.
[2] Institute for Nuclear Research, Moscow.
[3] The Lebedev Institute AN USSSR, Moscow.

In analysing the scattering of electrons on nuclein great hopes are connected with the so-called model-independent method (MIM) which is expected to provide us with an "experimental" charge density distribution (ChaDD) including its radial variations. However, along this line some difficulties appear. Thus, it is known that, first, MIM can be applied only when the precise experimental data, especially at large momentum transfers, exist. For this reason, beginning from [1,2], where MIM was developed and applied to the Calcium isotopes [3], in addition only four nuclei were successfully investigated in this manner (^{4}He [4] , ^{12}C [5] , ^{208}Pb [6]). Second, the results of MIM, in principle, are rather dependent on the particular mathematical procedure used, especially on a choice of the ChaDD trial functions. For the present the set of the Fermi-function derivatives [1,2] , the Gaussian, the sine and some other sets of functions have been used in the MIM analysis. But the results of these variants of MIM have never been compared for one and the same nucleus. Therefore even for the mentioned nuclei one cannot be sure that the obtained ChaDD, including just the indicated theoretical errors, may indeed be considered as experimental ones.

However, whereas MIM is in progress we would like to revive the interest to the traditional phenomenological method, where the simple forms of ChaDD are used for comparison with experiments. This method is known to give us the basic geometrical characteristics of nuclei

such as the mean-square radius, diffuseness and the shape of nuclear surface. The new aspect, to which we want to draw attention in connection with the phenomenological method, is that the method can give us some additional information on general, but not particular, features of ChaDDs, especially on some relative effects in investigating subsequently the nuclei along wide region of atomic numbers, provided each nucleus should be analysed with the help of the same ChaDD function. However, the main problem in realizing this idea in practice is an absence, up to now, of the same form of ChaDD for the whole group of nuclei investigated. Indeed, the Fermi type of ChaDDs with realistic exponential asymptotic behaviour is usually applied only to the medium and heavy nuclei while the Gaussian type of functions with the nonrealistic asymptotic behaviour is used for the light ones. This is because the Fermi function of ChaDD

$$\rho_F(r,R) = \rho_0 \left[1 + \exp\frac{r-R}{b}\right]^{-1} \qquad (1)$$

has the non-zero derivative in the center of a nucleus. This fact is of large significance for the light nuclei, which have comparatively small radii and very developed surface. On the other hand it is not important for heavier nuclei with large radii when this derivative is approximately equal to zero.

Here we present the results of joint phenomenological analysis of thirteen experimental form factors for the nuclei ^{4}He up to ^{208}Pb in the framework of the same (in other words, universal) so-called symmetrized Fermi ChaDD:

$$\rho_{SF}(r) = \rho_0 \left[\left(1+\exp\frac{r-R}{b}\right)^{-1} + \left(1+\exp\frac{-r-R}{b}\right)^{-1} - 1\right] = \rho_0 \frac{\mathrm{sh}\,R/b}{\mathrm{ch}\,R/b + \mathrm{ch}\,r/b}, \qquad (2)$$

$$\rho_0 = \frac{3}{4\pi R^3}\left[1 + \left(\frac{\pi b}{R}\right)^2\right]^{-1}. \qquad (3)$$

This function has many advantages /7/.

(i) For light nuclei its behaviour inside of a nucleus is similar to the Gaussian function, but for large distances it has a realistic exponential behaviour according to the quantum mechanics requirement for nuclei as finite systems. For heavier nuclei ($R \gg 1$) it practically coincides with the very known Fermi-fucntion (1) which many years was the most popular for analysis of experiments.

(ii) Despite the fact that for the Fermi function the derivative is nonvanishing at $r=0$, the suggested function (2) has zero derivative at this point. This fact gives some additional advantages.

(iii) Thus, the Born form factor and the mean square radius are calculated now in an obvious way /7/

$$F^B(q) = -\frac{4\pi^2 bR\rho_0}{q\,\mathrm{sh}\,\pi bq}\left(\cos qR - \frac{\pi b}{R}\sin qR\,\mathrm{cth}\,\pi bq\right) \qquad (4)$$

$$\langle r^2\rangle^{1/2} = R\sqrt{\frac{3}{5}}\sqrt{1+\frac{7}{3}\left(\frac{\pi b}{R}\right)^2}. \qquad (5)$$

This result makes very easy the calculations and also understanding of physical features of the process. For example, it is possible to see from eq. (4), that the positions of the form factor dips (in experiment, the first one) are directly connected with the shape parameters of a nucleus

$$\mathrm{tg}\,q_m R = \frac{R}{\pi b}\,\mathrm{th}\,\pi b q_m. \qquad (6)$$

(iv) Then, using the analytic properties of (2) one can calculate form factors in the high-energy approximation /8,9/, expressing the result through the corresponding residues in the pole points $r_n = R \pm i\pi b(2n+1)$; $n = 0, 1, 2, \ldots$ So this yields exactly (if $q > \sqrt{\frac{k}{R}}$)

$$F^{HE}(q) = -\frac{4\pi^2 bR\rho_0}{q\,\mathrm{sh}\,\pi bq} D\left[\cos(qR+\Phi) - \frac{\pi b}{q}\sin(qR+\Phi)\,\mathrm{cth}\,\pi bq\right], \qquad (7)$$

where the distortion effects are taken into account by the functions $D(q)$ and $\Phi(q)$, which are expressed via those given in ref. /10/. It was chacked /10/ that the high energy approximation gives the form factors for the medium and heavy nuclei very close to those calculated

when solving numerically the Dirac equation. Therefore, eq. (7) is very convenient in those physical investigations, where the needed accuracy is not better than $\chi^2 \approx 3$ per one experimental point (when all points are taken into account).

The result of applications of eq. (7) to the known experimental data [11] are shown in the Figure. The corresponding parameters are given in the Table. To composite one can say the following:

(i) Beginning from ^{12}C both the experimental and theoretical (7) form factors, given in the scale of $x = q_{eff} A^{1/3}$, where $q_{eff} = q\left(1 + \frac{4Ze^2}{3\langle r^2\rangle^{1/2} E} \frac{1}{137}\right)$ turn out to be invariants under changing q and A, i.e., $F(x,q,A) \approx F(x)$. An exception to this rule is ^4He and ^6Li due to very important role of number surface in the lightest nuclei.

(ii) In all cases an agreement with experiments is achieved up to $x < x_0 \approx 7.7\ fm^{-1}$ ($\pm 20\%$). . Roughly speaking, this means that the ChaDD under consideration correctly reproduces an average charge density behaviour in the interior of nuclei and especially its behaviour near and out of the nuclear surface.

Moreover, in more accurate consideration it can be assumed that the quantity x_0 is correlated by a character of filling of nuclear shells. Thus, for the s-shell nucleus ^4He we have $x_0 \sim 5.8$, for the p-shell nuclei ^6Li, ^{12}C, ^{16}O, $x_0 \sim 6.6$. Then, the nuclei ^{24}Mg and ^{28}Si fill the d-shell and have the near values $x_0 \sim 7.1$, while the ^{32}S and ^{40}Ca fill the $2s$-shell and $x_0 \sim 9.2$.

In this connection, it would be interesting to extend the measurements of the elastic form factors at large transfer momenta to heavier nuclei.

(iii) The region of $q > q_0 \approx 7.7 A^{-1/3}\ fm^{-1}$ may be called the region of the large momentum transfers, where form factors become sensitive to particular features of nuclear structure, in our case, to the radial variations of ChaDD in moving step by step from one nucleus to another. For instance, an appearance of additional dips or diminishing of slopes in experimental form factors of $^{12}C \div ^{40}Ca$ at $x \approx 8\ fm^{-1}$ can be caused by the alpha-cluster nature of these nuclei [12].

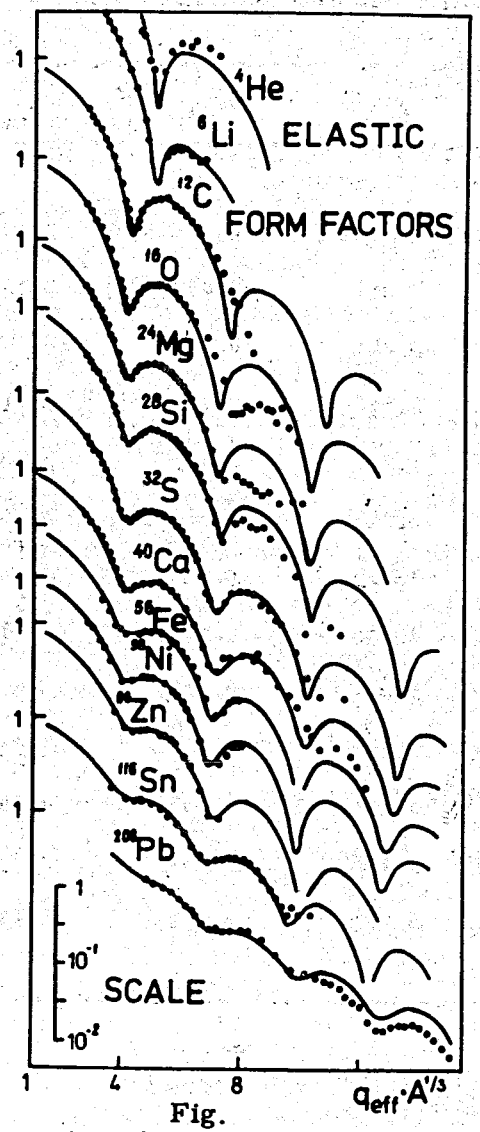

Fig. Form factors of nuclei calculated with the help of eq. (7) for symmetrized Fermi charge density distributions (2).

Table
The parameters of the fitted symmetrized Fermi charge density distributions.

	R	b	R_{rms}
^{4}He	1.238	0.380	1.708
^{6}Li	1.364	0.620	2.535
^{12}C	2.214	0.488	2.496
^{16}O	2.562	0.497	0.711
^{24}Mg	2.934	0.569	3.105
^{28}Si	3.085	0.563	3.175
^{32}S	3.255	0.601	3.370
^{40}Ca	3.556	0.578	3.493
^{56}Fe	4.054	0.600	3.853
^{58}Ni	4.153	0.566	3.844
^{66}Zn	4.200	0.663	4.081
^{116}Sn	5.486	0.561	4.734
^{208}Pb	6.557	0.515	5.427

References

1. V.K.Lukyanov, I.Ž.Petkov, Yu.S.Pol'. Jadernaja Fizika, 9, 349 (1969), (transl. in Sov. Journ. of Nucl.Phys. 9, 204 (1969)).
2. Yu.S.Pol'. Jadernaja Fizika, 10, 771 (1969) (transl.in Sov. Journ. of Nucl.Phys., 10, 771 (1969)).
3. J.B.Bellicard, P.Bounin, R.F.Frosch et al.Phys.Rev. Lett., 19, 9 (1967).
4. I.Sick. Proc. of Int. Conf. on Photonuclear Reactions and Appl. v. II, ed. by L.Berman, Dak Ridge, 1973.
5. I.Sick. Nucl.Phys., A218, 509 (1974).
6. J.L.Friar, J.W.Negele. Nucl.Phys., A212, 93 (1973).
7. Yu.N.Eldyshev, V.K.Lukyanov, Yu.S.Pol'. Jadernaja Fizika, 16, 506 (1972).
8. D.R.Yennie, F.L.Boos, D.G.Ravenholl. Phys.Rev., 137, B882 (1965).
9. Yu.S.Pol', V.K.Lukyanov, I.Z.Petkov. Acta Phys.Polonica, 34, 19 (1968).
10. I.Z.Petkov, V.K.Lukyanov, Yu.S.Pol'. Jadernaja Fizika, 4, 57 (1966) (transl. in Sov. Journal of Nucl.Phys., 4, 41 (1967)).
11. R.F.Frosch, J.S.McCarthy, R.E.Rand, M.R.Yearian. Phys.Rev., 160, 874 (1967);
    G.C.Li, I.Sick, R.R.Whitney, M.R.Yearian. Nucl.Phys., A162, 583 (1971).
    I.Sick, J.S.McCarthy. Nucl.Phys., A150, 631 (1970);
    G.C.Li, I.Sick, M.R.Yearian. Phys.Lett., 37B, 282 (1971); Preprint HEPL 691, Stanford, 1973;
    V.N.Khvastunov, N.G.Afanasyev et al. Phys.Lett., 28B, 1919 (1968);
    J.Heisenberg, R.Hofstadter, J.S.McCarthy. Phys.Rev. Lett., 23, 1402 (1969).
    V.D.Afanasyev, N.G.Afanasyev, V.M.Medjnik et al. Jadernaja Fizika, 12, 673 (1970).
    A.S.Litvinenko, N.G.Shevchenko, J.Yu.Buli et al. Jadernaja Fizika, 14, 40, 479 (1971).
12. E.V.Inopin, V.K.Lukyanov, Yu.S.Pol'. Preprint P4-7350, JINR, Dubna, 1973; Jadernaja Fizika, 19, 987 (1974).
    E.V.Inopin, V.S.Kinchakov, V.K.Lukyanov, Yu.S.Pol'. Preprint P4-7741, Preprint P4-7874, JINR, Dubna, 1974.

Received by Publishing Department
on June 18, 1974.