сообщения
объединенного
института
ядерных
исследований
дубна

E4-80-527

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A CLASSICAL STATISTICAL MODEL OF HEAVY ION COLLISIONS.

I. The Basic Formulae

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1980

1. INTRODUCTION

Newton-type equations of motion can be useful tool for treating the relative motion of two nuclei [1-8]. From classical trajectory calculations many basic features of heavy ion reactions can be estimated, e.g., the amount of the kinetic energy loss, the amount of dissipated orbital angular momentum, the deflection angle and the critical angular momentum for fusion. In addition, mean interaction times obtained from such dynamical calculations have been successfully used to investigate the mass transport in deep inelastic heavy ion collisions (DIC) on the basis of a Fokker-Planck equation (FPE) [4] with mass transport coefficients from a microscopic model [5]. Newton-type equations of motion can also serve as a starting point for more refined statistical theories. In this manner various multi-differential cross-sections of DIC have been analysed [4,6-9] on the basis of a FPE in the phase space of collective degrees of freedom [10] with mean values obtained from classical trajectories. So it appears that more sophisticated classical descriptions of the relative motion of two nuclei are of interest.

In the first part of this paper we present a two-dimensional friction model of heavy ion collisions. The mass transfer between the two nuclei is included and treated dynamically. The deformation energy in the exit channel is taken into account. The friction tensor is modified to improve the classical description of the orbital angular momentum loss. The radial and tangential friction parameters and the deformation constant are connected via a relation which gives a constraint on variations of these parameters, so that for a wide range of projectile-target combinations the calculated values for the fusion cross-section as well as the total energy loss do not change, if these parameters are varied.

Including statistical fluctuations around the classical mean values as they follow from the solution of a FPE in the phase space of collective degrees of freedom [10] we calculate multi-differential cross-sections of deep inelastic reaction products. In the second part of the paper we describe the computer code TRAJEC. The code solves the classical Newton-type equations together with the FPE numerically and calculates the mean trajectory characteristics (interaction time, turning point, deflection angle, angular momentum and energy loss, scission radius, mean mass and charge number in the exit channel, critical angular momentum for fusion) as well as the multi-differential cross sections of DIC ($d\sigma/d\Omega$, $d^2\sigma/d\theta dE$, $d\sigma/dZ$, $d^2\sigma/d\theta dZ$, $d^2\sigma/d\theta sZ$, $d^3\sigma/dE\,d\theta dZ$).

2. THE MODEL

In this section we present the basic formulae of the model and state the parameters. A detailed discussion of the influence of various degrees of freedom and the variation of the model parameters on the reaction dynamics can be found elsewhere /4,8,9, 11-13/.

We consider Newton-type equations of motion in polar coordinates (r, θ) with friction terms

$$\mu \ddot{\vec{r}} = - \frac{\partial U}{\partial \vec{r}} + \overleftrightarrow{\gamma} \dot{\vec{r}} : \qquad (1)$$

The reduced mass μ, the interaction potential U and the friction tensor $\overleftrightarrow{\gamma}$ are assumed to be time dependent via the mean mass asymmetry $\langle A_1\rangle(t)$ of the system as measured by the mass number A_1 of the projectile-like fragment. The potential U and the friction tensor $\overleftrightarrow{\gamma}$ depend explicitly also on the actual distance between the centres of the two ions $r(t)$ and $\overleftrightarrow{\gamma}$ is assumed to be dependent from the actual orbital angular momentum $\ell(t) = \mu r^2 \dot{\theta}$ too:

$$\begin{aligned} \mu &= \mu(\langle A_1\rangle(t)) = A^{-1} \langle A_1\rangle \cdot (A - \langle A_1\rangle), \\ U &= U(\langle A_1\rangle(t), r(t)), \\ \overleftrightarrow{\gamma} &= \overleftrightarrow{\gamma}(\langle A_1\rangle(t), r(t), \ell(t)). \end{aligned} \qquad (2)$$

The total mass of the system is denoted by $A = A_1 + A_2$.

2.1. Mass Transport

The time dependence of the mass asymmetry A_1 of the system we get from the solution of the FPE

$$\frac{\partial P(A_1,t)}{\partial t} = - \frac{\partial}{\partial A_1}[v_A(A_1, t)\, P(A_1,t)] + \frac{\partial^2}{\partial A_1^2}[D_A(A_1,t)\, P(A_1,t)] . \qquad (3)$$

The diffusion coefficient D_A and the drift coefficient v_A have been calculated in a microscopic model by Nörenberg et al.[5]

$$D_A \approx 0.462\,[E_\ell^*/A]^{1/4}\left[\frac{(A_1A_2)^{1/3}}{A_1^{1/3}+A_2^{1/3}}\right]^2 , \tag{4}$$

$$v_A \approx -\frac{D_A}{T(E_\ell^*)}\,\frac{\partial U_\ell(A_1)}{\partial A_1}, \tag{5}$$

in units $10^{22}s^{-1}$. They depend on the mass fragmentation A_1, the total mass A, the relative angular momentum ℓ and the ℓ-dependent excitation energy E_ℓ^*, which defines a temperature $T(E^*) = 3.46[E^*/A]^{1/2}$ MeV. In the ground state energy $U_\ell(A_1)$ of the system with the mass asymmetry A_1 shell effects are neglected. The coupling between (1) and (3) is given by the mean values and the time variable. So we do not make use of analytical solutions of (3) with mean transport coefficients but calculate the time dependence of $\langle A_1\rangle(t)$ according to

$$\frac{d\langle A_1\rangle}{dt} = -\langle v_A\rangle(t) \approx -v_A(\langle A_1\rangle(t)), \tag{6}$$

that is, we calculate the transport coefficients at the actual mean values of the mass asymmetry. We neglect the mass transfer within the very short approach phase[11] and include a form factor which linearly decreases the diffusion coefficient if the excitation energy at the classical turning point is less than 20 MeV.

Assuming strong correlation between neutron and proton transfer, one obtaines the transport coefficients for protons as

$$v_z = \left(\frac{Z}{A}\right) v_A, \qquad D_z = \left(\frac{Z}{A}\right)^2 D_A \tag{7}$$

and the corresponding mean value $\langle Z_1\rangle(t)$. Z denote the total charge of the system.

2.2. The Interaction Potential and the Deformation Energy

For the nucleus-nucleus potential we take the proximity potential[14] V^I in the entrance channel

$$V^I = 2\pi(\gamma_1+\gamma_2)\,\frac{R_1\cdot R_2}{R_1+R_2}\,s_0 \begin{cases} \frac{5}{3}\left(1+\frac{s}{s_0}\right)e^{-1.6s/s_0} & \text{if } s>0 \\ \frac{5}{3}-\frac{s}{s_0}-\frac{1}{2}\left(\frac{s}{s_0}\right)^2 & \text{if } s<0, \end{cases} \tag{8}$$

$$\gamma_1 = 0.9517(1-1.78(1-2Z_1/A_1)^2) \ \text{MeV/fm}^2,$$
$$R_1 = 1.17 A_1^{1/3} \ \text{fm}, \quad s = r-(R_1+R_2), \quad s_0 \approx 1 \ \text{fm}.$$

(For simplicity we do not denote the time dependence of A_1 and Z_1 explicitly in this section). The Coulomb part of the interaction is taken $U_c = Z_1 Z_2 e^2/r$. To be able to account for the observed energy damping far below the Coulomb barrier of spherical fragments and to compute the effect of dynamical deformations on the mean interaction time, we simulate the deformation of the fragments by modifying the nuclear interaction V^I in the channel according to

$$V_E = V^I + E_d^{(\ell)} [1-g(r)]. \tag{9}$$

Here we follow the basic idea of ref. /3/, but use the proximity rather than a Woods-Saxon potential for V^I, and in addition propose a different form of the ℓ-dependent deformation energy

$$E_d^{(\ell)} = \alpha(A_1^{2/3}+A_2^{2/3})\, b_{surf} \left\{ \left| \frac{V^I(R_{ret}(\ell))}{V^I_{min}} \right| - \left| \frac{V^I(R_{ret}(\ell_{gr}))}{V^I_{min}} \right| \right\}. \tag{10}$$

It is proportional to the surface energy of the initial unperturbed system ($b_{surf} = 17$ MeV). The minimum value of the proximity potential is V^I_{min}. The postulated dependence on the distance of closest approach R_{ret} introduces an ℓ-dependence: close collisions penetrate deeply and lead to large deformations. The last term in eq. (10) ensured vanishing deformation for grazing collisions ($\ell = \ell_{gr}$).

The form factor $g(r)$ has the form

$$g(r) = \exp\left[-\left(\frac{r-R_{ret}}{b}\right)^2\right] \tag{11}$$

with $b = R_{12} - R_{ret} + \Delta$. The sum of the effective sharp radii in the proximity formula is $R_{12} = R_1 + R_2$ and the parameter $\Delta = 8$ fm is chosen such that the fusion cross section for the systems used to determine the friction parameters does not change significantly. The parameter α determines the overall strength of the deformation energy.

2.3. The Friction Tensor

The friction tensor $\overleftrightarrow{\gamma} = \{\gamma_{rr}, \gamma_{\theta\theta}\}$ is diagonal in polar coordinates

$$\gamma_{rr} = a_r f(r), \quad \gamma_{\theta\theta} = a_\theta f(r) r^2 F_{st}(\ell). \tag{12}$$

Here $f(r)$ denotes the radial form factor of the type given originally by Gross and Kalinowski [1] $f(r) = (\partial U^{I}/\partial r)^2$. The tangential or sticking form factor $F_{st}(\ell)$ is introduced in order to express several successive stages in the angular momentum dissipation (sliding, rolling, sticking) expected from a classical picture. It is chosen in such a way that the angular momentum transfer terminates when the classical sticking limit is reached for the orbital angular momentum loss [11]

$$F_{st}(\ell) = 1 - \exp[-(\frac{\ell(t)-\ell_{st}}{\Delta\ell})^2] \quad (13)$$

with

$$\ell_{st} = \ell_i [J_{rel} / (J_{rel} + J_1 + J_2)], \quad (14)$$

where ℓ_i, J_k ($k = 1, 2$) and J_{rel} denote the initial relative angular momentum, the classical intrinsic and relative moments of inertia, respectively. The parameter $\Delta\ell = 5$ determines the range of partial waves in which the form factor $F_{st}(\ell)$ effectively reduces the strength of the tangential friction.

2.4. Friction and Deformation Parameters

A reasonable set of parameters is

$$a_R = 12 \text{ fm/c MeV}, \quad a_\theta = 0.22 \text{ fm/c MeV}, \quad a = 0.17. \quad (15)$$

This values for friction constants a_R, a_θ and the deformation parameter a are connected with those using earlier [8,4] according to the relations

$$a_\theta = a_R (a_R - 4.26)\ 414, \quad a = -0.023 a_R + 0.448 \quad (16)$$

(friction constants in fm/c MeV). These useful relations, which have been found empirically, give a constraint on variations of the parameters a_R, a_θ, a, so that for a wide range of projectile-target combinations the calculated values for the fusion cross section as well as the total energy loss do not change if these parameters are varied (details will be published elsewhere) [12]. A physical reasonable range for a_R is given by $a_R = 6 ... 15$ fm/c MeV.

2.5. The Interaction Time, Interaction and Scission Radii

To extract mean interaction times as function of the initial angular momentum ℓ_i, the values of interaction radius R_{int}

and scission radius R_{sc} have to be known. The interaction radius R_{int} can be taken from the quarter-point analyses of elastic scattering or estimated according to

$$R_{int} = 0.5 + 1.36 (A_1^{1/3} + A_2^{1/3}) \ . \qquad (17)$$

Since in our treatment of the deformation shapes are not specified, the determination of the scission radius involves uncertainty. We determine R_{sc} from the condition that the calculated exit-channel kinetic energy E_f is equal to the Coulomb plus exit-channel centrifugal energy at the scission radius [4]

$$R_{sc} = (2E_f)^{-1} \{ Z_1 Z_2 e^2 + [(Z_1 Z_2 e^2)^2 + 2E_f \ \ell_f^2/\mu]^{1/2} \} \qquad (18)$$

with the reduced mass μ and the exit-channel orbital angular momentum ℓ_f.

2.6. Statistical Fluctuations in the Relative Motion (Cross Sections)

Statistical fluctuations in the relative motion can be calculated by solving a generalized FPE in the phase space of the collective coordinates $\{r, \theta\}$ and corresponding conjugate momenta $\{P_r, P_\theta\}$, as obtained by Hofmann and Siemens [10] within linear response theory. The solution $d(t, r, \theta, P_r, P_\theta)$ of this FPE is a Gaussian with mean values as they have been obtained from the solution of the Newton-type equation (1). For the second moments a coupled set of first order differential equations can be derived [6,8]. The coupling between the internal degrees of freedom and the collective variables is described by a time dependent temperature (cf. sect.2.1) which is calculated with the internal excitation energy $E^*_\ell(t)$ produced by friction and increasing in time.

The total distribution function including the mass asymmetry degree of freedom as discussed in sect.2.1 can be written as

$$\sigma(t) = d(t, r, \theta, P_r, P_\theta, \langle A_1 \rangle (t)) \, P(t, A_1, \langle P_r \rangle, \langle P_\theta \rangle) \qquad (19)$$

provided relative motion and nucleon transfer are statistically uncorrelated. The coupling between relative motion and nucleon diffusion is given by the mean values and the time variable t.

From eq. (19) one obtains triple differential cross sections as

$$\frac{d^3\sigma}{dE\,d\theta\,dZ_1} = \frac{2\pi}{k^2}\left(\frac{\mu}{2E_f}\right)^{1/2} \int_{\ell_{cr}}^{\ell_{max}} \ell d\ell\, P_\ell\,(t = r_{int}\,(\ell)) \int d_\ell(t\to\infty)\, dr\, dP_\theta \,. \quad (20)$$

Here E_f is the c.m. energy of the fragments, k denotes the wave number of the relative motion, ℓ_{cr} is the calculated critical angular momentum for fusion, and ℓ_{max} is the maximum value of ℓ that contributes. It should be chosen that the integrated cross section σ agrees with the experiment. Other cross sections are derived by integrating over the corresponding variables.

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Received by Publishing Department
on July 21 1980.