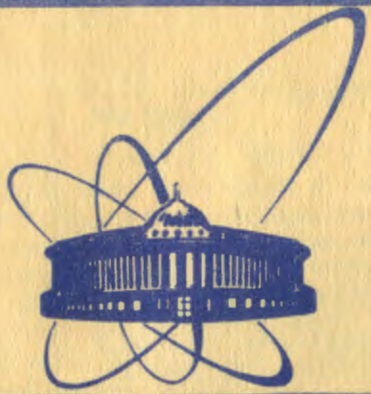

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UNITARY APPROACH TO THE DESCRIPTION OF THE PION-NUCLEUS ELASTIC SCATTERING

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1. Most approaches to the theory of elastic pion-nucleus scattering have used a certain form of the multiple scattering theory (MST)[1] in which the nuclear target is treated as a system containing elementary subsystems (i.e., nucleons). The basic physical quantity calculated in MST is the submatrix T_0 of the general scattering matrix T, which describes the elastic scattering channel. For calculation of the matrix T_0 one usually introduces the idea of a theoretical optical potential[1-4]. The optical potential is expressed in terms of the basic pion-nucleon collision matrix.

In general, the optical potential is a non-Hermitian operator. Its non-Hermitian part arises due to the contribution of possible inelastic channels to the elastic one. In practical construction of the optical potential one usually makes a set of approximations, like the impulse, the approximation linear with respect to the π-nucleon t-matrix, and so on. The optical potential thus calculated leads to the scattering amplitude which does not satisfy the general requirements[4,5] following from the unitarity condition. Therefore, there arises a complicated problem of studying the content of the non-Hermitian part of the optical potential[6].

In the description of π-nucleus interaction one must very carefully take into account the unitarity condition, because the pions can be absorbed in nuclear matter (unlike, e.g., the nucleons). So, if any theory of π-nucleus scattering even at the potential level leads to the nonunitary scattering amplitude, then the problem of incorporation of the absorption in such a scheme cannot be resolved in a consistent way*. In this situation it will be helpful to apply to the approach of description of π-nucleus elastic scattering recently presented in[7]. This approach is based on the so-called method of evolution with respect to coupling constant (CCE) (see review[8]). It is important that in the CCE method one can directly derive an equation for the phase shifts and binding energies. In this aspect the CCE-approach is similar to

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*This remark has nothing to do with the analysis of the πd-scattering based on the Faddeev equations where this problem can be solved exactly.

the well-known phase function method in potential scattering theory [9,10]. One of the main results of [7] is the construction of a new iteration scheme for the calculation of the π-nucleus scattering amplitude. In this scheme, in contrast to the MST [1], the scattering matrix is unitary of each stage of successive approximations. The analysis of the simplest example of low-energy πd-scattering given in [7] shows a fast convergence of this iterative series for the πd-scattering length to its exact value obtained by solving the Faddeev equations.

The calculation procedure of the π-nucleus phase shifts given in [7] is applicable to the description of processes of the quasi two-particle nature, i.e., to the low-energy region. The present paper is aimed at generalizing the approach given in [7] to arbitrary energies of the projectile. As in [7], we shall present here the nonrelativistic variant of the theory.

In Sec.2 we present the basic equation of the CCE-method. In Sec.3 the calculational scheme of the submatrix T_0 is constructed. In full analogy with the optical model method in the MST [1-4] we show that in the present approach the many-body problem of π-nucleus scattering can be reduced to the two-body one. In Sec.4 the pi-nucleus phase shifts in the first-order with respect to the so-called two-particle pi-nucleon u-matrix is obtained. In the present approach this approximation corresponds to the first-order optical potential in MST [1-4]. The spin-isospin structure of the problem and the static limit of the theory is discussed in Sec.5. It is shown that the pi-nucleus phase shifts can be expressed in terms of the pi-nucleon ones. In Sec.6 we discuss the results.

2. In the problem of π-nuclear interaction with the phenomenological πN-potential considered in the framework of the CCE method the Hamiltonian for the system is [7]:

$$H = H_0 + V + \lambda U, \quad U = \sum_{i=1}^{A} U^i , \tag{1}$$

where H_0 is the free Hamiltonian; V describes the NN-interaction; U^i, the pion interaction with an i-th nucleon, and λ plays the role of the pion-nucleon coupling constant. The parameter λ is taken unity at the end of calculations.

We shall assume the solution of the pure nuclear problem with the Hamiltonian

$$h = H_0 + V \tag{2}$$

to be known. So, the system evolution in coupling constant from $\lambda=0$ with the π-nuclear interaction switched-off to the realistic value $\lambda=1$ must be considered.

Let eigenfunctions of H(1) be $|\mu>, |\nu>$, etc., and matrix elements of the potential U over them $U_{\mu\nu}$, etc. The scattering S-matrix of the transition from state $|\mu>$ to $|\nu>$ (see ref. [8]) obeys the equation:

$$\frac{d}{d\lambda} S_{\mu\nu} = -2\pi i \sum_{\sigma} S_{\mu\sigma} U_{\sigma\nu} \delta(E_{\sigma} - E_{\nu}), \quad E_{\mu} = E_{\nu}, \tag{3}$$

with the boundary condition: $S_{\mu\nu}(\lambda=0) = \delta_{\mu\nu}$. For the T-matrix defined on the isoenergetic surface $E_{\mu}=E_{\nu}$ by the relation:

$$S_{\mu\nu} = \delta_{\mu\nu} + 2\pi i \delta(E_{\mu} - E_{\nu}) T_{\mu\nu}$$

the equation is:

$$\frac{d}{d\lambda} T_{\mu\nu} = -U_{\mu\nu} - 2\pi i \sum_{\sigma} T_{\mu\sigma} U_{\sigma\nu} \delta(E_{\sigma} - E_{\nu}) \tag{4}$$

with the boundary condition $T_{\mu\nu}(\lambda = 0) = 0$. The potential matrix element $U_{\mu\nu}$ in (3) and (4) obeys the relation:

$$\frac{d}{d\lambda} U_{\mu\nu} = \sum_{\sigma} U_{\mu\sigma} U_{\sigma\nu} \left(\frac{1}{E_{\mu} - E_{\sigma} - i\delta} + \frac{1}{E_{\nu} - E_{\sigma} + i\delta}\right). \tag{5}$$

The boundary condition for $U_{\mu\nu}$ at $\lambda = 0$ is the matrix element of the potential U over the eigenfunctions of Hamiltonian h(2). The energies $E_{\mu,\nu,\sigma}$ is the exact eigenvalue of H defined by the known equation $dE_{\mu}/d\lambda = U_{\mu\mu}$.

In [7] some iteration procedure for solving eq. (5) was developed. The expansion of $U_{\mu\nu}$ obtained in [7] can be presented in the form:

$$U_{\mu\nu}(\lambda) \equiv <\psi_{\mu}|\hat{U}(\lambda)|\psi_{\nu}>, \tag{6}$$

where the operator $\hat{U}(\lambda)$ is

$$\hat{U}(\lambda) = \sum_{n,m} |n>_0 O_{nm} {}_0<m|, \tag{7}$$

$$O_{nm}(\lambda) = \sum_{i=1}^{A} u^{i}_{nm}(\lambda) + \sum_{i\neq j=1}^{A} \sum_{s} \int_0^{\lambda} d\lambda_1 \times$$

$$\times u^{i}_{ns}(\lambda_1) u^{j}_{sm}(\lambda_1) \left(\frac{1}{E_n - E_s - i\delta} + \frac{1}{E_m - E_s + i\delta}\right) + \ldots . \tag{8}$$

The vector states $|\psi_{\mu}>$, $|\psi_{\nu}>$, etc., and $|m>_0$, $|n>_0$ in eqs. (6) and (7) are, respectively, eigenfunctions of the Hamilto-

nian h (2) and the free Hamiltonian H_0. The eigenfunctions and the eigenvalues of the channel Hamiltonian $(H_0+\lambda U)$ are denoted by Latin indices. The $u^i_{mn}(\lambda)$ is the exact two-body matrix element of the pion interaction with an i-th nucleon. The remaining nuclear nucleons are supposed to be free. Below we shall call this object briefly as a two-body u-matrix. It obeys an equation like (5) and on the isoenergetic surface $E_n = E_m$ defines the pi-nucleon phase shifts by the relation*:

$$\delta(\kappa) = -\pi\epsilon_2(\kappa)\int_0^1 d\lambda\,\{u_{mn}(\lambda)\}, \qquad (9)$$

where $|m>=|\vec{\kappa},-\vec{\kappa}>$, $|n>=|\vec{\kappa}',-\vec{\kappa}'>$ are solutions of the two-body scattering problem, $\epsilon_2(\kappa)=\mu\kappa/2\pi^2$ is the level density, μ, the reduced mass, $\vec{\kappa}$ and $\vec{\kappa}'$ are momenta in the c.m.s. before and after collision $(\kappa=|\vec{\kappa}|=|\vec{\kappa}'|)$. In (8) we present only two first iterations. One can easily calculate the next ones. Graphically the iteration series (8) is given in [7]. Each term in (8) is Hermitian and has correct analytic properties with respect to energy variables.

Let us obtain now some useful in the future representations of the S- and T-matrices. In view of (6) the matrix elements $S_{\mu\nu}$ and $T_{\mu\nu}$ can be presented in the form:

$$S_{\mu\nu}(\lambda)=<\psi_\mu|\hat{S}(E,\lambda)|\psi_\nu>,\ T_{\mu\nu}(\lambda)=<\psi_\mu|\hat{T}(E,\lambda)|\psi_\nu> \qquad (10)$$

on the isoenergetic surface $E=E_\mu=E_\nu$. Substituting (6) and (10) into eqs. (3) and (4) gives the equations for the operators $\hat{S}$ and $\hat{T}$:

$$d\hat{S}(E,\lambda)/d\lambda = -2\pi i\hat{S}(E,\lambda)\delta(E-h)\hat{U}(\lambda), \qquad (11)$$

$$d\hat{T}(E,\lambda)/d\lambda = -\hat{U}(\lambda)-2\pi i\hat{T}(E,\lambda)\delta(E-h)\hat{U}(\lambda), \qquad (12)$$

with the boundary conditions $\hat{S}(E,\lambda=0)=\hat{I}$ and $\hat{T}(E,\lambda=0)=0$. Here we take into account the completeness of the set of vector states $|\psi>$ and their independence of λ. The Hamiltonian h is defined in (2).

Eqs. (11) and (12) can be solved by iteration method expanding operators $\hat{S}$ and $\hat{T}$ in powers of the interaction operator $\hat{U}$:

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*Here and in Secs.3-5 we omit indices of the momentum, spin,... in relations like (9). The braces in (9) denote an appropriate partial harmonics of the matrix element.

$$\hat{S}(E,\lambda) = T_\lambda \exp[-2\pi i \int_0^\lambda d\lambda_1 \delta(E-h)\,\hat{U}(\lambda_1)], \qquad (13)$$

$$\hat{T}(E,\lambda) = -T_\lambda \int_0^\lambda d\lambda_1 U(\lambda_1) \exp[-2\pi i \int_{\lambda_1}^\lambda d\lambda_2 \delta(E-h)\hat{U}(\lambda_2)], \qquad (14)$$

where T_λ is the operator anti-ordering with respect to the variable λ[11]. Under this symbol the arguments λ of the operators must increase from left to right. Operators $\hat{S}$, $\hat{T}$ and $\hat{U}$ are defined in the space of eigenfunctions of the channel Hamiltonian h(2). In this space $\hat{S}$ is a unitary operator owing to (13) where operators h and $\hat{U}$ are Hermitian. It should be noted that its unitarity is conserved in the framework of iteration procedure (8).

3. The calculation of the phase shifts by eq. (9) is appropriate in the case of two-particle scattering[8]. This was used in ref.[7] in the study of low-energy pion-nucleus scattering. But in the general case the calculation of the pi-nucleus phase shifts by an equation like (9) is invalid. This can be easily seen from the r.h.s. of eqs. (3) and (4), where the summation goes over all possible π-nucleus system states at a given energy. Here we present a generalization of a simple relation like (9) to the case of the multichannel problem.

We introduce now a more detailed notation for the meson-nucleus states: $|\psi\rangle \equiv |\vec{k}, n\rangle$, where $\vec{k}$ labels the meson momentum (in the pi-nucleus c.m.) and n labels the properly antisymmetrized nuclear state ($n = 0,1,2,\ldots$; $n = 0$ denotes the ground state). Our goal is to investigate the elastic scattering process, i.e., the transitions from state $|\vec{k}, 0\rangle$ to the $|\vec{k}', 0\rangle$, where $\vec{k}$ and $\vec{k}'$ are the pion momenta before and after collision. To do this, let us introduce the projection operator for the ground nuclear state $\hat{P} = |0\rangle\langle 0|$ and the projection operator $\hat{Q} = 1 - \hat{P} = \sum_{n\neq 0} |n\rangle\langle n|$. Then the operators $\hat{S}_0$ and $\hat{T}_0$ corresponding to $n \neq 0$ elastic channel can be defined as follows:

$$\hat{S}_0 = \hat{P}\hat{S}\hat{P} \quad \text{and} \quad \hat{T}_0 = \hat{P}\hat{T}\hat{P}. \qquad (15)$$

Let us also introduce a new effective operator $\hat{U}_0$ that must define $\hat{S}_0$ and $\hat{T}_0$ matrix by the following equations:

$$\frac{d}{d\lambda}\hat{S}_0(E,\lambda) = -2\pi i\,\hat{S}_0(E,\lambda)\delta(E-h)\,\hat{P}\hat{U}_0(E,\lambda), \qquad (16)$$

$$\frac{d}{d\lambda}\hat{T}_0(E,\lambda) = -\hat{U}_0(E,\lambda) - 2\pi i\,T_0(E,\lambda)\delta(E-h)\hat{P}\hat{U}_0(E,\lambda), \qquad (17)$$

with the boundary conditions $\hat{S}_0(\lambda=0)=\hat{I}$ and $\hat{T}_0(\lambda=0)=0$. Thus, the intermediate states in (16) and (17) are restricted by $\hat{P}$ to be the nuclear ground state only. Consequently, (16) and (17) are simply two-body scattering equations for the pi-nucleus system in the CCE-approach. The formal solution of eqs. (16) and (17) can be obtained from (13) and (14) by replacing $\hat{U}$ by $\hat{P}\hat{U}_0$, i.e.,

$$\hat{S}_0(E,\lambda)=T_\lambda \exp[-2\pi i \int_0^\lambda d\lambda_1 \delta(E-h)\hat{P}\hat{U}_0(E,\lambda)]. \qquad (18)$$

The effects of nuclear excitations are incorporated into the definition of the effective operator $\hat{U}_0$. By the direct substitution one can verify that the relations (15) will be satisfied by solutions of eqs. (16) and (17) if $\hat{U}_0(E,\lambda)$ obeys the equation:

$$\hat{U}_0=\hat{U}+2\pi i\hat{S}_0^{-1}\hat{P}\hat{T}\delta(E-h)\hat{Q}\hat{U}, \qquad (19)$$

where the operator $\hat{S}_0^{-1}$ inverse to $\hat{S}_0$ is:

$$\hat{S}_0^{-1}(E,\lambda)=T_\lambda^{-1}\exp[2\pi i\int_0^\lambda d\lambda_1\delta(E-h)\hat{P}_0\hat{U}_0(E,\lambda)].$$

The symbol T_λ^{-1} is the operator "ordering" with respect to λ. Under it the arguments of the operators decrease from left to right. Operators $\hat{U}$ and $\hat{T}$ are defined respectively in (7) and (14). In the matrix form eq. (19) is as follows:

$$\langle\vec{k},0|\hat{U}_0|\vec{k}',0\rangle=\langle\vec{k},0|\hat{U}|\vec{k}',0\rangle+2\pi i\sum_{n\neq 0}\int\frac{d\vec{k}_1}{(2\pi)^3}\times$$

$$\times\int\frac{d\vec{k}_2}{(2\pi)^3}\langle\vec{k},0|\hat{S}_0^{-1}|\vec{k}_1,0\rangle\langle\vec{k}_1,0|\hat{T}|\vec{k}_2,n\rangle\delta(E(k)-E_n(k_2))\langle\vec{k}_2,n|\hat{U}|\vec{k}',0\rangle, \qquad (20)$$

where $E(k)=E(k')=k^2/2\mathfrak{M}-\epsilon_0$ is the collision energy in the pi-nucleus center-of-mass system, $-\epsilon_0$ is the nuclear ground state energy, $\mathfrak{M}$ denotes the reduced mass and $E_n(k)=k^2/2\mathfrak{M}-\epsilon_n$ $(n=1,2,3,\ldots)$, where $-\epsilon_n$ is the energy of the n-th excited nuclear state.

By eqs. (16)-(20) the many-body scattering problem is reduced to the two-body one. Thus, if one passes from these operator equations the matrix ones, as it has been done in (20), and makes a partial wave decomposition, then one gets the following simple relation for the pi-nucleus phase shifts:

$$\delta(k) = -\pi\epsilon_A(k)\int_0^1 d\lambda\{\langle\vec{k},0|\hat{U}_0(E,\lambda)|\vec{k}',0\rangle, \quad (21)$$

where $\epsilon_A(k) = \mathfrak{M}\cdot k/2\pi^2$ denotes the level density, $\mathfrak{M}$ is the reduced pion-nucleus mass, and k, k' are the momenta of pion before and after scattering.

Eq. (19) can be solved by iteration method by expanding $\hat{U}_0(E,\lambda)$ in powers of the known operator $\hat{U}(7)$. Two first terms of this series are:

$$\hat{U}_0(\lambda) = \hat{U}(\lambda) - 2\pi i\int_0^\lambda d\lambda_1\,\hat{U}(\lambda_1)\delta(E-h)\hat{Q}\hat{U}(\lambda). \quad (22)$$

For the phase shifts (21) in this approximation we get:

$$\delta(k) = -\pi\epsilon_A(k)\int_0^1 d\lambda\{\langle\vec{k},0|\hat{U}(\lambda)|\vec{k}',0\rangle - \quad (23)$$

$$-2\pi i\sum_{n>0}\int_0^\lambda d\lambda_1\int\frac{d\vec{k}''}{(2\pi)^3}\langle\vec{k},0|\hat{U}(\lambda_1)|\vec{k}'',n\rangle\times$$

$$\times\langle\vec{k}'',n|\hat{U}(\lambda)|\vec{k}',0\rangle\delta(E(k)-E_n(k'')),$$

(with the same notation as in (20)).

From eq. (19) it follows that $\hat{U}_0$-operator is in general the non-Hermitian operator. Hence, $\hat{S}_0$ (18) is the non-unitary operator. The non-Hermitian part of $\hat{U}_0$ represents the contribution of inelastic to the elastic channel (by virtue of $\hat{Q}$ in (19)). However, the contribution of the second term in the r.h.s. of eqs. (19), (20) and (22) disappears when the projectile cannot excite any nuclear state, i.e., $k^2 < \sqrt{2\mathfrak{M}(\epsilon_0-\epsilon_1)}$, where $-\epsilon_1$ is the first nuclear excited state energy. At this limit $\hat{U}_0 = \hat{U}$ becomes the Hermitian operator and $\hat{S}_0$ the unitary one. Consequently, in the low-energy limit the two-body unitarity condition is justified in our approach. As the pion energy will increase the second term in the r.h.s. of (19), (20) and (22) will give a contribution. In this way, the imaginary part of the phase shift arises.

4. In view of the basic expansion (8) of $\hat{U}$ it is natural to obtain a similar expansion for the phase shifts directly:

$$\delta(k) = \sum_{n=1}^{\infty}\delta^{(N)}(k), \quad (24)$$

where N denotes the power of the two-body u-matrix. One can easily get the expressions for $\delta^{(N)}$ by substituting (8) into (23) and grouping terms at a given power of u-matrix.

Here we shall consider the first term $\delta^{(1)}(k)$. In the first approximation for the nucleus phase shifts using antisymmetrized target wave function we have

$$\delta^{(1)}(k) = -A\pi\epsilon_A(k) \int_0^1 d\lambda \{\langle \vec{k}, 0| u^1(\lambda) |\vec{k}', 0\rangle\}, \tag{25}$$

where the two-body u-matrix $u^1(\lambda)$:

$$u^1(\lambda) = \sum_{n,m} |n\rangle_0 u^1_{nm}(\lambda) {}_0\langle m|, \tag{26}$$

describes the pion interaction with the first nucleon.

The matrix element in (25) has the same structure as the first-order optical potential (see refs.[1-4]). The latter arises from substitution of the two-body t-matrix for the u-matrix. In order to compute this matrix elements, it is advantageous to introduce the Jacobi coordinates:

$$\begin{aligned}
&\vec{p}_n = \frac{A-n}{A-n+1}\left(\frac{\vec{P}_n}{A-n} - k_n\right), \\
&\vec{P}_n = \sum_{m=0}^{A-n-1} \vec{k}_{A-m}, \qquad n = 1,2,\ldots,A-1; \\
&\vec{p}_0 = \mathcal{M}\left(\frac{\vec{P}_0}{AM} - \frac{\vec{k}_0}{m}\right), \quad \vec{P}_0 = \sum_{m=1}^{A} \vec{k}_m, \quad \vec{P} = \vec{P}_0 + \vec{k}_0,
\end{aligned} \tag{27}$$

where $\vec{k}_0$ labels the pion momentum, $\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_A$ label the nucleon momenta. Thus p_0 in (27) denotes the relative pion-nucleus momentum, p_1 the relative momentum of the first nucleon and the remaining (A-1) nucleons, etc. In the π-nucleus c.m. system the total momenta $P=0$ and the wave functions in (25) are as follows:

$${}_0\langle m|\vec{k}, 0\rangle = (2\pi)^3 \delta(\vec{p}_0^{(m)} - \vec{k})\, \psi_0(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_{A-1}). \tag{28}$$

To express the two-particle matrix element in (26), we introduce the Jacobi coordinates corresponding to scheme [(A-1), (1, 0)]. In this scheme the relative pion-first nucleon momentum and the momentum of this pair relative to the remaining (A-1) nucleons are defined as:

$$\vec{\kappa} = \mu\left(\frac{\vec{k}_1}{M} - \frac{\vec{k}_0}{m}\right), \quad \vec{Q} = \mathcal{M}_{A-1}\left(\frac{\vec{P}_1}{(A-1)M} - \frac{\vec{k}_0 + \vec{k}_1}{M+m}\right), \tag{29}$$

where the reduced masses are: $\mu \equiv mM/(m+M)$, $\mathfrak{M}_{A-1} \equiv$ $\equiv (A-1)M(m+M)/(m+AM)$, the momentum $\vec{P}_1 = \sum_{i=2}^{A} \vec{k}_i$ and the remaining nucleon Jacobi coordinates $p_2, p_3, \ldots, p_{A-1}$ are defined in (27). In these variables for the two-boby matrix element u^1_{nm} we have:

$$u^1_{nm}(\lambda) = (2\pi)^3 \delta(\vec{Q}^{(n)} - \vec{Q}^{(m)}) \langle \vec{\kappa}^{(n)} | u(\lambda) | \vec{\kappa}^{(m)} \rangle \times$$
$$\times \prod_{i=2}^{A-1} (2\pi)^3 \delta(\vec{p}_i^{(n)} - \vec{p}_i^{(m)}) . \qquad (30)$$

The sum in (26) means the integration over the momenta of particles in intermediate states*: $\sum_m = (2\pi)^{-3A} \int \prod_{i=0}^{A-1} d\vec{p}_i^{(m)}$

Using (28) and (30), the matrix element in (25) is:

$$\langle \vec{k}, 0 | u^1(\lambda) | \vec{k}', 0 \rangle = \int \frac{d\vec{p}}{(2\pi)^3} F_{00}(\vec{p} - \frac{A-1}{A}\vec{k}, \vec{p} - \frac{A-1}{A}\vec{k}') \times$$
$$\times \langle \vec{k} - \omega\vec{p} | u(\lambda) | \vec{k}' - \omega\vec{p} \rangle , \qquad (31)$$

where the overlap function F_{00} is:

$$F_{00}(\vec{p}, \vec{p}') = \int \frac{d\vec{p}_2}{(2\pi)^3} \ldots \frac{d\vec{p}_{A-1}}{(2\pi)^3} \psi_0^*(\vec{p}, \vec{p}_2, \ldots, \vec{p}_{A-1}) \psi_0(\vec{p}', \vec{p}_2, \ldots, \vec{p}_{A-1}) \qquad (32)$$

and the parameter $\omega = m/(M+m)$. In (31) and (32) the summation over spin-isospin variables is also implied. The spin-isospin structure of the problem will be discussed in Sec.5.

The expression (31) contains the small parameter $\omega \simeq 0.13$. Let the static limit of theory /7/ be $\omega \to 0$. In this limit (31) becomes:

$$\langle \vec{k}, 0 | u^1(\lambda) | \vec{k}', 0 \rangle = \langle \vec{k} | u(\lambda) | \vec{k}' \rangle \rho(\vec{q}) , \qquad (33)$$

where

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* This is correct if the pion-nucleon "bound-states" (isobar) are not taken into account.

$$\rho(\vec{q}) = \int \prod_{i=1}^{A-1} \frac{d\vec{p}_i}{(2\pi)^3} \psi_0^*(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_{A-1}) \times \tag{34}$$

$$\times \psi_0(\vec{p}_1 + \frac{A-1}{A}\vec{q}, \vec{p}_2, \ldots, \vec{p}_{A-1})$$

is the nuclear form factor, $\vec{q} = \vec{k} - \vec{k}'$ denotes the transfer momentum. Note that the two-body u matrix in (33) is defined on the isoenergetic surface ($|\vec{k}| = |\vec{k}'|$). Substituting (33) into (25) gives the following expression for the phase shifts:

$$\delta^{(1)}(k) = -A\pi\epsilon_A(k) \{\rho(\vec{q}) \int_0^1 d\lambda \langle \vec{k}|u(\lambda)|\vec{k}'\rangle . \tag{35}$$

5. The spin and isospin dependence of the pion-nucleon two-body u-matrix can be expressed in terms of spin and isospin operators:

$$O_0 = 1, \quad O_1 = \vec{t}\vec{\tau}, \quad O_2 = i\vec{\sigma}\vec{n}, \quad O_3 = i(\vec{\sigma}\vec{n})(\vec{t}\vec{\tau}), \tag{36}$$

where $\frac{1}{2}\vec{\sigma}$ is the nucleon spin operator, $\vec{t}$ and $\frac{1}{2}\vec{\tau}$ are the meson and nucleon isospin operators and unity vector $\vec{n} = [\vec{\kappa} \times \vec{\kappa}']/(\kappa \cdot \kappa')$. Thus, we can isolate the spin and isospin dependence of u by writing:

$$\langle \vec{\kappa}|U(\lambda)|\vec{\kappa}'\rangle = \sum_{\alpha=0}^{3} \langle \vec{\kappa}|u_\alpha(\lambda)|\vec{\kappa}'\rangle O_\alpha . \tag{37}$$

The matrix elements u_α are expressed in terms of the spin-dependent u_s^I and spin-independent u_c^I operators at a given isotopic πN-state $I = 1/2, 3/2$ as

$$\langle \vec{\kappa}|u_0(\lambda)|\vec{\kappa}'\rangle = \frac{1}{3}[2\langle \vec{\kappa}|u_c^{3/2}(\lambda)|\vec{\kappa}'\rangle + \langle \vec{\kappa}|u_c^{1/2}(\lambda)|\vec{\kappa}'\rangle],$$

$$\langle \vec{\kappa}|u_1(\lambda)|\vec{\kappa}'\rangle = \frac{1}{3}[\langle \vec{\kappa}|u_c^{3/2}(\lambda)|\vec{\kappa}'\rangle - \langle \vec{\kappa}|u_c^{1/2}(\lambda)|\vec{\kappa}'\rangle]. \tag{38}$$

Similar expressions for $u_{2,3}$ arise from that of $u_{0,1}$ by substitution of u_s^I for u_c^I.

In view of (37) for (33) we have:

$$\langle \vec{k}, 0|u^1(\lambda)|\vec{k}', 0\rangle = \sum_{\alpha=0}^{3} \langle \vec{k}, 0|u_\alpha(\lambda)|\vec{k}', 0\rangle \rho_\alpha(q) , \tag{39}$$

with the nuclear form factors:

$$\rho_\alpha(\vec{q}) = \int \prod_{i=1}^{A-1} \frac{d\vec{p}_i}{(2\pi)^3} \psi_0^*(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_{A-1}) O_\alpha \psi_0(\vec{p}_1 + \frac{A-1}{A}\vec{q}, \vec{p}_2, \ldots, \vec{p}_{A-1}). \quad (40)$$

In (40) the nuclear ground state wave functions are assumed to contain the spin-isospin part of the whole pion-nucleus system wave functions.

Let us consider now the pion scattering on nuclei with zeroth total spin and isospin. In this case the isospin $\vec{t}\cdot\vec{\tau}$, spin-flip $(\vec{\sigma}\cdot\vec{n})$ and spin-isospin flip $(\vec{\sigma}\vec{n})(\vec{t}\vec{\tau})$ terms do not contribute to (39). Hence, in view of (38), the pion-nucleus phase shifts (35) are:

$$\delta_L^{(1)}(k) = -A\pi\epsilon_A(k)\{\rho_0(\vec{q}) \int_0^1 d\lambda [\frac{2}{3}\langle\vec{k}|u_c^{3/2}(\lambda)|\vec{k}'\rangle + \frac{1}{3}\langle\vec{k}|u_c^{1/2}(\lambda)|\vec{k}'\rangle]\}_L, \quad L = 0,1,2,\ldots, \quad (41)$$

where $\rho_0(\vec{q})$ is the Fourier transform of the nuclear density, the symbol $\{f(x)\}_L \equiv \frac{1}{2}\int_{-1}^{1} dx P_L(x) f(x)$, where $P_L(x)$ are Legendre polynomials, $x = \hat{\vec{k}}\cdot\hat{\vec{k}}'$.

Pion-nucleus phase shifts (41) can be expressed in terms of the elementary pion-nucleon ones. To do this, it is necessary to make the partial wave decomposition of the matrix elements in (41). This decomposition is given for each meson-nucleon isospin state (labelled by I) as:

$$\langle\vec{k}|u_c^I(\lambda)|\vec{k}'\rangle = \sum_{\ell, j=\ell\pm\frac{1}{2}} (j+\frac{1}{2}) u_{Ij}^{\ell}(k,k;\lambda) P_L(\cos\theta).$$

The nuclear form factor $\rho(\vec{q})$ can also be expanded in partial waves:

$$\rho(\vec{q}) = \sum_{\ell} (2\ell+1)\rho_\ell(k) P_\ell(\cos\theta),$$

where $\cos\theta \equiv \hat{\vec{k}}\cdot\hat{\vec{k}}'$. Substituting these expressions into (41) and integrating over variable λ with the help of the basic relation (9) we obtain the desired result:

$$\delta_L^{(1)}(k) = A\frac{\mathfrak{M}}{\mu}\sum_{\ell,\ell',\, j=\ell\pm\frac{1}{2}}(j+\frac{1}{2})\begin{pmatrix} L & \ell' & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 \rho_{\ell'}(k)\times$$

$$\times\left[\frac{2}{3}\delta_{\ell_j}^{3/2}(k) + \frac{1}{2}\delta_{\ell_j}^{1/2}(k)\right], \qquad (42)$$

where $\delta_{\ell_j}^{I}$ denotes pion-nucleon phase shifts in each eigenchannel (I,ℓ,j), the pion-nucleus and pion-nucleon reduced masses are, respectively, $\mathfrak{M}$ and μ. The expression (42) is quite similar to that of the first order optical potential (see, e.g., (2.36) in ref.[12]). The latter arises from (42) by substitution of collision matrix $t_{\ell_j}^{I}$ for $\delta_{\ell_j}^{I}$. The pi-nucleus c.m. scattering amplitude is constructed by summing the contributions from various partial waves

$$f(\theta) = \frac{1}{k}\sum(2L+1)\exp[i\delta_L(k)]\sin(\delta_L(k))P_L(\cos\theta).$$

The differential cross section is then $d\sigma/d\Omega = |f(\theta)|^2$.

6. The main result of the present paper is the system of equations (16)-(21) for the calculation of pion-nucleus phase shifts. In full analogy with the optical model method in the multiple scattering theory [1-4] it reduces in the CCE-approach the many-body problem of the pi-nucleus scattering to the two-body one. By eq. (21) partial phase shifts are defined through the matrix element of the effective interaction operator $\hat{U}_0$. It obeys the exact equation (19) and plays here the role of the optical potential.

The considered here method of solution of a given equations consists in expansion in powers of the two-body pi-nucleon matrix u. One may believe in a relatively fast convergence of these iteration series. Indeed, each term of the basic expansion (8) has correct analytic properties with respect to energy variables and is Hermitian. On the other hand, the higher terms of this series the further singularities of S-matrix which give a subsequently decreasing contribution [8]. The consideration of simplest examples of πd- and nd-scattering discussed in [7,8] supports this expectation.

In Secs. 4 and 5 we have considered in detail the first order approximation to pi-nucleus phase shifts. In this approximation partial phases are expressed through the matrix element $\langle\vec{k},0|\sum u^1|\vec{k}',0\rangle$ similar in its structure to the first-order optical potential [1-4]. Unlike the latter we must know it only on the energy shell ($|\vec{k}| = |\vec{k}'|$). Hence, with the present approach, there is no an important problem of the off-shell

continuation of this matrix element. We show here (42) that the pi-nucleus phase shift can be expressed in terms of elementary pion-nucleon ones. This admits the semi-phenomenological analysis of pi-nucleus scattering in the present approach based on the experimentally defined πN-phase shifts and nuclear form factor. Results of such a description of experimental data will be presented in a subsequent paper.

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