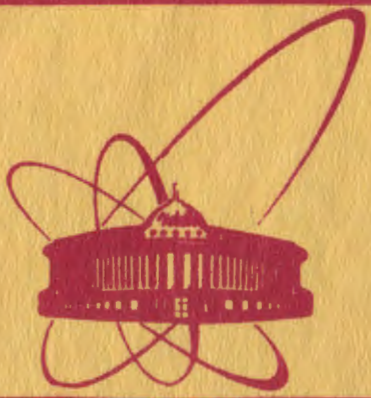

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E4-80-837

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POSSIBLE GENERALIZATIONS OF THE METHOD OF EVOLUTION IN THE COUPLING CONSTANT

Submitted to ЯФ

1980

The method of evolution in the coupling constant (ECC) proposed in ref.[1] turned out to be highly effective [2] for treating the many-body systems at low relative energies of particles[3].

In the π-nuclear scattering problems the simplicity of this approach is caused by a rapid convergence of iterations of the equations of the ECC-method for the scattering phases and by a comparatively simple calculation of the iterations themselves.

The starting point of the ECC-method is the statement that the description of evolution of the dynamic system in time is equivalent to the description of evolution with respect to the coupling constant. Hence, it is clear that the "natural" boundary condition to solve the differentil equations of the ECC-method should be the one at $g = 0$, where g is the coupling constant. Thus, evolution of the system starts from a free motion of all particles of the system. Just this point of the ECC-method admits generalization.

The generalization, which is the aim of this paper, consists in that the problem of evolution of the system turns out to be also solvable, if the considered system starts evolution from a certain value of the coupling constant $g = g_0$ which provides an exact solution of the problem, rather than from a free motion. In terms of the theory of multiple scattering this means that as a boundary condition with respect to g we use the problem in which a partial summation over a series of multiple scattering has already been performed.

Let us illustrate these reasons by two estimating examples. In both the cases we shall discuss the three-body problem, namely the πd-scattering at zeroth pion energy.

1.

Instead of the system Hamiltonian $\pi 2N$ we introduce an auxiliary Hamiltonian $H(g)$:

$$H(g) = H_0 + V_\pi + gh . \qquad (1)$$

Here H_0 is the kinetic energy of the relative motion of a pion and of the center of mass of two nucleons, $V_\pi = V_{\pi N_1} + V_{\pi N_2}$

is the interaction potential of π-meson with nucleons, h is the subsystem Hamiltonian 2N and g is the parameter which can take the values in the interval [0, 1].

It is obvious that H(1) is the Hamiltonian of the real system π2N and H(0) is the Hamiltonian of the model of fixed centers. Therefore, the scattering on fixed nucleons rather than a free motion is the boundary condition in this case. It should be mentioned that inclusion of the term gh in the total Hamiltonian of the system means that the contributions of the discrete and continuous spectra of the target Hamiltonian are taken into account simultaneously. This is an important distinction of the method from the known approximations of the theory of multiple scattering, in which the contribution of the continuous spectrum of the target Hamiltonian is neglected as a rule * .

Using the equations of the ECC-methad we find the corrections to the scattering length, which is exactly calculated in the model of fixed centers.

Following the ECC-method /2/ we shall proceed from the equation for the scattering length in the from

$$\frac{da}{dg} = -\frac{\mu'}{2\pi} \langle \Psi_{\vec{k}'=0} | \bar{h} | \Psi_{\vec{k}=0} \rangle^{c} , \qquad (2)$$

where $|\Psi_{\vec{k}}\rangle$ is the eigenfunction of the Hamiltonian (1), $\langle \Psi_{\vec{k}'} | \bar{h} | \Psi_{\vec{k}} \rangle = \langle \Psi_{\vec{k}'} | h | \Psi_{\vec{k}} \rangle - E\delta(\vec{k}'-\vec{k})$, index " c " implies a connected part of the matrix element, E is the total energy of the system, which is equal to the deuteron binding energy in the case under consideration.

For the matrix element in the right-hand side of eq. (2), we have

$$\langle \Psi_{\vec{k}'} | \bar{h} | \Psi_{\vec{k}} \rangle^{c} = h^{(1)} + h^{(2)} ,$$

$$h^{(1)} = \int \frac{d\vec{q}_1 d\vec{q}_2}{(2\pi)^6} [\phi_d(\vec{q}_1) \frac{\langle \vec{k}'\vec{q}_1 | h | \vec{k}'\vec{q}_2 \rangle}{Z - \frac{k'^2}{2\mu} - \frac{q_2^2}{m'}} \eta(\vec{k}', \vec{q}_2; \vec{k}) +$$

* In the Faddeev equations the contributions of both continuous and discrete spectra of target Hamiltonian are taken into account.

$$+\eta^*(\vec{k},\vec{q}_1;\vec{k}')\frac{\langle\vec{k}\vec{q}_1|\bar{h}|\vec{k}\vec{q}_2\rangle}{Z'^*-\frac{k^2}{2\mu}-\frac{q_1^2}{m'}}\phi_d(\vec{q}_2)], \qquad (3)$$

$$h^{(2)}=\int\frac{d\vec{k}_1 d\vec{q}_1 d\vec{k}_2 d\vec{q}_2}{(2\pi)^{12}}\eta^*(\vec{k}_1,\vec{q}_1;\vec{k}')\times$$

$$\times\frac{\langle\vec{k}_1\vec{q}_1|\bar{h}|\vec{k}_2\vec{q}_2\rangle}{Z'^*-\frac{k_1^2}{2\mu}-\frac{q_1^2}{m'}}\frac{\eta(\vec{k}_2,\vec{q}_2;\vec{k})}{Z-\frac{k_2^2}{2\mu}-\frac{q_2^2}{m'}}, \qquad (4)$$

$$m'=\frac{m}{g}, \qquad Z=\frac{k^2}{2\mu}+\frac{\alpha^2}{m'},$$

$$\eta(\vec{k}_1,\vec{q}_1;\vec{k})\equiv\int\frac{d\vec{q}}{(2\pi)^3}\langle\vec{k}_1\vec{q}_1|T|\vec{k}\vec{q}\rangle\phi_d(\vec{q}), \qquad (5)$$

T is the transition operator for the system $\pi 2N$ and ϕ_d is the deuteron wave function. To derive expressions (3) and (4), we have used the following representation of the wave function of the πd system:

$$\Psi_{\vec{k}'}(\vec{k}_1,\vec{q}_1)=(2\pi)^3\delta(\vec{k}'-\vec{k}_1)\phi_d(\vec{q}_1)+$$

$$+(Z'-\frac{k_1^2}{2\mu}-\frac{q_1^2}{m'})^{-1}\int\frac{d\vec{q}_1'}{(2\pi)^3}\langle\vec{k}_1\vec{q}_1|T(Z')|\vec{k}'\vec{q}_1'\rangle\phi_d(\vec{q}_1'), \qquad (6)$$

which is valid at low energies of π-meson. It follows from definition of $\bar{h}$ that the matrix element $h^{(1)}$ vanishes. The matrix element $h^{(2)}$ can be found by calculating function $\eta(\vec{k}_1,\vec{q}_1;\vec{k})$ in the approximation $g=0$*. As a result for $h^{(2)}$, we get

$$h^{(2)}=-a_0^2\frac{\gamma}{2\sqrt{g}}, \qquad (7)$$

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* Using the results of ref.[1] one can easily show the validity of the expansion in powers g for the function $\eta(\vec{k}_1,\vec{q}_1;\vec{k}')$.

where a_0 is the πd scattering length at $g=0$ and Y is the numerical constant ($Y \approx 0.05$). Substituting (7) into eq. (2) and taking into accout the boundary condition, we get the solution in the form

$$a(g) = a_0 - \sqrt{g}\, a_0^2 Y. \tag{8}$$

For the physical value $g = 1$, we have

$$a_{\pi d} = -0.06 \text{ fm}. \tag{9}$$

Thus, as is seen from expression (8) a correction to the πd scattering length a_0 obtained in the fixed centers approximation, is quadratic in a_0 and its contribution to the scattering length of the πd system is small because of a small value of a_0.

2.

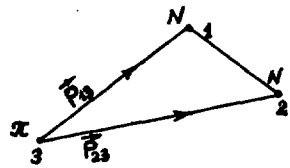
Now let us consider the second example. Instead of the physical Hamiltonian H of the system $\pi 2N$ (see the figure) we introduce an auxiliary Hamiltonian

$$H(g) = h_{13} + h_{23} + g h_{12}, \tag{10}$$

where

$$h_{ij} = \frac{p_{ij}^2}{2\mu_{ij}} + V_{ij}, \qquad (ij) = (13), (23),$$

$$h_{12} = \frac{1}{\mu_\pi} \vec{p}_{13} \cdot \vec{p}_{23} + V_{12}.$$



It is clear that H(1) is the physical Hamiltonian of the system $\pi 2N$. At a certain value of the constant $g=g_0$ the

πd -scattering length can be shown to be equal to the sum of elementary scattering lengths. This value of the constant g will be considered as a boundary condition in soloving the equations of ECC-method for the scattering length. To treat the physical meaning of the quantity g_0 we rewrite the Hamiltonian (10) in Jacobi variables $\vec{k} = \vec{p}_{13} + \vec{p}_{23}$,

$$\vec{q} = \frac{\vec{p}_{13} - \vec{p}_{23}}{2} :$$

$$H(\vec{k}, \vec{q}) = \frac{2m' + \mu'}{4m'\mu'} k^2 + \frac{q^2}{m'2} + V_{13} + V_{23} + gV_{12}, \qquad (11)$$

$$\mu' = \frac{\mu_\pi}{g}, \qquad m' = \frac{\mu_\pi}{1 - g + \mu_\pi / m} .$$

Thus, the Hamiltonian of the nucleon subsystem is

$$h_{NN} = \frac{q}{m'(g)} + gV_{12} . \qquad (12)$$

It turns out that at $g = g_0$ the Hamiltonian (12) has an eigenvalue equal to zero. This condition will be used to determine the numerical value of g_0. So, the Hamiltonian $H(g_0)$ describes the interaction of an incident particle of mass μ_π / g_0 with a bound system of two partices each having mass equal to $m'(g_0)$ and the binding energy equal to zero.

The equation for the scattering length has the form (2); in the right-hand side, we have to calculate the matrix element of the operator

$$h_{12} = V_{12} + \frac{1}{\mu_\pi}\left(\frac{\vec{k}^2}{4} - \vec{q}^2\right), \qquad (13)$$

$$\langle \Psi_{\vec{k}'} | h_{12} | \Psi_{\vec{k}} \rangle = \int \frac{d\vec{k}_1 \, d\vec{q}_1 \, d\vec{k}_2 \, d\vec{q}_2}{(2\pi)^{12}} \Psi^*_{\vec{k}'}(\vec{k}_1, \vec{q}_1) *$$

$$* \langle \vec{k}_1 \vec{q}_1 | h_{12} | \vec{k}_2 \vec{q}_2 \rangle \Psi_{\vec{k}}(\vec{k}_2, \vec{q}_2) . \qquad (14)$$

As in the previous case, we use representation (6) for the wave function of the system. As a result we get formulae (5) with change of $\tilde{h}$ by h_{12}.

The boundary condition for a(g) is

$$a(g = g_0) = \frac{\mu_{\pi d}(g_0)}{\mu_{\pi N}} 2a^{\circ}, \tag{15}$$

where a° is the isoscalar combination of the pion-nucleon scattering lengths,

$$\mu_{\pi d}(g) = \frac{2\mu_{\pi}}{1 + g + \frac{\mu_{\pi}}{m}}.$$

For the separable triplet NN potential V_{12} the numerical value of g_0 turned out to be equal to g_0=0.744. As this value is close to the physical one, g=1, a reasonable approximation to the exact value of $a_{\pi d}$ can be made by using a linear approximation for the function a(g)

$$a(g) = a(g_0) + (g - g_0)\, a^{(1)}(g_0). \tag{16}$$

Thus in approximation (16), we have to find the constant $a^{(1)}(g_0)$, i.e., the right-hand side of equation (2) at the point $g = g_0$. The contribution of $h^{(2)}(g_0)$ comprises several per cent only as compared with the contribution of $h^{(1)}(g_0)$; finally we get

$$a(g) = a(g_0)[1 + (g - g_0)(\frac{2}{g_0} + \frac{3}{2} \frac{1}{1 - g_0 + \mu_{\pi}/m_N})], \tag{17}$$

whence for $a_{\pi d} = a(1)$ we have

$$a_{\pi d} \approx 2.64 \cdot a(g\ ) = -0.066 \text{ fm}. \tag{18}$$

To define the estimates (9) and (18), we compare our results with the results of calculation of $a_{\pi d}$ using the Faddeev equations [4] and with the experimental value of this quantity

$$a_{\pi d}^{\text{Fadd}} = -0.061 \text{ fm}, \tag{19}$$

$$a_{\pi d}^{\text{exp}} = -0.073 \pm 0.030 \text{ fm} \ [5]. \tag{20}$$

The fact that the values of (9) and (18) are close to the "exact" value of (19) indicates the reliability of the obtained estimates.

Within the ECC-method one can easily evaluate the scattering length, which occurs due to the so-called indirect interaction of particles 1 and 2 (see the figure), i.e., due to the term $\frac{\vec{p}_{13}\cdot\vec{p}_{23}}{\mu_\pi}$.

The calculation of a(g) without this term gives

$$a(g) = a(g_0)\frac{g^2}{g_0^2}.$$

The comparison with the estimate taking into account the term $\frac{\vec{p}_{13}\cdot\vec{p}_{23}}{\mu_\pi}$ shows that its contribution to the scattering $a_{\pi d}$ length is 30%-40%.

CONCLUSION

So, in this paper two possible ways of improving approximate solutions of the three-particle equations describing the πd scattering at zeroth pion energy are realized in the framework of the ECC-method. Both approaches provide the value for the πd scattering length, which is close to that obtained by solving the Faddeev equations. A specific feature of both treatments is that the contributions of the continuous spectrum states of the target Hamiltonian are taken into account.

The considered examples show that the ECC-method can always be generalized when the "solvable part" of the many-particle Hamiltonian can be separated.

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Received by Publishing Department
on December 22 1980.