
$2461 / 2-80$

объединенныи
институт ядериых исследований

дубна

V,G.Soloviev, O.Stoyanova, Ch.Stoyanov

A NUMBER OF QUASIPARTICLES IN THE GROUND STATES OF SPHERICAL AND TRANSITIONAL NUCLEI

Submitted to "Извесмия AH CCCP, серия физическая".

The number of quasiparticles in the ground states of doubly even nuclei or the correlations in the ground states have been discussed in many papers ${ }^{1-4 /}$. The ground state is considered as a vacuum for the RPA-phonons, and an expression for the number of quasiparticles in this state is derived. However, the number of quasiparticles in the ground state has been calculated for the simple models only $/ 5 /$ and for the deformed nuclei ${ }^{/ 6 /}$. The analysis of the accuracy of calculations within the quasiparticle-phonon nuclear mode1/7/ necessitates recalculation of the number of quasiparticles in the ground states of doubly even nuclei. The RPA-phonons calculated with the multipole and spin-multipole forces are used as the basis of this model, i.e. ${ }^{\pi}+$ the phonons with $\lambda^{\pi}=1^{-}, 2^{+}, 3^{-}$, etc., and with $\lambda^{\pi}=1^{+}, 2^{-}, 3^{+}$, etc. The ground state of a doubly even nucleus is the vacuum for all these phonons. As it is noted in ref. , to analyze the accuracy of calculations within the quasiparticle phonon nuclear model, one should study the effect of the Pauli principle and to find the number of quasiparticles in the ground states of doubly even nuclei.

In this paper the number of quasiparticles is calculated in the ground states of doubly even spherical and transitional nuclei. A large number of roots of the secular equations for the RPA-phonons is taken into account. The calculations are performed for the multipole and spin-multipole phonons describing the collective low-lying quadrupole and octupole states and the giant resonances and also the weakly-collective and noncollective states.

1. QuAsIPARTICLES IN THE GROUND STATES

The quasiparticle-phonon nuclear model uses the RPA-phonons of the following form:

$$
\begin{equation*}
\left.Q_{\lambda \mu_{i}}=\frac{1}{2} \sum_{j_{1} j_{2}}\left\{\psi_{j_{1} j_{R}}^{\lambda_{1}} A\left(j_{1} j_{2} \lambda \mu\right)-(-)\right)^{\lambda-\mu} \phi_{j_{1} j_{2}}^{+} A^{+}\left(j_{1} j_{2} \lambda-\mu\right)\right\}, \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
A^{+}\left(j_{1} j_{2} \lambda \mu\right)=\sum_{m_{1} m_{2}}<j_{1} m_{1} j_{2} m_{2} \mid \lambda \mu>a_{j_{1} m_{1}}^{+} a_{j_{2} m_{2}}^{+} \tag{2}
\end{equation*}
$$

$a_{j m}^{+}$is the quasiparticle creation operator,

$$
\begin{equation*}
\psi_{j_{1} j_{2}}^{\lambda_{i}}=\frac{1}{\sqrt{2 Y\left(\lambda_{i}\right)}} \frac{f_{j_{1} j_{2}}^{(\lambda)}{ }^{u_{j_{1}} j_{2}}}{\epsilon_{j_{1}}+\epsilon_{j_{2}}-\omega_{\lambda_{i}}} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\phi_{j_{1} j_{2}}^{\lambda_{i}}=\frac{1}{\sqrt{2 Y(\lambda i)}} \frac{f_{j_{1} j_{2}}^{(\lambda)} u_{j_{1} j_{2}}}{\epsilon_{j_{1}}+\epsilon_{j_{2}}+\omega_{\lambda_{i}}} \tag{4}
\end{equation*}
$$

Here $f_{j_{1} j 2}^{(\lambda)}$ is the reduced single-particle matrix element; $\epsilon_{j}$ is the quasiparticle energy, $\omega_{\lambda_{i}}$ is the phonon energy;
$\mathbf{u}_{\mathrm{j}_{1} \mathrm{j}_{2}}=\mathrm{u}_{\mathrm{j}_{1}} \mathbf{v}_{\mathbf{j}_{2}}+\mathrm{v}_{\mathrm{j}_{1}} \mathrm{u}_{\mathrm{j}_{2}}$, where $\mathrm{u}_{\mathrm{j}}$ and $\mathbf{v}_{\mathbf{j}}$ are the Bogolubov coefficients, and $Y(\lambda i)$ is the normalization factor.

The phonon operators satisfy the commatation relations:

$$
\begin{align*}
& {\left[Q_{\lambda \mu i}, Q_{\lambda^{\prime} \mu^{\prime} i^{\prime}}^{+}\right]=} \\
& =\frac{1}{4}\left\{\sum_{\substack{j_{1} j_{2} \\
j_{3} j_{4}}} \psi_{j_{1} j_{2}}^{\lambda i} \psi_{j_{3} j_{4}}^{\lambda^{\prime} i^{\prime}}\left[A\left(j_{1} j_{2} \lambda \mu\right), A^{+}\left(j_{3} j_{4} \lambda^{\prime} \mu^{\prime}\right)\right]-\right.
\end{align*}
$$

The commutation relations of the operators $A, A^{+}$have a complicated form, though being averaged over the phonon vacuum, they are

$$
\langle 0|\left[A\left(j_{1} j_{2} \lambda \mu\right), A^{+}\left(j_{3} j_{4} \lambda^{\prime} \mu^{\prime}\right)\right]|0\rangle=
$$

$$
\begin{aligned}
& -\sum_{j_{1} j_{2}}(-)^{\lambda-\mu}(-){ }^{\lambda^{\prime}-\mu^{\prime}}{ }_{\phi_{i}}^{\lambda_{1} j_{2}}{ }^{\lambda_{j_{3}} \mathrm{j}_{4}}\left[\mathrm{~A}\left(\mathrm{j}_{1} \mathrm{j}_{2} \lambda-\mu\right), \mathrm{A}^{+}\left(\mathrm{j}_{3} \mathrm{j}_{4} \lambda^{\prime}-\mu^{\prime}\right)\right\} . \\
& \mathrm{j}_{3} \mathrm{j}_{\mathbf{4}}
\end{aligned}
$$

$$
\begin{align*}
& =\delta_{\lambda \lambda}-\delta_{\mu \mu}-\left[\delta_{j_{1} j_{3}} \delta_{j_{2} j_{4}}+(-)^{j_{3}-j_{4}+\lambda} \delta_{j_{1} j_{4}} \delta_{j_{2} j_{3}}\right]- \\
& -\delta_{\lambda \lambda}-\delta_{\mu \mu}-\delta_{j_{1} j_{2}} \delta_{j_{3} j_{4}} \delta_{j_{2} j_{3}}{ }^{2(2 j+1)^{-1 / 2}}<0 \| \mathrm{B}(\mathrm{jij} 00) \mid 0> \tag{6}
\end{align*}
$$

Where

$$
\begin{equation*}
B\left(j_{1} j_{2} \lambda_{\mu}\right)=\sum_{m_{1} m_{2}}(-)^{j_{2}^{+m} 2}<j_{1} m_{1} j_{2} m_{2} \| \mu_{\mu}>a_{j_{1} m_{1}}^{*} a_{j_{2}}-m_{2} \tag{7}
\end{equation*}
$$

The condition of orthonormalization of one-phonon states results in

$$
\begin{equation*}
<0\left|Q_{\lambda \mu \mathrm{i}}, Q_{\lambda \mu^{\prime} \mathrm{i}}^{+}, \eta 0\right\rangle=\delta_{\lambda \lambda^{\prime}} \delta_{\mu H} \cdot \delta_{\mathrm{ii}}, \tag{8}
\end{equation*}
$$

It is assumed within the RPA that
$\langle 0| B(\mathrm{j} 00)|0\rangle=0$.

Therefore, the following condition is imposed on the functions $\psi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}$ and $\phi_{\mathrm{j}_{1} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}=$

$$
\begin{equation*}
\sum_{j_{1} j_{2}}\left[\psi_{j_{1} j_{2}}^{\lambda_{i}} \psi_{j_{1} j_{2}}^{\lambda_{i}^{\prime}}-\phi_{j_{1} j_{2}}^{\lambda_{i}} \phi_{j_{1} j_{2}}^{\lambda_{i}^{*}}\right]=2 \delta_{i i} \tag{10}
\end{equation*}
$$

The wave function of the ground state is defined as a phonon vacuum as follows:

As has been shown earlier $/ 1,9 /$, it has the form

$$
\begin{align*}
& \Psi_{0}^{\lambda}=\frac{1}{\sqrt{N^{\prime}}} \exp i-\frac{1}{4} \sum_{\mu} \underset{j_{1} j_{2}}{\sum_{j_{3} j_{4}}}\left(\psi^{-1}\right)_{j_{3} j_{4}}^{\lambda_{i}} \phi_{j_{1} j_{2}}^{\lambda_{i}}(-)^{\lambda-\mu} \times  \tag{12}\\
& \left.\times A^{+}\left(j_{1} j_{2} \lambda_{\mu}\right) A^{+}\left(\mathrm{j}_{3} j_{4} \lambda-\mu\right)\right\}_{00} \text {, }
\end{align*}
$$

or

$$
\Psi_{0}^{\lambda}=\frac{1}{\sqrt{\mathrm{~N}^{\prime}}} \mathrm{e}^{-\hat{\mathrm{L}}_{\lambda}}
$$

in which the quasiparticle vacuum is determined as

$$
\begin{equation*}
a_{\lambda \mu i} \Psi_{00}=0 \tag{13}
\end{equation*}
$$

$N^{\prime}$ is the normalization factor.
Let us determine the number of quasiparticles with the single-particle quantum numbers $n \ell j$ (which are denoted by one index $j$ ) in the ground state which is the phonon vacuum of multipolarity $\lambda$

$$
\begin{equation*}
\mathrm{n}_{\mathrm{j}}^{\lambda}=(2 \mathrm{j}+1)^{-1}<0\left|\sum_{\mathrm{m}} a_{\mathrm{jm}}^{+} a_{\mathrm{jm}}\right| 0>, \tag{14}
\end{equation*}
$$

or using (7), we have

$$
\left.\mathrm{n}^{\lambda}{ }_{j}^{\lambda}=(2 \mathrm{j}+1)^{-1}<0|B(\mathrm{jj} 00)| 0\right\rangle
$$

It is seen from this formula that the condition of applicability of the RPA (9) results in $\mathrm{n}_{\mathrm{j}}^{\lambda}=0$.

Using expression (12) and formula

$$
\mathrm{e}^{\mathrm{A}} \mathrm{Be}^{-\mathrm{A}}=\mathrm{B}-[\mathrm{A}, \mathrm{~B}]+\cdots \cdots
$$

connecting the operators $A$ and $B$, we calculate

$$
\begin{aligned}
& \left.\mathrm{n}_{j}^{\lambda}=(2 j+1)^{-1 / 2}<0|B(j j 00)| 0\right\rangle= \\
& \left.=(2 j+1)^{-1 / 2}<0|[\hat{\mathrm{~L}} \lambda, B(j j 00)]| 0\right\rangle
\end{aligned}
$$

Using the commutation relation

$$
\begin{align*}
& {\left[B(\mathrm{j} 00), A^{+}\left(\mathrm{j}_{1} \mathrm{j}_{2} \lambda \mu\right)\right]=} \\
& =(2 \mathrm{j}+1)^{-1 / 2}\left[\mathrm{~A}^{+}\left(\mathrm{j}_{1} \mathrm{j} \lambda \mu\right)+\mathrm{A}^{+}\left(\mathrm{j} \mathrm{j}_{2} \lambda \mu\right)\right] \tag{15}
\end{align*}
$$

we get

$$
\begin{align*}
& { }_{n}{ }_{\mathrm{j}}^{\lambda}=(2 \mathrm{j}+1)^{-1 / 2}<0\left|\left[\hat{\mathrm{~L}}_{\lambda}, \mathrm{B}(\mathrm{jj} 00)\right]\right| 0> \\
& =(2 j+1)^{-1 / 2} \frac{1}{4} \sum_{\mu} \sum_{\substack{j_{1} j_{2} \\
j_{3} j_{4}}}(-)^{\lambda-\mu} \phi_{j_{1} j_{2}}^{\lambda i}\left(\psi^{-1} \cdot\right)_{j_{3} j_{4}}^{\lambda_{i}} \quad \times \\
& x<0\left|\left[A^{+}\left(j_{1} j_{2} \lambda \mu\right) A^{+}\left(j_{3} j_{4} \lambda-\mu\right), B(j j 00)\right]\right| 0>= \\
& \left.=(2 j+1)^{-1} \sum_{\mu} \sum_{\mathrm{j}_{2} \mathrm{j}_{3} \mathrm{j}_{4}}(-)^{\lambda-\mu} \phi_{\mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}\left(\psi^{-1}\right)_{\mathrm{j}_{3} \mathrm{j}_{4}}^{\lambda_{\mathrm{i}}}<0\left|A^{+}\left(\mathrm{jj}_{2} \lambda \mu\right) A^{+}\left(\mathrm{j}_{3} \mathrm{j}_{4} \lambda \mu\right)\right| 0\right\rangle= \\
& \left.=(2 \mathrm{j}+1)^{-1} \sum_{\mu} \sum_{\mathrm{i}} \sum_{\mathrm{j}_{2} \mathrm{j}_{3} \mathrm{j}_{4}}\left(\phi_{\mathrm{j} \mathrm{j}_{2}}^{\lambda_{\mathrm{i}}}\right)^{2}\left(\psi^{-1}\right)_{\mathrm{j}_{3} \mathrm{j}_{4}}^{\lambda_{\mathrm{i}}} \sum^{\prime}, \psi_{j_{3} \mathrm{j}_{4}}^{\lambda_{\mathrm{i}}{ }^{\prime}}<0\left|Q_{\lambda-\mu \mathrm{i}} Q_{\lambda-\mu \mathrm{i}}^{+},\right| 0\right\rangle= \\
& =(2 j+1)^{-1}(2 \lambda+1) \sum_{i} \sum_{j^{\prime}}\left(\phi_{j j^{\prime}}^{\lambda_{i}}\right)^{2} . \tag{16}
\end{align*}
$$

It is seen from this expression that the calculation of the number of quasiparticles in the ground state requires the summation over all the roots of the secular equation for the RPAphonon of multipolarity $\lambda$.

In the quasiparticle-phonon nuclear model the ground state wave function of a doubly even nucleus is the vacuum for all multipole and spin-multipole, ie.,

$$
\Psi_{0}=\prod_{\lambda} \Psi_{0}^{\lambda}
$$

or

$$
\begin{aligned}
& \Psi_{0}=\frac{1}{\sqrt{N^{\prime}}} e^{-\Sigma \hat{L}_{\lambda}} \equiv \\
& =\frac{1}{\sqrt{N^{\prime}}} \exp \left\{-\frac{1}{4} \sum_{\lambda \mu}^{\sum} \sum_{j_{1} j_{2}}^{j_{3} j_{4}} \phi_{j_{1} j_{2}}^{\lambda_{i}}\left(\psi^{-1}\right)_{j_{3} j_{4}}^{\lambda_{i}} \quad(-)^{\lambda-\mu} \times\right. \\
& \times A^{+}\left(j_{1} j_{2} \lambda \mu\right) A^{+}\left(j_{3} j_{4}{ }^{\lambda-\mu)\} \Psi_{00}}\right.
\end{aligned}
$$

Therefore the number of quasiparticles with quantum numbers $n \ell j$ in the ground state of a doubly even spherical nucleus is

$$
\begin{equation*}
n_{j}=\sum_{\lambda} n_{j}^{\lambda}=(2 j+1)^{-1} \sum_{\lambda_{i}}(2 \lambda+1) \sum_{j^{\prime}}\left(\phi_{j j^{\prime}}^{\lambda_{i}}\right)^{R} . \tag{17}
\end{equation*}
$$

Let us discuss expression (17) for $n_{j}$. It contains the square of the amplitudes $\phi_{j j}^{\lambda_{i}}$ (4). These functions are relatively large for the low-lying collective states. With increasing excitation energy $\omega$ the amplitudes $\phi_{\mathrm{jj}} \boldsymbol{\lambda}^{\prime}$, diminish due to the increase in the denominator. Therefore the highly excited collective states (giant resonances) are expected to give a small contribution to $\mathbf{n}_{j}$. Expression (17) is proportional to $(2 \lambda+1)$, therefore the phonon states with large $\lambda$ will perhaps give a pronounced contribution to $\mathbf{n}_{j}$.

The values of $\phi_{\mathrm{jj}}{ }^{\lambda_{j}}$, depend on the constants of multipole and spin-multipole forces generating the RPA-phonons. Some of these constants are determined from the experimental data, and the others are only estimated ${ }^{10 /}$. Therefore, using the RPA-phonons to describe the states with different momenta and parities (as in the quasiparticle-phonon model), one should verify how the constants of the forces generating these phonons agree with condition (9) which assumes the small values of $\mathbf{n}_{\mathbf{j}}$.

## 2. Details of Calculations and methodical results

The number of quasiparticles $n_{j}$ is calculated for some spherical and transitional nuclei. The nuclear Hamiltonian included, as in ref. ${ }^{10 /}$, the multipole forces generating the dipole, quadrupole, octupole phonons and the phonons with $\lambda=$ $=4,5$ and 6. The spin-multipole forces have been used for describing the phonon states with $\lambda^{\pi}=1^{+}, 2^{-}, 3^{+}$. The constants of all forces, except for the quadrupole and octupole, have been chosen according to the procedure of ref. ${ }^{107}$. The calculations have shown that the number of quasiparticles in the ground states $n_{j}$. depends strongly on the change of the constants of quadrupole and octupole forces. The calculations have used the values of the constants for which the electric transition probabilities $B\left(E 2,0_{\mathrm{g.s.}}^{+} \rightarrow 2_{1}^{+}\right)$and $B\left(E 3,0_{\mathrm{g} . \mathrm{s.}}^{+} \rightarrow 3_{1}^{-}\right)$,

Table 1.
Energies and transition probabilities of the $2_{1}^{+}$and $3_{1}$ states
Nucleus $\quad \mathrm{E}_{2_{1}^{+}} \mathrm{MeV} \quad \mathrm{B}(\mathrm{E} 2 \uparrow) \mathrm{e}^{2} \mathrm{~b}^{2} \quad \mathrm{E}_{3_{1}{ }_{1} \mathrm{MeV} \quad \mathrm{B}\left(\mathrm{E} 3^{\uparrow}\right) \mathrm{e}^{2} \mathrm{~b}^{3}}$
exp. theor. exp. theor. exp. theor. exp. theor.

| ${ }^{144} 4_{\mathrm{Sm}}$ | 1,660 | 2,15 | 0,25 | 0,24 | 1,810 | 2,3 | - | 0,14 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ${ }^{146} \mathrm{Sm}$ | 0,777 | 1,2 | - | 0,50 | 1,381 | 2,0 | - | 0,20 |
| ${ }^{148_{\mathrm{Sm}}}$ | 0,550 | 0,97 | 0,70 | 0,70 | 1,162 | 1,75 | 0,25 | 0,26 |
| ${ }^{150} \mathrm{Sm}$ | 0,334 | 0,59 | 1,37 | 1,36 | 1,071 | 1,6 | 0,30 | 0,30 |
| ${ }^{126_{\mathrm{Te}}}$ | 0,666 | 0,9 | 0,47 | 0,46 | 2,39 | 3,1 | 0,117 | 0,114 |
| ${ }^{118_{\mathrm{Sn}}}$ | 1,229 | 1,59 | 0,216 | 0,218 | 2,32 | 2,32 | 0,17 | 0,17 |
| $\boldsymbol{x}_{1}^{(4)} \mathrm{F}$ |  |  |  |  |  |  |  |  |
| $118_{\mathrm{Sn}}$ | 1,229 | 0,62 | 0,216 | 0,216 | 2,32 | 1,04 | 0,17 | 0,17 |
| $x_{1}^{(1)}=0$ |  |  |  |  |  |  |  |  |

calculated in the RPA, coincide with the experimental ones. Under such a choice of the energies of the $2_{1}^{+}$and $3_{1}^{-}$states are overestimated. These results are given in table 1, and the experimental data are taken from ref. ${ }^{11 / \text {. }}$

The multipole forces used in the calculations contain the isovector part in which the constant $\kappa_{1}^{(\lambda)}$ is connected with the isoscalar constant $\kappa_{0}^{(\lambda)}$ by $10 \%$,

$$
\begin{equation*}
\kappa_{1}^{(\lambda)}=-0,2(2 \lambda+1) \kappa{ }_{0}^{(\lambda)} . \tag{18}
\end{equation*}
$$

The summation over the roots of the secular equation for the phonons is performed up to an energy of about 20 MeV . Thus, the sum in (17) includes several dozens of roots of the collective and noncollective type.

Since the value of $n_{j}$ (14) is normalized to unity, the results of calculations by (17) in all the tables and figures are given in percent.

Now we study the influence of the $2_{1}^{+}$and $3 \overline{1}$ states on the number of quasiparticles in the sm isotopes. The result for the neutron $2 \mathrm{f}_{7 / 2}$ and proton $2 \mathrm{~d}_{5 / 2}$ states with the largest value of $n_{j}$ are shown in fig. 1. It is seen from the figure that the values of $n_{j}$ increase with increasing number of neutrons and have the maximal value for ${ }^{150} \mathrm{Sm}$. In all the isotopes the contribution of the $\boldsymbol{2}_{1}^{+}$and $\overline{3}_{\overline{1}}$ states to $n_{j}$ is large and except for ${ }^{144} \mathrm{Sm}$, it exceeds half of the value of $n_{j}$.


Fig. 1. Number of quasiparticles in the ground states for the neutron $2 f 7 / 2$ and proton $2 \mathrm{~d}_{5 / 2}$ states in Sm isotopes. The contribution of the first $2^{+}$phonons is the dashed line, of the first $2^{+}$and $3^{-}$phonons is the dashed-dotted line, and the total contribution of all phonons is the solid line.

This is due to the increase in the collectiveness of the $2_{1}^{+}$and $3_{1}^{-1}$ states along the samarium isotopic chain. A stronger collectiveness causes the increase in the number of large $\phi$ values. In the semimagic nucleus ${ }^{144} \mathrm{Sm}$ the $2_{1}^{+}$and $3_{1}^{-}$ levels are weakly collective. For the $2_{1}^{+}$state the maximal values of $\phi$ are equal to 0.1 for the neutron and 0.08 for the proton components. In ${ }^{150} \mathrm{Sm}$ the $2_{1}^{+}$and $3_{1}^{-}$states are the most collective. The number of components with $\phi>0.1$ enlarges up to 10 for the neutron and 7 for the proton systems, the maximal values become 0.6 and their contribution to $n_{j}$ is of several dozens per cent.

It is seen from fig. 1 that the total contribution to $n_{j}$ of the rest $2^{+}$and $3^{-}$states is not large. The contribution of satets with other values of $\lambda^{\pi}$ is from $1 / 5$ to $3 / 5$. The noncollective phonons give a small contribution to $n_{j}$ due to the small values of the functions $\phi$.

The values of $\phi$ are small for the high-lying collective states. Formula (4) shows that with increasing excitation energy $\phi \rightarrow 0$, since they do not contain a pole texm. Table 2

Table 2.
Contribution of phonons, forming the dipole and quadrupole resonances, to $n_{j}$ for the neutron $2 f_{7 / 2}$ and proton $2 \mathrm{~d}_{5 / 2}$ states

| Nucleus | Resonance multip. | $\begin{gathered} \overline{\mathrm{E}}, \\ \mathrm{MeV} \end{gathered}$ | contribution in EWSR \% | $\begin{gathered} \mathrm{N}: \mathrm{n}_{2 \mathrm{f}_{7 / 2}} \\ \% \end{gathered}$ | $\underset{8}{\mathrm{Z}: \mathrm{n}_{2 \mathrm{~d}}^{5 / 2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{144}{ }_{\text {SII }}$ | E1 | 15,12 | 62,75 | 0,116 | 0,105 |
|  | E2 | 11,68 | 23,94 | 0,12 | 0,45 |
| ${ }^{146}$ SII | E1 | 15,27 | 57,76 | 0,097 | 0,086 |
|  | E2 | 11,82 | 21,85 | 0,25 | 0,40 |
| $148_{\text {Sm }}$ | E1 | 15,92 | 61,28 | 0,093 | 0,09 |
|  | E2 | 12,12 | 23,95 | 0,4 | 0,3 |
| ${ }^{150}$ Sm | E1 | 15,57 | 63,3 | 0,09 | 0,09 |
|  | E2 | 12,13 | 21.15 | 0,3 | 0,3 |

gives the values of the number of quasiparticles $n_{j}$ in the ground states of the Sm isotopies, calculated for the phonons forming the giant dipole and isoscalar quadrupole resonances. For all the Sm isotopies the values of $n_{j}$ are very small and do not exceed $0,45 \%$. For the quadrupole resonance they are larger than for the dipole one.

The change of the quadrupole and octupole forces changes the collectiveness of the $2_{1}^{+}$and $3_{1}^{-}$states, thus influencing the number of quasiparticles in the ground state. We have performed the calculations with the constants which have been fitted to the experimental values of the $2_{1}^{+}$and $3_{1}$ states. These constants are by 10-15\% larger than those used for the calculations shown in table 1. This, on the average, increases twice the reduced E2 and E3-transition probability and $\mathrm{n}_{2 \mathrm{~d}_{5 / 2}}-$ and $n_{2 f 7 / 2}$ - values for the Sm isotopies in comparison with the values given in fig. 1.

Essential changes of the number of quasiparticles in the ground state can be obtain upon taking into account the anharmonic terms in the nuclear Hamiltonian $/ 12,13 /$. The anharmonic corrections have been studied in the Te isotopes ${ }^{14 / 4}$. The anharmonic corrections in ${ }^{126} \mathrm{Te}$ diminish the constants of the quadrupole and octupole forces (at that, $B(E 2)$ and $B(E 3)$ values are lower than the expeximental values). The values of the number of quasiparticles with the anharmonic values of the


Fig. 2. Number of quasiparticles $n_{j}$ in the ground state of ${ }^{128}$ Te for different subshells $n \ell j$. The upper fig, is the neutron and the bottom fig. is the proton systems. The subshell energies are given on the axis of coordinates; the arrow shows the position of the chem. potential. The height of the dasheddotted line denotes the contribution of the $2_{1}^{+}$phonon, it is added by the dashed line denoting the $3-1$ phonon contribution and finally it is added on by the solid line denoting the contribution of therest phonons.
quadrupole and octupole constants are less by $4-5$ times. For instance, the values of $n_{j}$ for the neutron states $2 d_{3 / 2}$ and $1 h_{11 / 2}$ are $4.5 \%$ and $3.7 \%$, respectively; some proton states have the following values of $n_{j}: n_{2 d 5 / 2}=5.3 \%_{1}, n_{1 g 7 / 2}=3.18$. They can be compared with the results in fig. 2, which have been calculated without taking into account the anharmonic corrections.

The influence of the isovector forces on the number of quasiparticles $n_{j}$ in the ground state has been studied for 118 Sn . It is known $/ 15 /$ that the inclusion of the isovector forces increases in the Sn isotopes the contribution of the proton system in the $2_{1}^{+}$and $3 \frac{1}{1}$ state structure, thus increasing considerably $B(E 2)$ - and $B(E 3)$-values. It is seen from table 1 that when the Hamiltonian includes the isovector forces (i.e., $\kappa_{1}^{(\lambda)} \neq 0$, the energies and electric transition probabilities of the $2_{1}^{+}$and $3_{1}^{\overline{1}}$ states agree satisfactorily with the experimental data. The $B(E 2)-$ and $B(E 3)$-values will be equal to the experimental ones at too low energies of the $2_{1}^{+}$and $(\lambda)^{\mathbf{1}}=$ sta-
tes, if the isovector forces are excluded (i.e., $\kappa_{1}^{-}=0$ ) (see table 1). This causes a sharp increase in $n_{j}$ for some states. For instance, the neutron states $3 s_{1 / 2}$ and $2 d_{3 / 2}$ have the values of $n_{j} 53 \%$ and $46 \%$, respectively.

The aforesaid results show that the largest contribution to the number of quasiparticles $n_{j}$ in the ground state is given by the components of the first quadrupole and octupole one-phonon states. The value of this contribution depends on the collectiveness of these states. The contribution of noncollective and also of high-lying collective states to $n_{j}$ is unessential.

## 3. RESULTS AND DISCUSSION

The results of calculation of the number of quasiparticles in the ground state of ${ }^{118} \mathrm{Sn}$ are given in table 3. The galculations have taken into account the isovector forces $\left(\kappa_{1}(\lambda) \neq 0\right)$. The high energy of the $2_{1}^{+}$state $(Z=50)$ predetermines a relatively low collectiveness of this state. Table 1 shows that in ${ }^{118} \mathrm{Sn}$ the $\mathrm{B}(E 2)$-value is small. Thexefore, neither of the states $n \ell j$ given in table 3 has $n_{j}>10 \%$. The majority of states with $n_{j}>5 \%$ are the neutron states. This testifies to the fact that in ${ }^{118} \mathrm{Sn}$ the proton shell is closed. The contribution of the $2 \overline{1}$ and $3 \overline{1}$ states to $n_{j}$ is small for the protons. The exception is the $2 d_{5 / 2}$ subshell. For the neutrons the contribution of the $2_{1}^{+}$and $3_{1}^{\frac{2}{1}}$ states is much larger and in many cases it is dominating.

Table 3.
Number of quasiparticles in the ground state $n_{j}$ in ${ }^{118} \mathrm{Sn}$

| Neutron system |  |  |  | Proton system |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n \ell j$ | $\begin{gathered} \mathbf{n}_{\mathrm{j}}, \\ \% \end{gathered}$ | contri- <br> bution $2_{1}^{+}, \%$ | contri- <br> bution <br> $3{ }_{1}^{-}$, \% | n $\ell \mathbf{j}$ | $\begin{gathered} \mathrm{n}_{\mathrm{j}} \\ \% \end{gathered}$ | contri- <br> bution $2_{1}^{+}, \%$ | contribution 3-1. \% |
| 1896 | 2,11 | 0,41 | 0,41 | 1/5/2 | 1,77 | 0,46 | 0,51 |
| 2d5/2 | 5,16 | 1,53 | 1,54 | $2 p_{3 / 2}$ | 2,37 | 0,50 | 0,73 |
| $1 \mathrm{~g} 7 / 2$ | 3,44 | 1,67 | 0,32 | $2 p^{1 / 2}$ | 2,99 | 0,29 | 1,26 |
| $3 s_{1 / 2}$ | 5,97 | 3,47 | 0,73 | $1 \mathrm{~g} 9 / 2$ | 6,67 | 0,30 | 1,48 |
| $2 d_{3 / 2}$ | 6,37 | 4,18 | 0,41 | $2 d s / 2$ | 4,11 | 2,24 | 0,74 |
| 1 hty | 6,92 | 3,60 | 1,12 | $197 / 2$ | 5,82 | 0,49 | 0,71 |
| $2 \mathrm{fr/2}$ | 2,04 | 0,40 | 0,59 | 1 hal/ | 2,80 | 0,76 | 0,96 |

The values of the number of quasiparticles $n_{j}$ in the ground state of ${ }^{126} \mathrm{Te}$ for different neutron and proton subshells are given in fig. 2. The figure shows the dependence of the $n_{j}$-values on the position of the subshell energy with respect to the chemical potential. The largest $n_{j}$-values have the subshells lying neax the chemical potential. It is seen from table 1 that the $2_{1}^{+}$state in ${ }^{126}$ Te is more collective than in ${ }^{118} \mathrm{Sn}$. Therefore, the number of quasiparticles in the ground state of ${ }^{126} \mathrm{Te}$ is larger than in ${ }^{118} \mathrm{Sn}$. The contribution of the $2_{1}^{+}$state to $\mathrm{n}_{\mathrm{j}}$ is increased. For instance, for the neutron $2 \mathrm{~d}_{3 / 2}$ and proton $2 \mathrm{~d}_{5 / 2}$ subshells the contribution of the $2_{1}^{+}$phonon to $\mathrm{n}_{\mathrm{j}}$ is dominating.

The results of calculation of the number of quasiparticles $n_{j}$ in the ground states of the Sm isotopes are given in table 4. The table shows the subshells lying near the chemical potential and having the largest values of $n_{j}$. The investigations have been performed for the $\operatorname{Sm}$ isotopes with the mass numbex A from 144 to 150 . In ${ }^{144} \mathrm{Sm}$ the neutron shell $\mathrm{N}=82$ is closed and the $2_{1}^{+}$and $3 \overline{1}$ states are weakly collectivized. The ${ }^{150}$ Sm nucleus contains evident collective properties of the 10 west excitations. It refers to the region of transitional nuclei. Table 1 shows the increase in the $B(E 2)-$ and $B(E 3)$-values in the Sm isotopes with increasing $A$. Therefore, the number of quasiparticles $n_{j}$ in the ground states also increases. It is seen from table 4 that $n_{j}$ for ${ }^{144} \mathrm{Sm}$ does not exceed 10\%. In ${ }^{150} \mathrm{Sm}$ several states have the values of $\mathrm{n}_{\mathrm{j}}$ exceeding 20\%.

Table 4.
Number of quasiparticles (in percent) in the ground states of the Sm isotopes


Based on the calculations, one can conclude that condition (9) is violated in the transitional nuclei ( ${ }^{150} \mathrm{Sm}$ ), and the RPA cannot be used for the calculation of the $2_{1}^{+}$and $3_{1}^{-} \ldots$ state properties. The increase in the number of quasiparticles $\mathrm{n}_{\mathrm{j}}$ in the ground states of the Sm isotopes with increasing $A$ is caused by the increase in the contribution to $n_{j}$ of the first quadrupole and octupole phonons, which is dominating. Therefore, in the transitional nuclei the $2_{1}^{+}$and $3_{1}^{-}$state characteristics should be calculated using a more correct approximation than the RPA.

## 4. CONCLUSION

The above calculations have shown that the number of quasiparticles $n_{j}$ in the ground states of doubly even nuclei is small in the nuclei with one closed shell and in the neighbouring nuclei. The main contribution to $n_{j}$ is given, as a rule, by the first quadrupole and octupole phonons.

A considerable decrease in the number of quasiparticles in the ground state isy caused by decreasing of collectiveness in
the lowest excited states. For instance, the inclusion of the isovector forces in ${ }^{118} \mathrm{Sn}$ and of the anharmonic corrections in ${ }^{126}$ Te caused a noticeable redistribution of the $n_{j}$-values. It would be interesting to study the influence of the interaction in the particle-particle channel on the number of quasiparticles in the ground state. The inclusion of the interaction in the particle-particle channel results mainly in the decrease of collectiveness of the $2_{1}^{+}$and $\mathbf{3}_{1}$ states. Therefore, its inclusion is expected to broaden the region of fulfillment of condition (9).

The ground states of transitional doubly even nuclei contain a large number of quasiparticles. This especially concerns the subshells lying neax the chemical potentials in the neutron and proton systems. In these nuclei one cannot assume that $\langle 0| B(\mathrm{ij} 00)|0\rangle=0$, and therefore, the RPA should not be used for calculating the low-lying state characteristics.

## ACKNOWLEGMENTS

The authors are grateful to A.I.vdovin for useful discussions.

## REFERENCES

1. Ken-ji Hara. Prog.Theor. Phys., 1964, 32, p.87. Ikeda K.. Udagawa T., Yamamura H. Prog. Theor. Phys., 1965, 33. p. 22.
2. Miyanishi Y., Yamamura M. Prog.Theor. Phys., 1967, 38, p. 332.
3. Rowe D.J. Phys.Rev., 1968, 175, p. 1283.
4. Johnson R.E., Dreizler R.M., Klein A. Phys.Rev., 1969, 186, p. 1289.
5. Parikh J.C., Rowe D.J. Phys.Rev., 1968, 175, p. 1293. Schalow G., Yamamura M. Nucl. Phys., 1971, A161, p.93.
6. Hernandez E.S., Plastino A. Phys.Lett. , 1972, 39B, p.163; Z. Physik, 1974, 268, p.337; Z.Physik, 1975, A273, p. 253.
7. Соловьев В. Г. ЭЧАЯ, 1978, 9, с.580; Nucleonika, 1978, 23, p. 1149.
8. Soloviev V.G. JINR, E4-12623, Dubna, 1979.
9. Soloviev V.G. Nuci.Phys., 1965, 69, p.1;

Atomic Energy Rev., 1965, 3, No.2, p. 117.
10. Soloviev V.G., Stoyanov Ch., Vdovin A.I. Nucl. Phys., 1977, A288, p. 376.
Ponomarev V.Yu. et al. Nucl.Phys., 1979, A323, p.446.
11. Nucl. Data Tables., 1972, 11, p. 281.
12. Соловьев В.Г. Теория сложных я्रдер. "Наука", М., 1971.
13. Вдовин А.И., Кырчев Г., Стоянов Ч. ТМФ, 1974, 21, с.137.
14. Вдовин А.И., Стоянов Ч. Изв. АН СССР (сер.физ.), 1974, 38, с. 2604.
15. Fedotov S.I. et al. JINR, D-9682, Dybna, 1976, v.1, p. 120. 16. Вдовин А.И. и др. Изв. АН СССР (сер.физ.) , 1976, 40, с. 2183.

