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L.N.Bogdanova,* V.E.Markushin,* V.S.Melezhik,
L.I.Ponomarev

NUCLEAR SYNTHESIS REACTION
IN THE MUONIC MOLECULE dtμ

*Institute for Theoretical and Experimental Physics,
Moscow.

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1. Introduction

Interest to the muon catalysis of the nuclear synthesis in the mixture of hydrogen isotopes[1-3] has been revived by the intensive theoretical [4-5] and experimental [6] investigations, the possibilities to take the practical advantage of this phenomenon having been discussed [7]. For the detailed description of the μ - catalysis kinetics in ($D_2 + T_2$) mixture it is necessary to know the rates λ_f of the nuclear synthesis reaction

$$dt\mu \xrightarrow{\lambda_f} {}^4He + n + \mu^-$$ (1)

from the different states (JV) of the rotational and vibrational motions of the $dt\mu$ molecule. In solving this problem one uses the experimental data on the reaction [8,9]

$$d + t \longrightarrow {}^4He + n + 17.6 \text{ MeV}$$ (2a)
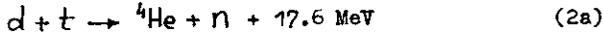
and the scattering [10]

$$d + t \longrightarrow d + t .$$ (2b)

Reaction (2a) has been studied in the dt CMS energy region 8 keV $< E <$ 12 MeV. The peculiarity is the nearthreshold resonance in the cross section $\sigma_{in}(E)$ at energy $E_R = 64$ keV with the halfwidth $\Gamma/2 \simeq 70$ keV, the cross section maximum $\sigma_{in}^{max} \simeq 5b$ being close to the unitary limit. All the experimental data suit the hypothesis that at energies $E < 200$ keV reaction (2a) proceeds from the dt state with the orbital angular momentum of the relative motion $l=0$ and the total angular momentum $J = 3/2$ through the intermediate excited state of ${}^5He$ ($3/2^+$). The other states ($L = 0$, $J^\pi = 1/2^+$ and $L \geq 1$) contribute less than 1% in the energy region discussed [9,11].

Ordinarily [1,12,13] the reaction (1) rate is calculated with formula

$$\lambda_f = A_o |\Psi^{JV}(0)|^2,$$ (3)

where $\Psi^{JV}(\vec{R})$ is the wave function of the dt relative motion in the mesomolecule $dt\mu$ (with the nuclear dt

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interaction not taken into account) and A_0 is the reaction constant

$$A_0 = \lim_{v \to 0} \left(\sigma_{in} v \, C_0^{-2} \right) \quad , \tag{4}$$

v being the dt-relative velocity and C_0 - Gamov factor for the S- wave scattering

$$C_0^2 = \frac{2\pi \zeta}{e^{2\pi \zeta} - 1} \quad , \quad \zeta = \frac{\alpha c}{v} \quad . \tag{5}$$

All the existing estimates [1,12] of the rate λ_f were obtained with formula (3) for the $dt\mu$ ground state ($\mathcal{J} = \mathcal{V} = 0$). They, however, cannot be believed without some additional consideration.

First of all, when calculating λ_f they usually exploited the quasiclassical approximations for the wave functions $\psi^{\mathcal{J}\mathcal{V}}(\vec{R})$ inside the nuclear forces range ($0 < R \lesssim R_i \approx 7$ fm). That, as we will see below, is quite inadequate for the excited rotational states with $\mathcal{J} \neq 0$. Besides, in some papers [14] the relevance of formula (3) was doubted because in the region $R \lesssim R_i$ the nuclear dt interaction radically changes the wave function $\psi^{\mathcal{J}\mathcal{V}}(\vec{R})$. Furthermore, it is not clear in advance to what extent formula (3) can be trusted in case when the reaction cross section is dominated by the nearthreshold resonance with large inelasticity.

In the present paper to calculate the mesic molecule levels' shifts and widths caused by the nuclear dt interaction we have solved the eigen-value problem for the Hamiltonian of $dt\mu$ system. For the calculation of the μ molecule wave functions we have used the adiabatic representation for the three-body problem [15] and the algorithms developed in papers [16-18]. The nuclear dt interaction $\widetilde{V}_1$ was chosen in the form which follows from the consideration of the coupled channels problem for $dt \to n^4He$ system. The anti- Hermitian part of $\widetilde{V}_1$ has the separable form and negligibly depends on energy near the threshold. The Hermitian part of $\widetilde{V}_1$ is also a smooth function of energy and can be approximated by a local potential. With the nuclear interaction of the kind we obtained a good description of the reaction (2a) and the elastic dt scattering

in the $^5He\left(\frac{3}{2}^+\right)$ resonance region and calculated the widths Γ^{Jv} and shifts ΔE^{Jv} of $dt\mu$ molecule levels Jv.

In the paper we have also determined the rate of nuclear reaction "in flight", i.e., without preceding formation of $dt\mu$ molecule

$$t\mu + d \rightarrow n + {}^4He + \mu^- \quad . \tag{6}$$

2. The Effective Hamiltonian of dt Interaction

Consider the two-channel problem with the Hamiltonian:

$$H = \begin{pmatrix} H_1^0 + V_1 & V_{12} \\ V_{21} & H_2^0 + V_2 \end{pmatrix} \quad . \tag{7}$$

Here H_1^0 and H_2^0, V_1 and V_2 are free Hamiltonians and interaction potentials in the channels dt and $n\,{}^4He$, correspondingly, $V_{12} = V_{21}^+$ is the potential coupling the channels dt and $n\,{}^4He$.

To find the dt scattering amplitude $f^{11}(E)$ we exploit the method of the "generalized optical potential" /19/ which enables one to reduce the two-channel problem with Hamiltonian (7) to the one-channel problem with the non-local and energy-dependent Hamiltonian $\widetilde{H}_1$ of the form

$$\widetilde{H}_1 = H_1^0 + V_1 + V_{12}\left(E - H_2^0 - V_2\right)^{-1}V_{21} \equiv H_1^0 + \widetilde{V}_1 \quad , \tag{8}$$

Making use of the spectral representation for the Green function of $n\,{}^4He$ channel*)

$$\left(E - H_2^0 - V_2\right)^{-1} = \frac{(2m_2)^{3/2}}{\hbar} \int_0^\infty \frac{|\varepsilon\rangle\langle\varepsilon|}{E + \Delta - \varepsilon}\sqrt{\varepsilon}\, d\varepsilon \quad , \tag{9}$$

where $|\varepsilon\rangle$ is the eigen-function of the Hamiltonian $H_2^0 + V_2$, $\Delta = 17.6$ MeV the distance between the thresholds dt and $n\,{}^4He$, we obtain the following expression for the genera-

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*) Here and below we consider the state $J^\pi = 3/2^+$ with the quantum numbers: $L = 0$, $S = 3/2$ in dt channel (reduced mass m_1) and $L = 2$, $S = 1/2$ in $n\,{}^4He$ channel (reduced mass m_2) /11/.

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lized optical potential $\widetilde{V}_1$ in the physical region of dt scattering ($E = 0$ corresponds to the dt channel threshold)

$$\widetilde{V}_1(E+i0) = V_h + V_a \quad ,$$

$$V_h = V_1 + \frac{(2m_2)^{3/2}}{\pi} \int_0^\infty \frac{V_{12}|\varepsilon\rangle\langle\varepsilon|V_{21}}{E+\Delta-\varepsilon}\sqrt{\varepsilon}\,d\varepsilon \quad , \tag{10}$$

$$V_a = -i\,(2m_2)^{3/2}\,(E+\Delta)^{1/2}\,V_{12}|\varepsilon\rangle\langle\varepsilon|V_{21} \ .$$

The anti-Hermitian part V_a of the potential $\widetilde{V}_1$ is of a separable form. Near the dt threshold at $E \ll \Delta$ the E dependence of V_a can be neglected and V_a can be written as

$$V_a = -i\beta|\xi\rangle\langle\xi| \quad , \tag{10a}$$

β being the real constant and functions $\langle R|\xi\rangle$ being localized inside the range of nuclear forces. As it will be seen below, their specific form is inessential.

The Hermitian part V_h of the potential $\widetilde{V}_1$ includes, besides the Coulomb repulsion $V_c = \alpha/R$, the superposition V_n of the diagonal nuclear dt interaction and the Hermitian part of dt interaction due to the $dt \to n^4 He$ channel coupling. The latter weakly depends on energy E , since in the case of interest $E \ll \Delta$ and E is small compared to the characteristic range of integration $\varepsilon_0 \sim (m_1 R_1^2)^{-1} \sim 1$ MeV in expression (10) for V_h ($R_1 \simeq 7$ fm - the range of nuclear interaction in dt channel). In the following we assume that V_h can be approximated with local and energy-independent potential $U(R)$.

Partition (10) allows one to express the scattering amplitude $f^{11}(E)$ of the initial two-channel problem in terms of the solutions of the one-channel dt scattering problem with potential $V_h = U(R)$. Let known be: Green function $G_1 = (E-H_1^0-U)^{-1}$, scattering amplitude $f_L(E)$, Jost function $f_L(E)$ and the regular solution $\psi_{LE}(R)$ for the partial wave with the orbital angular momentum L . Remind that

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$$\psi_{LE}(R) \underset{R\to 0}{\longrightarrow} R^L \qquad f_L(E) = |f_L(E)|\, e^{-i\bar\delta_L(E)} ,$$

$$\bar\delta_L(E) = \delta_L^c(E) + \delta_L(E) \qquad f_L(E) = \frac{e^{2i\bar\delta_L(E)}-1}{2iK} \qquad (11a)$$

and the normalized solution of the scattering problem is of the form:

$$(KR)^{-1}\chi_{LE}(R) = K^L |f_L(E)|^{-1}\psi_{LE}(R) \underset{R\to\infty}{\longrightarrow} (KR)^{-1}\sin\!\left(KR-\gamma\ln 2KR-\tfrac{L\pi}{2}+\bar\delta_L\right) (11b)$$

where $K = m_1 v = (2m_1 E)^{1/2}$ is momentum in the dt channel, $\delta_L^c(E) = \arg\Gamma(L+1+i\gamma)$ is Coulomb phase shift, $\delta_L(E)$ is additional phase shift due to potential V_n.

With the two-potential formula (see e.g. /19/) scattering amplitude $f^{11}(E)$ can be expressed in terms of the introduced quantities as follows /20/ (below we restrict ourselves to the case $L = 0$ and suppress the index L):

$$f^{11}(E) = f - 2m_1\, f^{-2}(E)\, \langle\psi_E|V_a\left(1-G_1 V_a\right)^{-1}|\psi_E\rangle . \qquad (12)$$

Making use of the separable form (10a) of potential V_a we can calculate the matrix element (12) explicitly:

$$\langle\psi_E|V_a\left(1-G_1 V_a\right)^{-1}|\psi_E\rangle = \frac{-i\beta\,|\langle\psi_E|\xi\rangle|^2}{1+i\beta\langle\xi|G_1|\xi\rangle} . \qquad (13)$$

Then the S- matrix element corresponding to the dt scattering can be written as follows:

$$S^{11}(E) = e^{2i\bar\delta(E)}\left(1 - \frac{4m_1 K\beta\,|f(E)|^{-2}\,|\langle\xi|\psi_E\rangle|^2}{1+i\beta\langle\xi|G_1|\xi\rangle}\right) . \qquad (14)$$

Using the spectral representation for the Green function of dt channel

$$G_1(E+i0) = \frac{(2m_1)^{3/2}}{\pi}\int_0^\infty \frac{|\psi_\varepsilon\rangle\langle\psi_\varepsilon|}{E-\varepsilon+i0}\,|f(\varepsilon)|^{-2}\sqrt{\varepsilon}\,d\varepsilon \qquad (15)$$

we rewrite the matrix element S^{11}:

$$S^{11}(E) = e^{2i\bar\delta(E)}\,\frac{1-2m_1 K\Lambda\,|f(E)|^{-2}+i F(E)}{1+2m_1 K\Lambda\,|f(E)|^{-2}+i F(E)} , \qquad (16)$$

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where

$$\Lambda = \beta |\langle \psi_E | \mathfrak{z} \rangle|^2 \quad , \tag{17a}$$

$$F(E) = \frac{(2m_1)^{3/2}}{\pi} \int_0^\infty \frac{\beta |\langle \psi_E | \mathfrak{z} \rangle|^2}{E - \varepsilon} |f(\varepsilon)|^{-2} \sqrt{\varepsilon} \, d\varepsilon \quad . \tag{17b}$$

At $0 < E < 200$ keV $\Lambda \simeq const$ since in this energy region the regular solution $\psi_E(R)$ depends weakly on energy inside the nuclear forces range.

The integration region in (17 b) can be divided into two parts: $0 \leqslant \varepsilon \leqslant \varepsilon_0$ and $\varepsilon_0 \leqslant \varepsilon < +\infty$ in such a way that at $\varepsilon \leqslant \varepsilon_0$ the proximity $\Lambda \simeq const$ is satisfied, while the E-dependence of the integral over the region $\varepsilon_0 \leqslant \varepsilon < +\infty$ can be neglected:

$$F(E) = \frac{(2m_1)^{3/2}}{\pi} \Lambda \int_0^{\varepsilon_0} \frac{|f(\varepsilon)|^{-2} \sqrt{\varepsilon}}{E - \varepsilon} d\varepsilon + F_0 \tag{17c}$$

(As the numerical calculations show $\varepsilon_0 \simeq 0.5$ MeV is appropriate.)

Thus, to describe the cross sections of the elastic dt scattering and the reaction $dt \to n\,{}^4He$ near dt threshold two constants Λ and F_0 and Jost function $f(E)$ for potential $U(R)$ at energies $E < \varepsilon_0$ will suffice. The behaviour of $f(E)$ at $E > \varepsilon_0$ is irrelevant and we will exploit this circumstance when choosing the potential $U(R)$. As it will be shown in Sect.3 the knowledge of the quantities Λ, F_0 and $U(R)$ is sufficient for calculation of nuclear shifts and widths of $dt\mu$ molecule levels.

We have chosen potential $U(R)$ in the form (see Fig.1):

$$U(R) = V_c + V_n = \begin{cases} -V_0 + \dfrac{m_1 \omega^2}{2} R^2 \quad , & 0 \leqslant R \leqslant R_1 \\[2mm] 1/R & , \quad R > R_1 \end{cases} \tag{18}$$

provided $U(R_1) = V_c(R_1)$. Here and below the system of units $e = \hbar = m_\alpha = 1$ ($m_\alpha = m_\mu m_t (m_\mu + m_t)^{-1} \approx 199.3 m_e$) is used. Value of m_1 and the units of energy ε_1 and of

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Fig. 1.
The sketch of potential
$$U_\mu(R) = V_\mu(R) + V_n(R) \quad ;$$
the depth of $V_\mu(R)$ is
~ 0.6 keV $\ll E_R$,
$R_1 = 0.025, \quad V_0 = 15.8$ MeV,
$U_\mu(R_1) \simeq 200$ keV.

Fig. 2a. Cross section $\sigma_{in}(E)$
of $d + t \rightarrow n + {}^4He$ reaction: $\tilde{\xi}$ –
experiment[8], $\bullet$ – the survey[9],
theoretical curve was calcula-
ted with (20a'), (16) at para-
meters (21) of potential (18).

Fig. 2b. Quantity $\zeta(E)$(20b) for
(2b): experimental points
from[10], theoretical curve was
calculated with (20b), (16).
$\chi^2 = 16$ when simultaneous fit-
ting to σ_{in}(13 points) ans ζ
(11 points).

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length a_1 equal correspondingly

$$m_1 = 11.045\, m_\alpha \; ,$$

$$\varepsilon_1 = m_e e^4/\hbar^2 = 5.422 \text{ keV}, \qquad a_1 = \frac{\hbar^2}{m_e e^2} = 2.655 \cdot 10^{-11} \text{cm} \; . \tag{19}$$

Parameters V_0 , ω , Λ , F_0 were chosen so that the quantities calculated with the formulae[*]

$$\sigma_{in} = \frac{2}{3}\frac{\pi}{\kappa^2}\left(1 - |S^{11}|^2\right) \; , \tag{20a}$$

$$\zeta = \frac{1}{3} + \frac{2}{3}\left| e^{-2i\eta \ln\sin\theta/2} - \frac{i}{2\eta}\left(1 - S^{11}\right)\right|^2 \tag{20b}$$

give the best fit to the experimental data. Here

$$\zeta(E) = \frac{d\sigma_d(E,\theta)}{d\Omega} \Big/ \frac{d\sigma_c(E,\theta)}{d\Omega} \bigg|_{\theta = \pi/2} \tag{20c}$$

is the ratio of the differential cross sections of the elastic dt scattering and of the Coulomb scattering at $\theta_{CM} = \pi/2$. S- matrix element $S^{11}(E)$ was obtained with formulae (16), (17) with Jost function $f(E)$ found from the numerical solution of the one-channel scattering problem with potential (18). The experimental data are from papers [8-10].

The best fit (χ^2/n_Φ = 16/20 ; 24 experimental points and 4 parameters) to the experimental points $\sigma_{in}(E)$ (13 points) and $\zeta(E)$ (11 points) in the energy region $0 < E < 200$ keV is achieved with the following parameters (see Fig.2a and 2b):

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[*] Coefficients 2/3 and 1/3 in formulae (20) take into account the fact that the cross sections $\sigma_{in}(E)$ and $\sigma_{el}(E)$ have been measured for unpolarized d and t , while the S- matrix has been calculated for the state with definite total spin of the nuclei $S = 3/2$.

$$\Lambda = 5.45 \cdot 10^{-6} \left(\mathcal{E}_1 a_1 / \hbar \right) \; = 8.34 \cdot 10^{-19} \mathrm{cm}^3 \mathrm{sec}^{-1}$$

$$F_0 = 0.025 \quad \text{at} \quad \mathcal{E}_0 = 0.5 \text{ MeV} \tag{21}$$

$$V_0 = 2917 \, \mathcal{E}_1 = 15.82 \text{ MeV}$$

$$\omega = 917 \, a_1^{-1} = 3.45 \, \text{fm}^{-1} \quad (R_1 = 0.0252 \, a_1 = 6.70 \, \text{fm})$$

In Fig. 3 functions $|f(E)|^2$, $|f(E) C_0|^2$ and $F(E)$ calculated with potential (18) and parameters (21) are presented.
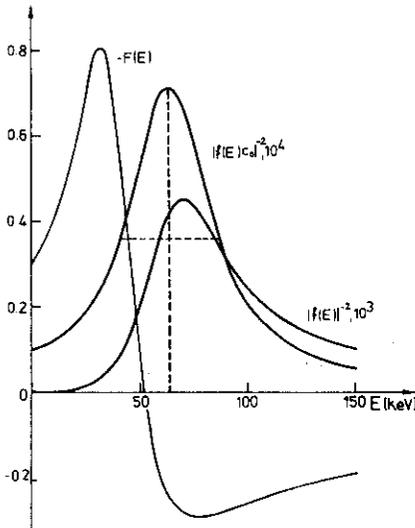
Fig. 3. Functions $|f(E)|^{-2}, |f(E) C_0|^{-2}$ and $F(E)$ for potential (18) with parameters (21).

Some remarks about the choice procedure of potential $U(R)$ and constants Λ and F_0 . For the given shape of the potential $U(R)$ the χ^2 criterium enables one to find potential parameters, however, the shape itself does not follow uniquely from the scattering and reaction data. There exists a set of potentials $U(R)$ having similar Jost functions at $0 < E < \mathcal{E}_0$ and, hence, allowing to describe successfully the experimental data on reactions (2), since formulae (16) – (17) involve not the potential but its Jost function $f(E)$. In connection with the said, potential (18) should be regarded as a representative of the definite class of the potentials but not as the true nuclear dt interaction potential, the real form of which is inessential as far as we deal with calculation of the mesic molecule levels' widths.

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3. Widths and Shifts of $dt\mu$ Molecule Levels

When finding the shifts $\Delta\mathcal{E}_{\mathcal{J}\upsilon}$ and widths $\Gamma^{\mathcal{J}\upsilon}$ of $dt\mu$ molecule levels $\mathcal{J}\upsilon$ the first problem is to calculate the energy levels $E_{\mathcal{B}}$ and the wave functions $\Psi_{\mathcal{B}}$ of the mesic molecule with the Hermitian part (10) of the nuclear dt interaction V_n taken into account. The interaction Hamiltonian H_μ in this case differs from the Coulomb mesic molecular Hamiltonian $H_\mu^0 + V_\mu$ /15,16/ by the short-ranged nuclear potential V_n (see Fig.1):

$$H_\mu = H_\mu^0 + V_\mu + V_n ,$$

$$(H_\mu - E_\mathcal{B})|\Psi_\mathcal{B}\rangle = 0 . \tag{22}$$

The energy $E_\mathcal{B} = \mathcal{E}_{\mathcal{J}\upsilon} + \Delta\mathcal{E}_{\mathcal{J}\upsilon}^{(n)}$ of the $\mathcal{J}\upsilon$ state of $dt\mu$ molecule is measured from the $t\mu + d$ channel threshold, $\mathcal{E}_{\mathcal{J}\upsilon}$ being the energy of $dt\mu$ molecule level for Hamiltonian $H_\mu^0 + V_\mu$.

The shift $\Delta E = \Delta\mathcal{E}_{\mathcal{J}\upsilon}^{(a)}$ and the width $\Gamma = \Gamma^{\mathcal{J}\upsilon}$ of the energy level $E_\mathcal{B}$ due to the absorbing potential V_a are found from the Schrödinger equation with the Hamiltonian $\widetilde{H}_\mu$:

$$\widetilde{H}_\mu = H_\mu + V_a ,$$

$$(\widetilde{H}_\mu - E)|\Psi\rangle = 0 , \tag{23}$$

$$E = E_\mathcal{B} + \Delta E - i\Gamma/2 ,$$

where potential V_a is defined by relation (10a).

Equation (23) can be transformed:

$$|\Psi\rangle = (E - H_\mu)^{-1} V_a |\Psi\rangle . \tag{24}$$

Write the spectral representation of the Green function:

$$(E - H_\mu)^{-1} = \sum_n \frac{|\Psi_\mathcal{B}^{(n)}\rangle\langle\Psi_\mathcal{B}^{(n)}|}{E - E_\mathcal{B}^{(n)}} + \frac{(2m_1^*)^{3/2}}{\pi} \int_0^\infty \frac{|\Psi_\mathcal{E}^\mu\rangle\langle\Psi_\mathcal{E}^\mu|}{E - \mathcal{E}} \sqrt{\mathcal{E}}\, d\mathcal{E} , \tag{25}$$

where $\Psi_\mathcal{B}^{(n)}$ and $\Psi_\mathcal{E}^\mu$ are, correspondingly, the wave functions of the $dt\mu$ molecule state $n = (\mathcal{J}\upsilon)$ and

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$(t\mu + d)$ system at scattering energy ε, m_i^* is the reduced mass of $(t\mu + d)$ system. Not written is the contribution from the continuum of $d\mu + t$ channel. Here we have neglected the contributions from the channels corresponding to high excitations and dissociation of $d\mu$ and $t\mu$ atoms.

In the adiabatic representation of the three body problem /15,16/ the wave functions $\Psi_8^{(n)} = \psi_{m_J}^{J\upsilon}(\vec{r}, \vec{R})$ of $d t \mu$ molecule states $J\upsilon$ are decomposed over the adiabatic basis /21/:

$$\psi_{m_J}^{J\upsilon}(\vec{r}, \vec{R}) = \sum_{j}^{N_0} \phi_j(\vec{r}, R) \chi_j^{J\upsilon}(R) \mathcal{D}_{m m_J}^{J}(\Phi, \theta, 0) \quad , \tag{26}$$

where N_0 — the number of decomposition terms which provides the required accuracy, $\mathcal{D}_{m m_J}^{J}(\Phi, \theta, 0) = [2(1+\delta_{m0})]^{-\frac{1}{2}}[D_{-m m_J}^{J}(\Phi, \theta, 0) + \varepsilon(-1)^{m} D_{m m_J}^{J}(\Phi, \theta, 0)]$.

In this decomposition functions $\phi_j(\vec{r}, R)$ describe the μ - meson motion in the state with quantum numbers $j = (N\ell m)$ in the field of the fixed at distance R nuclei d and t. The algorithms of their calculation are known /21/. The functions $\chi_j^{J\upsilon}(R)$ present the relative dt motion. They satisfy the system of differential equations

$$\left\{ \frac{d^2}{dR^2} + 2m_1\left(\varepsilon_{J\upsilon} + \Delta\varepsilon_{J\upsilon}^{(n)} - V_n(R) - \frac{J(J+1)-2m^2}{R^2} - U_{jj}^{J}(R)\right)\right\} \chi_j^{J\upsilon} =$$
$$= \sum_{j' \ne j}^{N_0} U_{jj'}^{J}(R) \chi_{j'}^{J\upsilon}(R) \tag{27}$$

$$\chi_j^{J\upsilon}(0) = 0 \qquad \chi_j^{J\upsilon}(R) \xrightarrow[R \to \infty]{} 0$$

which follows from Schrödinger equation (22) after the substitution of decomposition (26) and the elimination of the meson coordinates $\vec{r}$ and the angular variables θ, Φ of vector $\vec{R}$ /15,16/.

To calculate the matrix elements in (24) one has to know functions $\psi_8^{(n)}(\vec{r}, \vec{R})$ and $\Psi_\varepsilon^\mu(\vec{r}, \vec{R})$ in the region $0 \le R \le R_1 \ll 1$. In the limit $R \to 0$ functions $\phi_j(\vec{r}, R)$ transform to wave functions $\psi_{N\ell m}(\vec{r})$ of $(\mu\,{}^5He)$ mesic atom states $j = (N\ell m)$, defined in the coordinate frame rotating with vector $\vec{R}$ /22/, asymptotics of the total wave function becoming

$$\Psi^{\mathcal{JV}}_{m_{\mathcal{J}}}(\vec{r},\vec{R}) \underset{R\to 0}{=} \sum_{N\ell m}^{N_o} \psi_{N\ell m}(r,\theta,\psi)\, R^{-1}\chi^{\mathcal{JV}}_{N\ell m}(R)\,\mathcal{D}^{\mathcal{J}}_{mm_{\mathcal{J}}}(\Phi,\theta,0). \quad (28)$$

V_α being projection operator to the $L \neq 0$ state of the relative dt motion, it is necessary to extract the corresponding part from the wave function (28), what is achieved by transition to the laboratory reference frame $\vec{r}=(r,\theta,\varphi) \to \tilde{\vec{r}}=(r,\tilde{\theta},\tilde{\varphi})$ /22/. This transition is given by the following formulae /22/

$$\Psi^{\mathcal{JV}}_{m_{\mathcal{J}}}(\vec{r},\vec{R}) = R^{-1}\sum_{N\ell}\sum_{m=0}^{\min(\ell,\mathcal{J})}\sum_{L=|\mathcal{J}-\ell|}^{\mathcal{J}+\ell}\sum_{m_\ell=-\ell}^{\ell} C^{\mathcal{J}m_{\mathcal{J}}}_{\ell m_\ell L m_L}\cdot \quad (29)$$

$$\cdot\, \psi_{N\ell m_\ell}(r,\tilde{\theta},\tilde{\varphi})\, Y_{Lm_L}(\theta,\phi)\, C^{\ell\mathcal{J}}_{mL}\,\chi_{N\ell m}(R)$$

where

$$G^{\ell\mathcal{J}}_{mL} = (-1)^{\ell+m}\,\frac{1+(-1)^{\mathcal{J}-\ell-L}}{[2(1+\delta_{mo})]^{1/2}}\, C^{LO}_{\ell m\, \mathcal{J}-m} \quad (30)$$

Chang-Fano transformation /23/ from the rotating to the rest frame of coordinates, $C^{LO}_{\ell m\, \mathcal{J}-m}$ and $C^{\mathcal{J}m_{\mathcal{J}}}_{\ell m_\ell L m_L}$ are Clebsch-Gordan coefficients /24/.

It is important that in the new representation (29) the dependence from the angles θ and ϕ is governed not by the total angular momentum $\mathcal{J}$ of the three-body system, as it were in the initial decomposition (28), but the orbital angular momentum L of the relative dt motion in the mesic molecule.

The $R \to 0$ asymptotics of the solutions of the set of equations (27) has been constructed in paper /22/ and has the form:

$$R^{-1}\chi^{\mathcal{JV}}_{N\ell m}(R) \underset{R\to 0}{=} \sum_{L=|\mathcal{J}-\ell|}^{\mathcal{J}+\ell} G^{\ell\mathcal{J}}_{mL}\, A^{L\mathcal{JV}}_{N\ell}\, \varphi^{M}_{LB}(R), \quad (31)$$

where matrices $G^{\ell\mathcal{J}}_{mL}$ are defined by relation (30), φ^{M}_{LB} is regular solution of the system (27)

$$\varphi^{M}_{LB}(R) \underset{R\to 0}{\longrightarrow} R^{L} \quad (32)$$

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and the values of coefficients $\mathcal{A}_{N\ell}^{\mathcal{I}\mathcal{U}}$ are found from relations (31) and (32) with the calculated from eqs.(27) functions $\chi_j^{\mathcal{I}\mathcal{U}}(R)$ normalized by the condition:

$$\sum_{j}^{N_0} \int_0^{\infty} \left[\chi_j^{\mathcal{I}\mathcal{U}}(R)\right]^2 dR = 1 \quad .\tag{33}$$

In the sum (31) only the term with $L = 0$ should be taken into account, since V_α is the projector to S- state. Using the properties of Clebsch-Gordan coefficients

$$C_{\ell m_\ell \, 00}^{\mathcal{I}m_\mathcal{I}} = \delta_{\ell \mathcal{I}} \, \delta_{m_\ell m_\mathcal{I}} \qquad C_{\mathcal{I}m \; \mathcal{I}-m}^{00} = (2\mathcal{I}+1)^{-1/2} (-1)^{\mathcal{I}-m} \tag{34}$$

and orthogonality condition:

$$\sum_m G_{mL}^{\ell\mathcal{I}} \, G_{mL}^{\ell\mathcal{I}} = 1 \tag{35}$$

we obtain with formulae (29) and (31)

$$R^{-1}\chi_{N\mathcal{I}m}^{\mathcal{I}\mathcal{U}}(R) \underset{R\to 0}{=} G_{mo}^{\mathcal{I}\mathcal{I}} \, \mathcal{A}_{N\mathcal{I}}^{0\mathcal{I}\mathcal{U}} \, \psi_{0B}^{H}(E) \quad ,$$

$$\Psi_{m_\mathcal{I}}^{\mathcal{I}\mathcal{U}}(\vec{r},\vec{R}) \underset{R\to 0}{=} \sum_{Nm} G_{mo}^{\mathcal{I}\mathcal{I}} \, \psi_{N\mathcal{I}m_\mathcal{I}}(r,\tilde{\theta},\tilde{\varphi}) Y_{00}(\theta,\varphi) R^{-1}\chi_{N\mathcal{I}m}^{\mathcal{I}\mathcal{U}}(R) = \tag{36}$$

$$= \frac{1}{\sqrt{4\pi}} \sum_N \psi_{N\mathcal{I}m_\mathcal{I}}(r,\tilde{\theta},\tilde{\varphi}) \, \mathcal{A}_{N\mathcal{I}}^{0\mathcal{I}\mathcal{U}} \, \psi_{0B}^{H}(R) \quad . \tag{37}$$

Coefficient $\mathcal{A}_{N\mathcal{I}}^{0\mathcal{I}\mathcal{U}}$ can be found from relation (31):

$$\mathcal{A}_{N\mathcal{I}}^{0\mathcal{I}\mathcal{U}} = \left(G_{00}^{\mathcal{I}\mathcal{I}}\right)^{-1} \lim_{R\to 0} R^{-1} \chi_{N\mathcal{I}0}^{\mathcal{I}\mathcal{U}}(R) =$$

$$= \sqrt{2\mathcal{I}+1} \lim_{R\to 0} R^{-1}\chi_{N\mathcal{I}0}^{\mathcal{I}\mathcal{U}}(R) \quad . \tag{38}$$

From the numerical calculations it follows that coefficients $\mathcal{A}_{N\mathcal{I}}^{0\mathcal{I}\mathcal{U}}$ at $N = \mathcal{I} + 1$ exceed coefficients with $N \neq \mathcal{I}+1$ by an order of magnitude. Thus for the wave function Ψ_B the following expression is valid within $\sim 10\%$ accuracy:

$$\psi_{m_\mathcal{I}}^{\mathcal{I}\mathcal{U}}(\vec{r},\vec{R}) \underset{R\to 0}{=} \frac{1}{\sqrt{4\pi}} \, \psi_{\mathcal{I}+1,\mathcal{I} \, m_\mathcal{I}}(r,\hat{\theta},\hat{\varphi}) \, B^{\mathcal{I}\mathcal{U}} \psi_{0B}^{H}(R) \quad , \tag{39}$$

where we have introduced notation: $B^{\mathcal{I}\mathcal{U}} = \mathcal{A}_{\mathcal{I}+1,\mathcal{I}}^{0\mathcal{I}\mathcal{U}}$.

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Function ψ_E^M in representation (25) is that of the scattering problem $t\mu + d \to t\mu + d$ at $R \to 0$ and has the form analogous to (29) and (39) with $\chi_{\ell m}^{\gamma}(K,R)$ substituting $\chi_{N\ell m}^{\gamma\nu}(R)$. Functions $\chi_{\ell m}^{\gamma}(K,R)$ of the continuum $(\varepsilon = E = K^2/2m_1)$ satisfy eqs. (27) with the boundary conditions

$$\chi_{\ell m}^{\gamma}(K,R) \xrightarrow[R \to \infty]{} \sin\left(KR - \frac{\gamma\pi}{2} + \delta_{\gamma}\right), \tag{40a}$$

$$(KR)^{-1} \chi_{\ell m}^{\gamma}(K,R) \underset{R \to 0}{=} \sum_{L=|\gamma-\ell|}^{\gamma+\ell} G_{mL}^{\ell\gamma} K^4 |f_\mu(E)| \psi_{LE}^{-1\,M}(R),$$

$\psi_{LE}^M(R) \xrightarrow[R\to 0]{} R^L$ is the regular solution (27). Jost function $f_\mu(E)$ for potential $U_\mu(R) = V_\mu(R) + V_n(R)$ at $L = 0$ is defined, similarily to (38)

$$\left|f_\mu(E)\right|^{-1} = \sqrt{2J+1} \lim_{R \to 0} (KR)^{-1} \chi_{J0}^{\gamma}(K,R). \tag{40b}$$

Using (25), (39), (40) and separable form of V_α, eliminate the unknown function $|\psi\rangle$ from Eq.(24):

$$i = \frac{\beta |\langle\psi_{0B}^M|B\rangle|^2 |B|^2}{E - E_B} + \frac{(2m_1^*)^{3/2}}{\pi} \beta \int_0^\infty \frac{|\langle\psi_{0\varepsilon}^M|B\rangle|}{E - \varepsilon} \left|f_\mu(\varepsilon)\right|^{-2} \sqrt{\varepsilon}\, d\varepsilon. \tag{41}$$

(Here we take into account only the leading term in the spectral representation (25) of the Green function, because the contribution from other $dt\mu$ molecule levels is negligible near the pole $E = E_B$.

From eq.(41) follow the expressions for the shift and width of $dt\mu$ molecule level[*])

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[*]) This result is valid always except for the case when there exists a narrow nuclear level in the potential V_n, its energy being close to the energy of the level in the molecular potential V_μ and hence the rearrangement of the spectrum of $dt\mu$ system occurs /26/. If it is the case, when solving Eq.(41) the dependence $f_\mu(E)$ on E should be taken into account. In $dt\mu$ system such situation is not realized. In more detail this question will be discussed in other paper (see also Appendix).

$$\Gamma = \frac{2\beta |\langle \varphi_{0B}^M | \xi \rangle|^2 \Gamma \cdot |B^{J\upsilon}|^2}{1 + F_\mu^2 (E_B)} \;, \quad \Delta E = -\frac{\Gamma}{2} F_\mu (E_B) \;, \qquad (42)$$

where

$$F_\mu (E) = \frac{(2m_1^*)^{3/2}}{\pi} \beta \int\limits_0^\infty \frac{|\langle \varphi_{0E}^M | \xi \rangle|^2}{E - \varepsilon} |f_\mu (\varepsilon)|^{-2} \sqrt{\varepsilon}\, d\varepsilon \;. \qquad (43)$$

As the calculations show the regular solution $\varphi_{0B}^M (R)$ for Hamiltonian H_μ coincides with the regular solutions $\varphi_{0E} (R)$ and $\varphi_{0E}^M (R)$ for Hamiltonians H_1 and H_μ at energy E up to 200 keV, the precision being $\sim 10^{-3}$. Taking this fact and relation (17a) into account we obtain as a result:
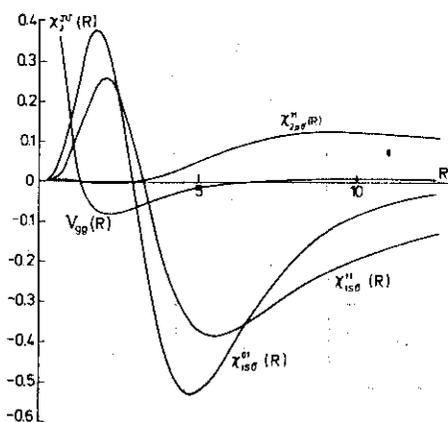
Fig. 4. Wave functions $\chi_{N\ell m}^{J\upsilon} (R)$ of different states of $dt\mu$ molecule (see Table I). $V_{gg} (R)$ — the leading term in the matrix $U_{ji}^\eta (R)$ of effective potentials in (27).
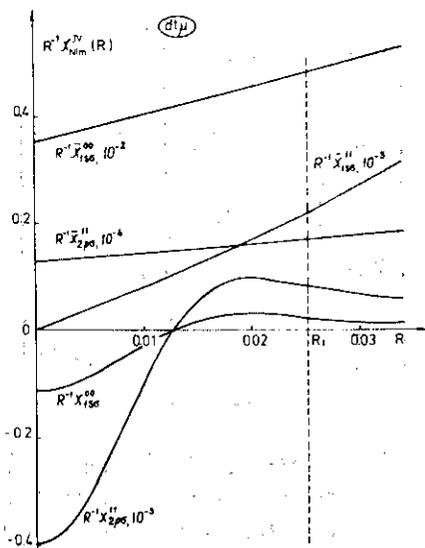
Fig. 5. $dt\mu$ molecule wave functions $\chi_{N\ell m}^{J\upsilon} (R)$ (for $V_\mu + V_n$) and $\bar{\chi}_{N\ell m}^{J\upsilon} (R)$ (for V_μ) inside the range of nuclear forces.

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$$\Gamma^{\mathcal{JV}} = \frac{2\Lambda \cdot |B^{\mathcal{JV}}|^2}{1 + F_\mu^2(E_B)} \quad ,$$

$$\Delta \mathcal{E}_{\mathcal{JV}}^{(a)} = \Delta E = -\frac{1}{2}\Gamma^{\mathcal{JV}} F_\mu(E_B) \ . \tag{44}$$

Coefficients $B^{\mathcal{JV}}$ are presented in Table I along with coefficients $B_o^{\mathcal{JV}}$ which have been calculated from the system of equations (27) without potential V_n of nuclear dt interaction. It is easy to see that coefficients $B^{\mathcal{JV}}$ and $B_o^{\mathcal{JV}}$

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The main characteristics of ($\mathcal{JV}$) states and rates of nuclear reaction in $dt\mu$ molecule*)

($\mathcal{JV}$)	(00)	(01)	(10)	(11)	(20)
Nlm	$1s\sigma$	$1s\sigma$	$2p\sigma$	$2p\sigma$	$3d\sigma$
$-\mathcal{E}_{\mathcal{JV}}$ eV	319.2	34.9	232.4	0.64	102.5
$B_o^{\mathcal{JV}}$	$0.374 \cdot 10^{-2}$	$0.340 \cdot 10^{-2}$	$0.210 \cdot 10^{-4}$	$0.130 \cdot 10^{-4}$	$0.519 \cdot 10^{-6}$
$B^{\mathcal{JV}}$	-0.112	-0.102	$-0.634 \cdot 10^{-3}$	$-0.393 \cdot 10^{-3}$	$-0.147 \cdot 10^{-4}$
$\lambda_{of}^{\mathcal{JV}}$ s⁻¹	$1.1 \cdot 10^{12}$	$0.95 \cdot 10^{12}$	$1.1 \cdot 10^{8}$	$4.2 \cdot 10^{7}$	$1.1 \cdot 10^{5}$
$\lambda_{f}^{\mathcal{JV}}$ s⁻¹	$1.0 \cdot 10^{12}$	$0.80 \cdot 10^{12}$	$1.1 \cdot 10^{8}$	$4.2 \cdot 10^{7}$	$1.1 \cdot 10^{5}$

*) Quantities $B^{\mathcal{JV}}$ and $B_o^{\mathcal{JV}}$ are calculated for potential
(18) with parameters (21) and for $V_n(R) = 0$,respectively,
with $N_o = 12$, including 6 pairs of states $j = (Nlm)$ of
two-center problem: $1s\sigma_g$, $2p\sigma_u$; $2s\sigma_g$, $3p\sigma_u$;
$3d\sigma_g$, $4f\sigma_u$; $2p\pi_u$; $3d\pi_g$; $3p\pi_u$, $4d\pi_g$;
$3d\delta_g$, $4f\delta_u$. Quantities $\lambda_f^{\mathcal{JV}}$ and $\lambda_{of}^{\mathcal{JV}}$ are calculated by means of (47) and (48), A and A_o from (46)
and (4). λ_f^{∞} coincides by the order of magnitude
with estimates $\lambda \simeq 10^{12}$ s⁻¹ from /1,12/.

differ by more than an order of magnitude. Figs. 3 and 4 display the functions $\chi_{N\ell m}^{\mathfrak{IV}}(R)$ and $\bar{\chi}_{N\ell m}^{\mathfrak{IV}}(R)$ calculated from eqs.(27) with potential V_n and without it, correspondingly. It is evident that functions $\chi_{N\ell m}^{\mathfrak{IV}}(R)$ and $\bar{\chi}_{N\ell m}^{\mathfrak{IV}}(R)$ behave in quite different manner. In particular, functions $\chi_{N\ell m}^{\mathfrak{IV}}(R)$ have nodes in the region $R < R_1$ [*].

4. Calculation of Rates $\lambda_f^{\mathfrak{IV}}$

In the limit $E \to 0$ the cross section $\sigma_{in}(E)$ (20a) can be written in the form

$$v\sigma_{in} \underset{E\to 0}{=} \frac{16\pi}{3} \cdot \frac{\Lambda \, |f(E)|^{-2}}{1 + F^2(E)} .$$

(45)

From (44) and (45) it follows

$$\lambda_f^{\mathfrak{IV}} = \Gamma^{\mathfrak{IV}} = \frac{3}{2\pi} \lim_{v\to 0} \left(v\sigma_{in} |f(E)|^2 \right) |B^{\mathfrak{IV}}|^2 \frac{1 + F^2(0)}{1 + F_\mu^2(E_\delta)} .$$

Introducing notation

$$A = \lim_{v\to 0} \left(v\sigma_{in} |f(E)|^2 \right)$$

(46)

and separating the scale factor a_1^{-3} from $|B^{\mathfrak{IV}}|^2$ write $\lambda_f^{\mathfrak{IV}}$ in the form

$$\lambda_f^{\mathfrak{IV}} = \frac{3}{2} \frac{A}{4\pi a_1^3} |B^{\mathfrak{IV}}|^2 \frac{1 + F^2(0)}{1 + F_\mu^2(E_\delta)} .$$

(47)

Expression (47) coincides, but for the last factor, by its form with the "classical" expression (3), which, after elimination of the angular variables from $\psi(\vec{R})$, becomes (with the spin factors taken into account)

$$\lambda_{of}^{\mathfrak{IV}} = \frac{3}{2} \frac{A_o}{4\pi a_1^3} |B_o^{\mathfrak{IV}}|^2 .$$

(48)

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[*] Such behaviour of functions $\chi_j^{\mathfrak{IV}}(R)$ corresponds to the function of oscillator 2S- state, which well reproduces $3/2^+$ state of $^5He^*$ nucleus /25/.

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The similarity between formulae (47) and (48) is due to the identity of the methods used to derive them: in both cases the Hamiltonian $\tilde{H}_1 = H_1^0 + V_c + V_n + V_a$ of the effective one-channel problem is decomposed in two parts. For one of them the eigen-value problem is solved precisely, the other one is taken into account by somewhat means.

The relation (48) is based on the partition $\tilde{H}_1 = (H_1^0 + V_c) + (V_n + V_a)$, the short-range interaction $V_n + V_a$ being considered as perturbation in the scattering amplitude[27]*). The deviations from relation (48) have been studied in detail only for two body system with Coulomb interaction [28], for arbitrary long-ranged potential this question was scarcely investigated (see also[20] and references therein).

Relation (47) originates from partition $\tilde{H}_1 = (H_1^0 + V_c + V_n) + V_a$, where the anti-Hermitian part V_a, having a separable form, is a perturbation. This relation is valid everywhere, but in the region of the rearrangement of the molecular spectrum. The probability of the rearrangement is extremely small as it follows from general considerations[26]. The illustrating numerical calculation is given in Appendix

Reaction constants

$$A_0 = 1.3 \cdot 10^{-14} \text{ cm}^3 \text{ s}^{-1}$$
$$A = 1.3 \cdot 10^{-17} \text{ cm}^3 \text{ s}^{-1} \tag{49}$$

were found with formulae (4) and (46), formulae (20a) and (16) with parameters (21) having been used to extrapolate $\sigma_{in}(E)$ to $E = 0$. The A_0 magnitude is close to $A_0 = 1.1 \cdot 10^{-14} \text{cm}^3 \text{s}^{-1}$ found in paper [29] by Padé-approximation of cross section $\sigma_{in}(E)$ to $E = 0$.

To calculate integrals $F(0)$ and $F_\mu(E_s)$ it is necessary to find Jost functions $f(E)$ and $f_\mu(E)$, for potentials $U(R) = V_c + V_n$ and $U_\mu(R) = V_\mu + V_n$, the potentials differ-

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*) Note, that the nuclear potential V_n should not necessarily be small (in particular, nuclear levels of its own are possible), and the perturbation theory in the potential is, in general, inapplicable.

ing by their long-ranged terms V_c and V_μ. Jost function $f(E)$ was obtained by numerical solution of the one-channel scattering problem with potential $U(R)$. Function $f_\mu(E)$ was found from the relation

$$f_\mu(E) = f(E) C_c^{-1} \bar{f}_\mu(E) \ , \tag{50}$$

where $\bar{f}_\mu(E)$ is Jost function of purely Coulomb $t\mu+d$ problem, calculated with the algorithm [30] based on the method of phase functions [31]. (Here we have taken into account that μ – meson involved in $t\mu+d$ scattering slightly influences the position and width of the nuclear dt resonance.)

The numerical calculation gives $(1+F^2(0))/(1+F_\mu^2(E_s)) = 0.93$ for $\mathcal{J} = 0$ and 1.1 for $\mathcal{J} = 1$. Besides, as one can see from Table I, the ratio holds

$$|B^{\mathcal{J}\upsilon}/B_0^{\mathcal{J}\upsilon}|^2 \simeq A_0/A$$

hence, values of λ_f and λ_{of} found with formulae (47) and (48) agree within 10% accuracy.

Our investigation shows that when calculating $\lambda_f^{\mathcal{J}\upsilon}$ one can use formula (48) along with (47), despite the nearthreshold resonance in the (2a) cross section and the node of the wave function $\chi^{\mathcal{J}\upsilon}(R)$ in the region $R < R_1$ *).

The accuracy of the values $\lambda_f^{\mathcal{J}\upsilon}$ is estimated to be about 10%. This uncertainty includes the experimental errors of cross section (2) and the error in their extrapolation to $E = 0$. Besides, when calculating $\lambda_f^{\mathcal{J}\upsilon}$ we have kept only the leading term in the decomposition (29) of $\psi^{\mathcal{J}\upsilon}(\vec{r}, \vec{R})$. Inclusion of the remaining terms (the so-called nonadiabatic corrections) would somehow increase the values of $\lambda_f^{\mathcal{J}\upsilon}$. Further, we have taken into account only the state $\mathcal{J}^\pi = 3/2^+$, $L = 0$ dominating in

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*) In paper [32] the formula was used instead of (3) and (48)
$\lambda_f = A_0 /4\pi a_1^3 |R_0^{-1}\chi(R_0)|^2$, where $R_0 \lesssim R_1$ was varied to check up the stability of results. In reality, there is no problem of R_0 choice, if one keeps in mind the identity of regular solutions $\psi_{0E}(R)$ and $\psi_{08}(R)$ at $R \lesssim R_1$.

reaction (2a). To estimate the contribution from other states is a special problem.

Mention also, that the calculated values $\lambda_f^{\mathcal{J}\mathcal{V}}$ correspond to the rate of nuclear reaction from the hyper-fine structure of $dt\mu$ $\mathcal{J}\mathcal{V}$ levels with total spin of nuclei $S = 3/2$ [33]. This circumstance should be taken into account when calculating the μ - catalysis process kinetics.

The shifts $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(n)}$ and $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(a)}$ of $dt\mu$ mesomolecule levels due to nuclear dt interaction are given in Table 2. The total level shifts $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}} = \Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(n)} + \Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(a)}$ do not exceed 10^{-3} eV for all levels $(\mathcal{J}\mathcal{V})$.

<p style="text-align:center">T a b l e   2</p>

The shifts of $dt\mu$ molecule levels
due to dt nuclear interaction*)

$(\mathcal{J}\mathcal{V})$	(00)	(01)	(10)	(11)	(20)
$\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(n)}$	$-0.88\cdot10^{-3}$	$-0.74\cdot10^{-3}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-6}$
$\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(a)}$	$0.18\cdot10^{-3}$	$0.14\cdot10^{-3}$	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$
$\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}$	$-0.70\cdot10^{-3}$	$-0.60\cdot10^{-3}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-6}$

*) The shifts are given in eV. $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(n)}$ – the shift due to nuclear potential V_n , $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(a)}$ – the shift due to V_a , $\Delta\mathcal{E}_{\mathcal{J}\mathcal{V}} = \Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(n)} + \Delta\mathcal{E}_{\mathcal{J}\mathcal{V}}^{(a)}$.

5. The Rate of Reaction $t\mu + d \rightarrow n + {}^4He + \mu^-$

In our approach the reaction (6) cross section is calculated with the same formulae as the reaction (2a) cross section is. The only difference is that one should insert Jost function $f_\mu(E)$ of $t\mu + d$ scattering problem into formulae (20a) and (16) instead of dt scattering problem Jost function $f(E)$. It is evident that at large energies $E \gtrsim$ 1 MeV the difference between $f(E)$ and $f_\mu(E)$ becomes insignificant and cross sections of reactions (2a) and (6) equal each other. However, at $E \lesssim$ 10 keV this difference is noticeable, because there is no
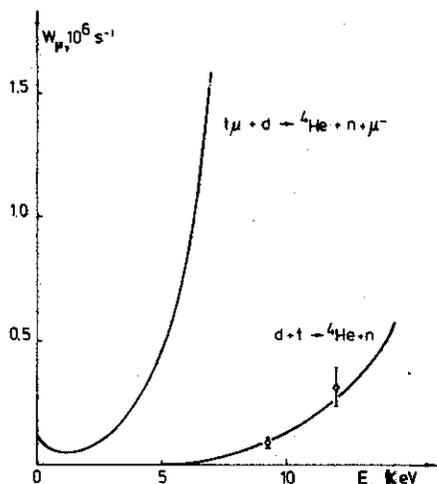
Fig. 6. The rates $W_\mu(E)$ (51) and $W(E)$ of reactions (6) and (2a) at liquid hydrogen density. Experimental points are from [9].

Coulomb repulsion at $R \gtrsim a_1 = 2.66 \cdot 10^{-11}$ fm in $t\mu + d$ system. The means of $f_\mu(E)$ calculation were discussed in the previous Section.

Figure 6 shows the E- dependence of reaction (6) rate W_μ

$$W_\mu = \sigma_{in} \cdot v \cdot N_0 \qquad (51)$$

(v is relative $t\mu \cdot d$ velocity, $N_0 = 4.25 \cdot 10^{22}$ cm^{-3} is the liquid Hydrogen density). The attention should be paid to the specific minimum at $E \simeq 0.5$ keV. At $E = 12$ keV the reaction (6) rate $W_\mu = 4.9 \cdot 10^6$ S^{-1} still significantly exceeds the rate $W = 2.7 \cdot 10^5$ s^{-1} of reaction (20).

The obtained results are in good agreement with the simple estimate of nuclear reaction "in flight" rate W_μ [1,3]

$$W_\mu \simeq \lambda_f^{dt\mu}(a_1/a_o)^3 \simeq 1.2 \cdot 10^{12}\left(\frac{2.6 \cdot 10^{-11}}{0.5 \cdot 10^{-8}}\right)^3 \approx 10^5 \text{ s}^{-1}.$$

6. Conclusions

The present investigation is the first detailed calculation of nuclear reaction rates from different (Jv) states of $dt\mu$ molecule. The method developed can be applied to calculation of nuclear reactions rates in other $\mu -$ molecules.

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It is shown that formulae (47) and (48) for calculation $\lambda_f^{\mathcal{J}\mathcal{V}}$ are approximately equivalent, despite the nearthreshold resonance.

Reaction (2a) was theoretically studied earlier in the framework of the two-channel potential model $/34,35/$ in order to describe the cross section $\sigma_{in}(E)$ in the wide energy region $E \leq 5$ MeV, the possibility to use the simple effective one-channel Hamiltonion being thus excluded.

In the energy region of $^5He^*(^3/_2^+)$ resonance ($0 < E < 200$ keV) we succeded in reproducing reactions (2a) and (2b) cross sections with the generalized optical potential.

Our approach is, in essense, model independent, in spite of the particular form (18) of potential. It would be, however, desirable to calculate the rate $\lambda_f^{\mathcal{J}\mathcal{V}}$ without concretization of potential $U(R)$. The possibility is given, e.g., by boundary condition model where the set of equations (27) is solved with boundary conditions at $R \sim R_1$ following from the analysis of reactions (2a) and (2b) in the framework of $\mathcal{R}$ - matrix theory $/36/$.

A p p e n d i x

Formulae (44) are valid when there is no rearrangement of $dt\mu$ molecule spectrum. To estimate the probability of such situation we have studied how the energies of $dt\mu$ molecule states depend on the depth V_0 of nuclear "well" V_n.

Figure 7 displays the V_0 - dependence of the energy $\mathcal{E}_{\mathcal{J}\mathcal{V}}$ of $dt\mu$ molecule state ($\mathcal{J} = 0$, $\mathcal{V} = 1$), found by numerical integration of eqs.(27).

The depth V_0 of nuclear potential increasing, the nuclear quasi-stationary level $E_R \approx 64$ keV corresponding to $3/2^+$ state of ^5He, goes down and at $V_0 = V_0^*$ the spectrum
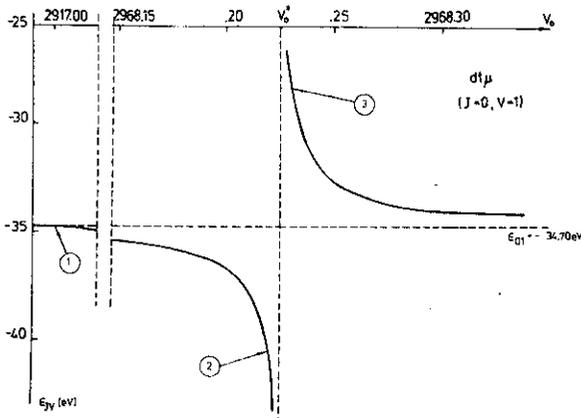
Fig. 7   The scheme of the rearrangement of $dt\mu$ system spectrum. The position of the levels ($3/2^+$ of $^5He^*$ and ($\mathcal{J} = 0$, $\mathcal{V} = 1$) of $dt\mu$ molecule) is shown versus V_0 - parameter in the potential (18). The magnitude $V_0^* = 2968.225$ essentially differs from $V_0 = 2917$ from the set (21). The range of rearrangement region $\Delta V_0 / V_0 \sim 10^{-5}$.

of $dt\mu$ system rearranges, the "ex"-level $3/2^+$ of the nuclear well V_n substitutes the μ molecule level ($\mathcal{J} = 0$, $\mathcal{V} = 1$), the latter takes the place of ($\mathcal{J} = 0$, $\mathcal{V} = 0$) level and this one becomes bound state in the nuclear well. The wave functions $\chi_j^{\mathcal{J}\mathcal{V}}(R)$ achieve additional nodes inside the range of nuclear forces $0 < R < R_1$ (see Fig.8). The calculations show that the strong interaction weakly influences the position of $dt\mu$ molecule levels, but in the close vicinity ΔV_0 of the point V_0^*, where the level shift can be of the order of the level energy: $|\Delta \mathcal{E}_{\mathcal{J}\mathcal{V}}| \sim |\mathcal{E}_{\mathcal{J}\mathcal{V}}|$. The calculations show that the critical region is very small $\Delta V_0 / V_0 \sim 0.01/10^3 \sim 10^{-5}$.

Parameter $V_0^* = 2968.225$ corresponding to the rearrangement region essentially differs from this (21), which provides the best fit to the experimental cross sections (see Fig.1a and 1b).

The cross sections $\sigma_{in}(E)$ and $\sigma_{el}(E)$ calculated with formulae (20) at $V_0 = V_0^*$ differ from the experimental
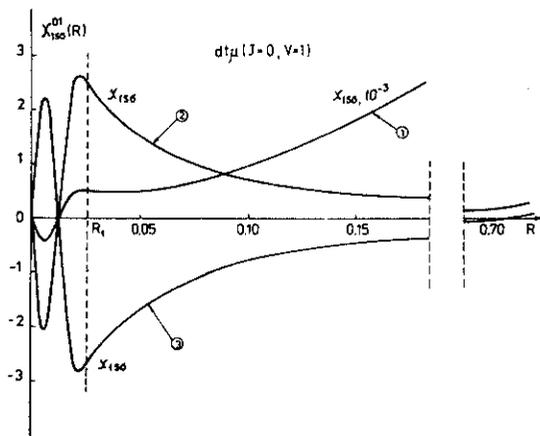
Fig. 8   The wave functions of $dt\mu$ system (with potential (18)) in the rearrangement region. The numbers correspond to Fig.7. Functions 2 and 3 exceed function 1 by three orders of magnitude. Function 3 (after the rearrangement) differs from function 2 (before the rearrangement) by its sign in the region $R \lesssim R_1$ and by additional node at $R \simeq 0.7$.

ones by two-three orders of magnitude and do not reproduce, in particular, their resonant behaviour. This fact evidences that the rearrangement of $dt\mu$ molecule spectrum does not occur.

Note that this study has been carried for the nuclear level of zero width. The rearrangement of $dt\mu$ molecule spectrum in general case needs additional consideration and should be presented elsewhere.

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